

ON $(gsp)^{**}$ - CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

*In this paper we have introduced a new class of sets called $(gsp)^{**}$ -closed sets which is properly placed in between the class of $(gsp)^*$ -closed sets and (gsp) -closed sets. Properties and characterization of $(gsp)^{**}$ -closed sets are investigated.*

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1. INTRODUCTION:

Levine [4] introduced the class of g – closed sets in 1970. Dontchev[2] introduced gsp - closed sets. Pauline Mary Helan [12] introduced $(gsp)^*$ - closed sets in 2014. Sindhu Surya [15] introduced strongly $(gsp)^*$ - closed sets in 2015. The Purpose of this paper is to introduce the concept of $(gsp)^{**}$ -closed sets and $(gsp)^{**}$ -open sets in topological space and some of their properties and characterization are investigated.

2. PRELIMINARIES

In this chapter (X, τ) represent non-empty topological spaces on which no separations axioms are assumed unless otherwise mentioned. (X, τ) will be replaced by X if there is no changes of confusion. For a subset A of a space (X, τ) , $cl(A)$ and $int(A)$ denote the closure and interior of A respectively.

The smallest semi-closed (resp. pre-closed and α -closed) set containing a subset A of (X, τ) is called the semi-closure (resp. pre-closure and α -closure) of A and is denoted by $scl(A)$ (resp. $pcl(A)$ and $\alpha cl(A)$).

Definition 2.1: A subset A of a topological space (X, τ) is called

- (i) pre-open set [8] if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$.
- (ii) semi-open set [5] if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$.
- (iii) semi-pre-open set [1] if $A \subseteq cl(int(cl(A)))$ and semi-pre-closed if $int(cl(int(A))) \subseteq A$.
- (iv) α -open set [10] if $A \subseteq int(cl(int(A)))$ and α -closed if $cl(int(cl(A))) \subseteq A$.
- (v) regular open set [8] if $A = int(cl(A))$ and regular closed if $A = cl(int(A))$.

Definition 2.2: A subset A of a topological space (X, τ) is called

- (i) a generalized closed set [6] (briefly g -closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open.
- (ii) a α -generalized closed set [7] (αg -closed) if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open.
- (iii) generalized-semi-pre-regular-closed [3] (briefly $gspr$ -closed) if $spcl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular open.
- (iv) generalized semi-closed [2] (briefly gs -closed) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is open.
- (v) generalized pre-closed [9] (briefly gp -closed) if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open.
- (vi) generalized semi- pre-closed [3] (briefly gsp -closed) if $spcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open.
- (vii) generalized pre-regular closed [4] (briefly gpr -closed) if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular open.
- (viii) weakly generalized closed [11] (briefly wg -closed) if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is open.

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- (ix) regular weakly generalized closed [14] (briefly rwg-closed) if $\text{cl}(\text{int}(A)) \subseteq U$, whenever $A \subseteq U$ and U is regular open.
- (x) g^* -closed [17] if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is g^* -open.
- (xi) g^{**} -closed set [13] if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is g^* -open.
- (xii) $(gsp)^*$ -closed [15] if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is (gsp) -open.
- (xiii) (g^*p) closed set [17] if $\text{pcl} \subseteq U$, whenever $A \subseteq U$ and U is g^* -open.
- (xiv) sg^{**} -closed set [16] if $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is g^{**} -open.
- (xv) Strongly $(gsp)^*$ closed set [11] if $\text{cl}(\text{int}(A)) \subseteq U$, whenever $A \subseteq U$ and U is (gsp) -open.

The complements of the above mentioned closed sets are their respective open sets.

Remark 2.3: [17] Jankovic and Reilly pointed out that every singleton $\{x\}$ of a space X is either nowhere dense or pre-open. This provides another decomposition $X = X_1 \cup X_2$, where $X_1 = \{x \in X / \{x\} \text{ is nowhere dense}\}$ and $X_2 = \{x \in X / \{x\} \text{ is pre-open}\}$.

Definition 2.4: [17] The intersection of all gb -open sets containing A is called the gb -kernel of A and it is denoted by $gb\text{-ker}(A)$.

Lemma 2.5: [17] For any subset A of X , $X_2 \cap \text{cl}(A) \subseteq gb\text{-ker}(A)$.

Theorem 2.6: For a topological space (X, τ) ,

- (i) Every $(gsp)^*$ open set is g -open.
- (ii) Every open set is $(gsp)^*$ -open.
- (iii) Every $(gsp)^*$ -open set is (gsp) -open.
- (iv) Every semi-open set is $(gsp)^*$ -open.

3. $(gsp)^{**}$ -closed sets

In this chapter, we introduce $(gsp)^{**}$ -closed sets in topological spaces and obtain some of their properties.

Definition 3.1: A subset A of a topological space (X, τ) is said to be a $(gsp)^{**}$ -closed set if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is $(gsp)^*$ -open. The family of all $(gsp)^{**}$ -closed sets in X is denoted by $(gsp)^{**}\text{-C}(X, \tau)$.

Theorem 3.2: Every closed set is $(gsp)^{**}$ -closed.

Proof: Let A be a closed set. Let $A \subseteq U$ and U is $(gsp)^*$ -open. Since A is closed, $\text{cl}(A) = A \subseteq U$. Thus, $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gsp)^*$ -open and therefore A is $(gsp)^{**}$ -closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.3: Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then $A = \{a, c\}$ is $(gsp)^{**}$ -closed but not closed.

Theorem 3.4: Every $(gsp)^*$ -closed set is $(gsp)^{**}$ -closed.

Proof: Let A be a $(gsp)^*$ -closed set. Let $A \subseteq U$ and U is $(gsp)^*$ -open. Since every $(gsp)^*$ -open set is (gsp) -open, U is (gsp) -open. Also since A is $(gsp)^*$ -closed, $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gsp)^*$ -open. Hence A is $(gsp)^{**}$ -closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.5: Let $X = \{a, b, c\}$; $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$, Then $A = \{a, b\}$ is $(gsp)^{**}$ -closed but not $(gsp)^*$ -closed.

Theorem 3.6: Every g^* -closed set is $(gsp)^{**}$ -closed.

Proof: Let A be a g^* -closed set. Let $A \subseteq U$ and U is $(gsp)^*$ -open. Since every $(gsp)^*$ -open set is g -open and also since A is g^* -closed, $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gsp)^*$ -open. Hence A is $(gsp)^{**}$ -closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.7: Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$, Then $A = \{b\}$ is $(gsp)^{**}$ -closed but not g^* -closed.

Theorem 3.8: Every (gsp)**-closed set is (gsp)-closed.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U is open. Since every open set is (gsp)*-open and also since A is (gsp)**-closed, $\text{spcl}(A) \subseteq \text{cl}(A) \subseteq U$. This implies that $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Hence A is (gsp)-closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.9: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$, Then $A = \{a\}$ is (gsp)-closed but not (gsp)**-closed.

Theorem 3.10: Every (gsp)**-closed set is gpr-closed.

Proof: Let A be a (gsp)**-closed. Let $A \subseteq U$ and U is regular-open. Since every regular-open set is (gsp)*-open and also since A is (gsp)**-closed, $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$. This implies that $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is r-open. Hence A is gpr-closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.11: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{a, b\}, X\}$, Then $A = \{a\}$ is gpr-closed but not (gsp)**-closed.

Theorem 3.12: Every (gsp)**-closed set is wg-closed.

Proof: Let A be a (gsp)**-closed. Let $A \subseteq U$ and U is open. Since every open set is (gsp)*-open and also since A is (gsp)**-closed, $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U$. This implies that $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open. Hence A is wg-closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.13: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{a, b\}, X\}$, Then $A = \{b\}$ is wg-closed but not (gsp)**-closed.

Theorem 3.14: Every (gsp)**-closed set is rwg-closed.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U is regular open. Since every regular open set is (gsp)*-open and also since A is (gsp)**-closed, $\text{cl}(\text{int}(A)) \subseteq \text{cl}(A) \subseteq U$. This implies that $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open. Hence A is rwg-closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.15: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$, Then $A = \{a, b\}$ is rwg-closed but not (gsp)**-closed.

Theorem 3.16: Every (gsp)**-closed set is α g-closed.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U is open. Since every open set is (gsp)*-open and also since A is (gsp)**-closed, $\text{cl}(A) \subseteq U$. Since $\alpha\text{cl}(A) \subseteq \text{cl}(A) \subseteq U$. This implies that $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Therefore A is α g-closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.17: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{a, b\}, X\}$, Then $A = \{b\}$ is α g-closed but not (gsp)**-closed.

Theorem 3.18: Every (gsp)**-closed sets is gspr-closed.

Proof: Let A be a (gsp)**-closed. Let $A \subseteq U$ and U is regular open. Since every regular open set is (gsp)*-open and also since A is (gsp)**-closed, $\text{cl}(A) \subseteq U$, $\text{spcl}(A) \subseteq \text{cl}(A) \subseteq U$. This implies that $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open. Hence A is gspr-closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.19: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$; Then $A = \{a\}$ is gspr-closed but not (gsp)**-closed.

Theorem 3.20: Every (gsp)**-closed set is gs-closed.

Proof: Let A be a (gsp)**-closed set. Let $A \subseteq U$ and U is open. Since every open set is (gsp)*-open and also since A is (gsp)**-closed, $\text{scl}(A) \subseteq \text{cl}(A) \subseteq U$. This implies that $\text{scl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open. Hence A is gs-closed.

The converse of the above proposition need not be true in general, as shown in the following example.

Example 3.21: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$, Then $A = \{a\}$ is gs-closed but not (gsp)**-closed.

Theorem 3.22: Every (gsp)**-closed set is gp-closed.

Proof: Let A be a (gsp)**-closed. Let $A \subseteq U$ and U is open. Since every open set is (gsp)*-open and also since A is (gsp)**-closed, $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$. This implies that, $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open. Hence A is gp-closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.23: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{a, b\}, X\}$, Then $A = \{b\}$ is gp-closed but not (gsp)**-closed.

Theorem 3.24: Every regular-closed set is (gsp)**-closed set.

Proof: let A be a regular closed. Let $A \subseteq U$ and U is (gsp)**-open. Since every regular closed set is closed and by Theorem 3.2, A is (gsp)**-closed.

The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.25: Let $X = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b, c\}, X\}$, Then $A = \{b\}$ is (gsp)**-closed but not regular closed.

Remark 3.26: (gsp)**-closed sets is independent of sg** -closed sets.

Example 3.27: Let $X = \{a, b, c\}$; $\tau = \{\phi, X, \{a\}, \{a, c\}\}$, Then $A = \{b\}$ is sg** -closed but not (gsp)**-closed.

Example 3.28: Let $X = \{a, b, c\}$; $\tau = \{\phi, X, \{a\}, \{sb, c\}\}$, Then $A = \{b\}$ is (gsp)**-closed but not sg** -closed.

4. Basic properties (gsp)**-closed sets

In this chapter we obtain some of its basic properties in topological spaces.

Theorem 4.1: If A is a (gsp)**-closed set of (X, τ) such that $A \subseteq B \subseteq \text{cl}(A)$ then B is also a (gsp)**-closed set (X, τ) .

Proof: Let U be (gsp)*-open set in X such that $B \subseteq U$. Then $A \subseteq U$ Since A is (gsp)**-closed, $\text{cl}(A) \subseteq U$. Also, since $B \subseteq \text{cl}(A)$. Therefore $\text{cl}(B) \subseteq \text{cl}(\text{cl}(A)) = \text{cl}(A) \subseteq U$. Thus, $\text{cl}(B) \subseteq U$ whenever $B \subseteq U$ and U is (gsp)*-open. The converse of the above theorem need not be true in general, as shown in the following example.

Example 4.2: Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Let $A = \{b\}$ and $B = \{a, b\}$. Then A and B are (gsp)**-closed set in X but $A \subseteq B \not\subseteq \text{cl}(A)$.

Theorem 4.3: If a subset A of X is (gsp)**-closed set in X . Then $\text{cl}(A) \setminus A$ does not contain any non-empty (gsp)*-closed set in X .

Proof: Suppose A is (gsp)**-closed set in X . Suppose U is any non – empty (gsp)*-closed set such that $\text{cl}(A) \setminus A \supseteq U$. Now, $U \subseteq \text{cl}(A) \setminus A$. Then $U \subseteq \text{cl}(A) \cap A^c$. This implies $U \subseteq X \setminus A$. Therefore, $A \subseteq X \setminus U$. Since U is (gsp)*-closed set, $X \setminus U$ is (gsp)*-open in X . Since A is (gsp)**-closed in X , $\text{cl}(A) \subseteq X \setminus U$. This implies $U \subseteq X \setminus \text{cl}(A)$. Also, $U \subseteq \text{cl}(A)$ and therefore $U \subseteq \text{cl}(A) \cap X \setminus \text{cl}(A) = \phi$. This implies that $U = \phi$. Which is a contradiction to U is non-empty. Hence $\text{cl}(A) \setminus A$ does not contain any non-empty (gsp)*-closed set in X .

Corollary 4.4: If a subset A of a space X is (gsp)**-closed in X . Then $\text{cl}(A) \setminus A$ does not contain any non-empty closed set in X .

Proof: Let A be a (gsp)** - closed subset of X . Suppose $\text{cl}(A) - A$ contains a non – empty closed set F . By Theorem 3.2, F is (gsp)** - closed. Thus we have, $\text{cl}(A) / A$ contains a non – empty (gsp)** - closed set. This contradicts the theorem 4.4. Hence $\text{cl}(A) / A$ does not contain any (gsp)**- closed set.

Theorem 4.5: If A is (gsp)*-open and (gsp)**-closed set of X then A is a closed set of X .

Proof: Let A be (gsp)*-open and (gsp)**-closed set. Then $\text{cl}(A) \subseteq A$ and obviously $A \subseteq \text{cl}(A)$. Therefore $A = \text{cl}(A)$. Hence A is closed.

Definition 4.6: Let X be a topological space and Y be a subspace of X . Then the subset A of Y is (gsp)* - open in Y if $A = G \cap Y$, where G is (gsp)* - open in X .

Theorem 4.7: Let $A \subseteq Y \subseteq X$ and if A is a (gsp)**-closed set in X . Then A is a (gsp)**-closed relative to Y .

Proof: Given that $A \subseteq Y \subseteq X$ and A is a (gsp)**- closed set in X . To prove that A is (gsp)** - closed set relative to Y . Let us assume that $A \subseteq Y \cap U$, where U is (gsp)* - open in X . since A (gsp)**- closed set in X , then $\text{cl}(A) \subseteq U$. That implies, $Y \cap \text{cl}(A)$ is the closure of A in Y and $Y \cap U$ is (gsp)*- open in Y . Therefore $\text{cl}(A) \subseteq Y \cap U$ in Y . Hence A is (gsp)**- closed set relative to Y .

Theorem 4.8: Let A be a (gsp)**-closed in (X, τ) . Then A is closed iff $\text{cl}(A) \setminus A$ is a (gsp)*-closed.

Proof: Suppose A is closed in X . Then $A = \text{cl}(A)$. Therefore $\text{cl}(A) \setminus A = \emptyset$. Hence, A is (gsp)*-closed. Conversely, Suppose $\text{cl}(A) \setminus A$ is (gsp)*-closed set in X . Since A is (gsp)*-closed, By Theorem 4.1, $\text{cl}(A) \setminus A$ does not contain any non-empty (gsp)*-closed set in X . This implies that $\text{cl}(A) \setminus A = \emptyset$. Thus $A = \text{cl}(A)$. Hence A is closed.

Theorem 4.9: If A and B are (gsp)**-closed sets then $A \cup B$ is also a (gsp)**-closed set.

Proof: Let A and B be (gsp)**-closed and $A \cup B \subseteq U$ and U is (gsp)*-open. Since A and B are (gsp)**-closed, $\text{cl}(A) \subseteq U$ and $\text{cl}(B) \subseteq U$. Since $\text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B) \subseteq U$. Therefore $\text{cl}(A \cup B) \subseteq U$ whenever $A \cup B \subseteq U$ and U is (gsp)*-open. Therefore $A \cup B$ is a (gsp)**-closed set.

Definition 4.10: The intersection of all (gsp)*-open sets containing A is called the (gsp)*-kernel of A and it is denoted by $(\text{gsp})^*\text{-ker}(A)$.

Theorem 4.11: A subset A of X is (gsp)**-closed iff $\text{cl}(A) \subseteq (\text{gsp})^*\text{-ker}(A)$.

Proof:

Necessity: Let A be a (gsp)*-closed subset of X and $x \in \text{cl}(A)$. Suppose $x \notin (\text{gsp})^*\text{-ker}(A)$. Then there exists a (gsp)*-open set U containing A such that $x \notin U$. Since A is (gsp)**-closed set, then $\text{cl}(A) \subseteq U$. This implies that, $x \in \text{cl}(A)$, which is a contradiction to $x \notin U$. Therefore $\text{cl}(A) \subseteq (\text{gsp})^*\text{-ker}(A)$.

Sufficiency: Suppose $\text{cl}(A) \subseteq (\text{gsp})^*\text{-ker}(A)$. If U is any sb*-open set containing A , then $(\text{gsp})^*\text{-ker}(A) \subseteq U$. That implies, $\text{cl}(A) \subseteq U$. Hence A is (gsp)**-closed in X .

Remark 4.12: For any subset A of X , $\text{gb-ker}(A) \subseteq (\text{gsp})^*\text{-ker}(A)$.

Theorem 4.13: For any subset A of X , $X_2 \cap \text{cl}(A) \subseteq (\text{gsp})^*\text{-ker}(A)$.

Proof: By Lemma 2.5 and Remark 4.12, $X_2 \cap \text{cl}(A) \subseteq (\text{gsp})^*\text{-ker}(A)$.

Theorem 4.14: A subset A of X is (gsp)**-closed if and only if $X_1 \cap \text{cl}(A) \subseteq A$.

Proof: Necessity: Suppose that A is (gsp)**-closed and $x \in X_1 \cap \text{cl}(A)$. Then $x \in X_1$ and $x \in \text{cl}(A)$. Since $x \in X_1$, then $\text{int}(\text{cl}(\{x\})) = \emptyset$. Therefore $\{x\}$ is semi-closed. By Theorem 2.4, $\{x\}$ is (gsp)*-closed. If x does not belongs to A , then $U = X - \{x\}$ is a (gsp)*-open set containing A and so $\text{cl}(A) \subseteq U$. Since $x \in \text{cl}(A)$, $x \in U$. This is a contradiction to x not in U . Hence $X_1 \cap \text{cl}(A) \subseteq A$.

Sufficiency: Let $X_1 \cap \text{cl}(A) \subseteq A$. Then $X_1 \cap \text{cl}(A) \subseteq (\text{gsp})^*\text{-ker}(A)$. Now, $\text{cl}(A) = X \cap \text{cl}(A) = (X_1 \cup X_2) \cap \text{cl}(A) = (X_1 \cap \text{cl}(A)) \cup (X_2 \cap \text{cl}(A)) \subseteq (\text{gsp})^*\text{-ker}(A)$. Then by Theorem 2.6, A is (gsp)**-closed.

Theorem 4.15: Arbitrary intersection of (gsp)**-closed sets is (gsp)**-closed.

Proof: Let $\{A_i\}$ be the collection of (gsp)**-closed sets of X . Let $A = \bigcap A_i$. Since $A \subseteq A_i$, for each i , then $\text{cl}(A) \subseteq \text{cl}(A_i)$. That implies, $X_1 \cap \text{cl}(A) \subseteq X_1 \cap \text{cl}(A_i)$. Since each A_i is (gsp)**-closed, then by Theorem 4.14, $X_1 \cap \text{cl}(A_i) \subseteq A_i$, for each i . Now, $X_1 \cap \text{cl}(A) = X_1 \cap \text{cl}(\bigcap A_i) \subseteq \bigcap (X_1 \cap \text{cl}(A_i)) \subseteq \bigcap A_i = A$. Again by Theorem 4.14, A is (gsp)**-closed.

Remark 4.16: The set of all (gsp)**- closed sets form a topology on X .

5. (gsp)** open sets

Definition 5.1. A subset A of (X, τ) is said to be (gsp)**-open set if its complement $X \setminus A$ is (gsp)**-closed in X . The family of all (gsp)**-open sets in X is denoted by (gsp)**- $O(X, \tau)$.

Theorem 5.1: For a topological space (X, τ)

- (i) Every open set is (gsp)** open set.
- (ii) Every g^* - open is (gsp)** open set.
- (iii) Every (gsp)* open set is (gsp)** open set.
- (iv) Every (gsp)** open set is gspr open set.
- (v) Every (gsp)** open set is (g^*p) open set.
- (vi) Every (gsp)** open set is rwg open set.
- (vii) Every (gsp)** open set is (αg) open set.
- (viii) Every (gsp)** open set is gs open set.
- (ix) Every (gsp)** open set is (gsp) open set.
- (x) Every (gsp)** open set is gpr open set.
- (xi) Every (gsp)** open set is wg open set.
- (xii) Every (gsp)** open set is gp open set.
- (xiii) (gsp)** open set is independent of sg^{**} open sets.

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