International Journal of Mathematical Archive-9(2), 2018, 129-134
Available online through www.ijma.info ISSN 2229 – 5046

DEPARTMENTS OF FUZZY GROUP AUTOMATA IN MONADS

M SURESH BABU[1,*], E. KESHAVA REDDY[2]

[1]Research Scholar, Department of Mathematics, JNTUA, Ananthapuramu, (A.P.), India.

[2]Professor, Department of Mathematics, JNTUA, Ananthapuramu, (A.P.), India.

(Received On: 08-10-17; Revised & Accepted On: 17-01-18)

ABSTRACT

The department of fuzzy group automata in monads (in twin form) is introduced. A monad of extensional fuzzy sets in sets with parallel relations and a monad of fuzzy objects in spaces with fuzzy partitions is introduced and relationships between fuzzy automata in sets with the parallel relation or in spaces with fuzzy partitions, on one hand, and fuzzy type automata in corresponding monads, on the other, are investigated.

Keywords: *Fuzzy Automata; Fuzzy morphism; Fuzzy states; Fuzzy Partitions; Fuzzy SPACE FP; Fuzzy Power set and Fuzzy algebra.*

INTRODUCTION

A view of fuzzy automata was introduced by several Authors and, move over several further generalizations of this notion appeared which seem to cover almost all known types of fuzzy and standard automata, including nondeterministic automata. On the other hand legal attempts still appear to extend various definitions of fuzzy automata in a way which could cover at least various value lattices of fuzzy set or existing special types of automata. In that way new variants of fuzzy automata have been proposed in different applications and different generalizations. It very early, categorical theory has been used to describe efficiently constructions in automata theory. It has led to a great development of studying automata using departmental methods. Monads over a department have been used in fuzzying mathematical objects, especially automata. The twin form of a category **K** is most suitable in this direction and empresses as basic idea of a fuzzyfication. The idea of using monads for fuzzyfication is based on extension of objects $\boldsymbol{X}$ of a department **K** to another object $(X) \in |\boldsymbol{K}|$, which may be regarded as an object of a " Twin of fuzzy stages" with a morphism $\eta: X \to T(X)$, representing " twin " states in the object of " fuzzy states ". Then a "fuzzy morphism" $f: X \to Y$ is simply a morphism $f: X \to T(Y)$ in the department **K** and a composition of fuzzy morphisms is defined by a special operation ◊. The result of these constructions is a triple T = (T,, η), which is known as fuzzy theory and which, in fact, is a monad or algebraic theory in a twin form. A system of objects of a department K with fuzzy morphisms and a composition ◊ of fuzzy morphisms is then a Kleisli category in the category K. Kleisli department was firstly used in definitions of non-deterministic automata by Manes and Arbib and also extended to the case of Q – valued fuzzy automata, with Q – valued transition function X ×∑→T(X) from states and inputs to fuzzy states. The manuscripts continue in the development of automata theory in monads in twin form. The definition of T – automaton, Where T is a monad in twin form in a department K and also a department Aut K (T) of T – automata and prove that classically defined departments of non-deterministic automata and Q – valued fuzzy automata and newly defined and departments of automata defined in spaces with fuzzy partitions are isomorphic to departments Aut set (T), for appropriate monads T in the department of set.

PRELIMINARIES

Q denotes a complete residuated lattice. The category set (Q) with objects Q-sets, i.e., couples (A, δ), where A is set and δ is a Q-valued similarity relation in A and with morphism f:(A, δ)→(B, γ) defined as a map f:A→B , such that γ(f(x),f(y)) ≥ δ(x, y), for all x, y ∈A. Within morphisms in the category Set (Q), we will are extensional fuzzy sets in (A, λ). Let F (A, λ) be the set of all extensional fuzzy sets in (A, λ). Then F: Set (Q)→ Set is a covariant functor. The Definition of Space FP, which is based on Q-valued fuzzy partitions.

Corresponding Author: M Suresh Babu[1,*],
[1]Research Scholar, Department of Mathematics, JNTUA, Ananthapuramu, (A.P.), India.

Definition I: Let X be a set. A system A = $\{A_\lambda : \lambda \in \}$of normal Q – valued fuzzy sets in X is a fuzzy partition of X, if $\{core\ (A_{\lambda)} : \lambda \in \}$ is a partition of X. A pair (X, A) is called a space with fuzzy partition. The index set of A will be denoted by $|A|$ .Now introduced the category **Space FP** of spaces with fuzzy partitions.

Definition II: The category **Space FP** is defined by **a)** Fuzzy partitions (X, A), as objects, **b)** Morhisms (g, σ) : $(X, \{A_\lambda : \lambda \in \}) \to (Y, \{B: \in \Omega\}$, such that i) $g: X \to Y\ is\ a\ map$, ii) $\sigma: \to \Omega\ is\ a\ map$, iii) $\forall\ \lambda$, A_λ(x)$\le$ $B_{\sigma(\lambda)}$(g(x)), for each $x \in X$. **c)** The composition of morphisms in **Space FP** is defined by $(h, \mathcal{T})o(g, \sigma) = (h\ o\ g, \mathcal{T}\ o\ \sigma)$.

We have to prove that there exists a full and Faithfull functor : $Set(Q) \hookrightarrow \boldsymbol{SpaceFP}$. If $\leftrightarrow$ is a bi-residuum operation in Q, then $(Q, \leftrightarrow)$. It can be $|Set(Q)|$ and put $(Q, \mathcal{Q}) = I(Q, \leftrightarrow)$. It can be proved that $|\mathcal{Q}| = Q$. By $R(X, A)$. We denote the set of all fuzzy sets $f: X \to Q$, for which there exists a fuzzy set [f]: $|A| \to Q$,such that $(f, [f]): (X, A) \to (Q, \mathcal{Q})$ is a morphism in **Space FP**. To prove that for any space with a fuzzy partition (X, A) it is possible to construct two Q-sets $(X, \delta_{X,A})$ and $(|A|, \rho_{X,A})$, with similarity relations called characteristic similarity relations of (X, A), such $f \in R(X, A)$ iff f is an extensional fuzzy set with $\delta_{X,A}$ respectively.

MONADS IN SOME CATEGORIES

Now Bring to mind the definitions and basic properties of monads (in twin form) and we also all some basic facts about these structures. We recall examples of the monad (in twin form) **P** based on the classical power set functor $P: set \to set$, such that P$(X) = 2^X$, and the monad (in twin form) Z, based on the Zadeh's power set functor Z : $set \to set$, such that Z $(X) = Q^X$, where Q is a complete residuated lattice. Moreover, in the departments Set (Q) and **Space FP** to construct new monads (in twin form), based on power set functors F and R.

Definition III: If **T** = (T, $\lozenge$, η) is an monad (is twin form) (or algebraic theory) in a category K, then the following conditions are satisfied:

(1) $T: |K| \to |K|$ is an object function,
(2) η is a system of K-morphism $\eta_A: A \to T(A)$, for any object A,
(3) For each pair of K-morphisms $f: A \to T(B), g: B \to T(C)$, there exists a composition $g \lozenge$ f: A $\to$ T(C),which is associative,
(4) For every K-morphism $f: A \to T(B), \eta_B \lozenge$ f = f ,
(5) $\lozenge$ is compatible with composition o of morphisms of K, i.e., for each K-morphisms $f: A \to B, g: B \to T(C)$, we have $g \lozenge (\eta_B$o f) = g o f.
It should be noted that if (T, $\lozenge$, η) is a monad (in twin form) in a category K, then $T: |K| \to |K|$ is a functor, such that for each morphism $f: A \to B, T(f) = (\eta_B$o f) $\lozenge$ $1_{T(A)}$. Moreover, η represents identities on both sides for $\lozenge$, that is, for each $A \to T(B), \eta_B \lozenge$ f = f, f $\lozenge\ \eta_A$=f. In that case $\eta: 1_K \to T$ is a natural transformation.

After that, the following object functions of power set fuzzy objects functors $K \to Set$.

(1) K = Set, P(A) = 2^A , A$\in |Set|$,
(2) K = Set, Z(A) = Q^A , A$\in |Set|$,
(3) K = Set(Q), F(A,δ) = Hom $_{Set(Q)}$ ((A,δ), $(Q, \leftrightarrow)$), (A,δ) $\in |Set(Q)|$,
(4) K = Space FP, $R(X, A)$ Hom $_{Space\ FP}$ $((X, A) \to (Q, \mathcal{Q})$, $(X, A) \in |Space\ FP|$.
For these object functions in a category K we provide monads in K. Algebraic theories corresponding to functors P and Z were introduced in many previous papers, recall,e.g.,

Proposition I: Let P = (P, $\lozenge$, η) be a structure, such that

(1) For each $X \in |Set|, P(X) = 2^X$,
(2) For each $X \in |Set|$, define $\eta_X: X \to P(X)$ by $\eta_X(x) = \{x\}$.
(3) For each $f: X \to P(Y), g: Y \to P(Z)$, define $g \lozenge$ f: X $\to$ P(Z), by $(g \lozenge$ f)(x) = $\bigcup_{y \in f(x)}$ g(y).

Then P is a monad (in dual form) in the department Set.
In that case, P : $set \to set$ is a factor, such that for any $f: X \to Y$ in Set, $f_P^{\to} = P(f): P(X) \to P(Y)$ is defined by $f_P^{\to}\ (S) \coloneqq (\eta_Y\ of) \lozenge 1_{R(X)}$ = f(S).

Proposition II: Let Z = (Z, $\lozenge$, χ) be a structure, such that

(1) For each $X \in |Set|$, $Z : |set| \to |Set|$ is defined by $Z(X) = Q^X$,
(2) For each $X \in |Set|, (\chi X): X \to Q^Z$ is a characteristic map $X \to Q\ of\ \{x\}$ in X,
(3) For each $f: X \to Q^Z\ by$ $[(g \lozenge$ f)(x)](z) = $\bigvee_{y \in Y}$(f(x)(y) $\otimes$ (g(y))(z). Then Z is a monad (in dual form) in a category Set. In that case, $Z\ : |set| \to |Set|$ is a functor, such that for each morphism $f: X \to Y$ in Set, $f_Z^{\to} = Z(f) = (\chi Y\ o\ f) \lozenge 1_{Z(x)}: Q^X \to Q^Y$ is defined by $f_Z^{\to}(s)(y) = \bigvee_{x \in X, f(x) = y} s(x)$.

Next Introduce a monad (in dual form) F in the category Set (Q), based on the object function of a power set of extensional fuzzy sets object functor $F: Set(Q) \to Set$, defined by $F(X, \delta) = \{s \in Q^X$: s is extensional with respect to $\delta\}$, and for each morphism $f: (X, \delta) \to (Y, \gamma), F(f)(s)(y) = \bigvee_{x\in} s(x) \otimes \gamma(f(x), y)$.

Proposition III: Let $F = (\mathcal{F}, \Diamond, \eta)$ be a structure, such that

(1) $\mathcal{F}: |Set(Q)| \to |Set(Q)|$ is defined by $\mathcal{F}(X, \delta) = (F(X, \delta).\sigma(X, \delta)$, where the similarity relation σ is defined by $\sigma_{(X,\delta)}(s, t) = \bigwedge_{x\in X} s(x) \leftrightarrow t(x)$, for each $x, t \in F(X, \delta)$,

(2) For each $(X, \delta) \in |Set(Q)|, \eta_{(X,\delta)}: (X, \delta) \to \mathcal{F}(X, \delta)$ is defined by $\eta_{(X,\delta)}(a)(x) = \delta(a, x)$, for each $a, x \in X$,

(3) For each $f: (X, \delta) \to \mathcal{F}(Y, \gamma)$, for each $g; (Y, \gamma) \to \mathcal{F}(Z, \omega) in\ Set(Q), define\ g \Diamond f: (X, \delta) \to \mathcal{F}(Z, \omega)$ $by\ [(g \Diamond f)(x)](z) = \bigvee_{y\in Z}\big(f(x)\big)(y) \otimes (g(y)(z)$.

Then F is a monad (in dual form) in the department Set (Q).

Next to construct a new monad (in dual form) in the department Space FP, which is based on the object function $R(X, A)$. To begin with a definition of the structure $R = (\boldsymbol{\mathcal{R}}, \quad, \boldsymbol{v})$. Let (X, A) be a set with a fuzzy partition and let $\sigma_{X,A}$ be a fuzzy relation on $R(X, A)$ defined by $\sigma_{(X,A)}(s, t) = \bigwedge_{x\in X} s(x) \leftrightarrow t(x)$, for each $s, t \in R(X, A)$. Then $\sigma_{X,A}$ is a similarity relation on $R(X, A)$ and we can consider a space with a fuzzy partition $\big(R(X, A), C_{\sigma(X,A)}\big) = I(R(X, A), \sigma_{(X,A)}$. It is clear that $|C_{\sigma(X,A)}| = R(X, A)\ and\ C_{\sigma(X,A)} = \{C_f^{X,A}: f \in R(X, A)\}\ where\ C_f^{X,A}(g) = \sigma_{X,A}(f, g)$.

AUTOMATA IN MONADS

We introduce a notion of a T-automaton in a category K, where T is a monad (in dual form) in a department K and we construct several examples of T-automata for different monads and departments. Moreover, for a monad T we introduce the department $Aut_K[T]$ of T-automata and we prove for various monads T, that this category is isomorphic to traditionally defined departments of non-deterministic automata, Q-valued fuzzy automata, newly defined departments of automata in sets with similarity relation, or automata in spaces with fuzzy partitions. We also introduced a department Monad of monads in various departments and we show that any morphism in Monad generates a functor between corresponding departments of T-automata.

The idea to use monads (in dual form), or Kleisli departments in automata theory is not new. Kleisli department was firstly used in definitions of non-deterministic automata by Manes and Arbib and also extended to the case of Q-valued fuzzy automata, with Q-valued transition function $X \times \sum \to T(X)$ from states and inputs to fuzzy states.

Definition IV.1: Let K be a category and let T = (T, $\Diamond, \eta$) be a monad (in dual form) in K. Then a T-automaton in a department K is a system (S,(M,*)δ) , such that

1) $S \in |K|$,
2) (M ,*) is a monoid,
3) $\delta: (M,*) \to (Hom_K\big(S, T(S)\big), \Diamond)$ is a monoid homomorphism, i.e.,
   i) for each $m \in M, \delta(m): S \to T(S)$ is a morphism in K,
   ii) $\delta(1_M) = \eta_S$, iii) for each $m, n \in M, \delta(m * n) = \delta(n) \Diamond \delta(m)$.
   Remember that if T is monad (in dual form) , then T is a function, such that for each morphism $f: A \to B, T(f) = (\eta_B o\ f) \Diamond 1_{T(A).}$ To classify a department of T-automata in a department K.

Defition IV.2: Let T = (T, $\Diamond$, η) be a monad (in dual form) in a department K. Then the category $Aut_K[T]$ of T-automata in a department K is defined by

(**1**) Objects of $Aut_K[T]$ are t-automata (S, (M,*)δ) in K,
(**2**) Morphisms (f, α): (S, (M,$*$)δ) $\to$ (R, (N,$\times$), ω) are defined by
   a) morphisms $f: S \to R\ in\ K$,
   b) Morphisms $\alpha: (M,*), \to$ (N,$\times$) of monoids,
   c) For each $m, \in M$, the commutative diagrams:

$$\begin{array}{ccc} S & \xrightarrow{\delta\ (m)} & T(s) \\ f \big\downarrow & & \big\downarrow T(f) = (\eta_R o\ f) \Diamond 1_{T(S).} \\ R & \xrightarrow{\omega(\alpha(m))} & T(R). \end{array}$$

   d) Composition of morphisms is defined by compositions of corresponding parts of morphisms.

Lemma IV.1: $Aut_K[T]$ is a category.

The examples of T-automata in the following well known types of automata, including non-deterministic automata and Q-valued fuzzy automata are, in fact, T-automata for some monad (in dual form) T and some departments. Moreover, we introduce new fuzzy type automata defined in spaces with fuzzy partitions and prove that also these new types of fuzzy automata are T-automata for some monads and categories.

1) Non-deterministic automata: Let set be the category of sets with maps as morphisms. Recall firstly, that a system $(\mathrm{S}, (\mathrm{M},*) d)$ is a non-deterministic automaton over a monoid (M,*), if $S \in |Set|$ and $d: S \times M \rightarrow 2^S$ is a non-deterministic transition function, such that
i) $\forall x \in X, d(x, 1_M) = \{x\}$,
ii)$\forall m, n \in M, x \in S, d(x, m * n) = \cup_{y \in d(x,m)} d(y, n)$.

The stands for department of non-deterministic automata with morphisms
$(f, \propto)$: $(\mathrm{S}, (\mathrm{M},*) d) \rightarrow (R, (N,\times), h)$, *such that*
i) $f: s \rightarrow R$ is a map and $\propto: (M,*) \rightarrow (N,\times)$ is a monoid homorphism,
ii)*for each* $x \in S, m \in M, f\big(d(x, m)\big) = h\big(f(x), \propto(m)\big)$. Then the following proposition holds.

Proposition IV.I: Let P = (P, ◊ , η) be the monad (in dual form) from proposition. Then the departments Non-deterministic automata $Aut_{Set}|P|$ are isomorphic.

2) Q-Valued fuzzy automata: Let Q be a complete residuated lattice. Recall that $(S, (M,*), d)$ is a (Q-valued) fuzzy automaton over a monoid (M,*), If S∈ $|Set|$ and $d: S \times M \times S \rightarrow Q$ is a fuzzy transition function, such that

(**i**) For each $s, t \in S, d(s, 1_M, t) = \begin{cases} 1_Q & \textit{if and only if } s = t \\ 0_Q & \textit{otherwise} \end{cases}$

(**ii**) For each $s, t \in S, m, n \in M, d(s, m * n, t) = \vee_{x \in s} d(s, m, x) \otimes \mathrm{d(x, n, t)}$.

By Fuzzy we denote the department of Q-valued fuzzy automata with morphisms $(f, \propto): (S, M,*), d) \rightarrow (R, (N,\times), h)$, such that
(i) $\propto: (M,*) \rightarrow (N,\times)$ is a monoid homomorphism,
(ii) *F*or each $s \in S, r \in f(s) \subseteq R, m \in M, h(f(s), \propto(m), r) = \vee_{x \in S, f(x)=r} d(s, m, x)$ holds.

Proposition IV.2: Let Z = (Z, ◊, η) be the monad (in twin form) in the department Set, then the departments Fuzzy and $Aut_{Set}|Z|$ are isomorphic.

Definition IV.3: $((S, \tau), (M,*), d)$ is a (Q-valued) automaton in a set with similarity relation, if
(i) $(S, \tau) \in |Set(Q)|, (M,*)$ is a monoid,
(ii) $d: S \times M \rightarrow F(S, \tau)$, such that
  a) For each $s, x \in S, d(s, 1_M)(x) = \tau(s, x)$,
  b) For each $s, t \in S, m, n \in M, d(s, m * n)(t) = \vee_{x \in s} d(s, m)(x) \otimes \mathrm{d(x, n)(t)}$,
  c) For each $s, t, x \in S, m \in M,\ d(s, m)(x) \otimes \tau(\mathrm{s, t}) \leq \mathrm{d(t, m)(x)}$.

By Similarity we denote the category of automata in sets with similarity relations with morphisms
$(f, \propto): ((S, \tau), (M,*), d) \rightarrow \big((R, \gamma), (N,\times), h\big)$, such that
(i) $f: (S, \tau) \rightarrow (R, \gamma)$ is a morphism in the department Set(Q),
(ii) $\propto: (M,*) \rightarrow (N,\times)$ is a monoid homorphism,
(iii) For $s \in S, r \in R, m \in M, \vee_{x \in S} d(s, m)(xf) \otimes \gamma(\mathrm{f(x), r}) = \mathrm{h\big(f(s), \propto(m)\big)(r)}$ holds.

Proposition IV.3: Let $\boldsymbol{F} = (\mathcal{F}, \lozenge, \eta)$ be the monad (in twin form) in the department Set (Q), Then the department Similarity and $Aut_{Set(Q)}|\boldsymbol{F}|$ are isomorphic.

Proof: A functor $\prod: Similarity \rightarrow Aut_{Set(Q)}|\boldsymbol{F}|$ is defined by
(i) For $((S, \tau), (M,*), d) \in |SIM|, \prod\big((S, \tau), (M,*,)d\big) = \big((S, \tau), (M,*), \delta\big)$
  where $\delta: (M,*) \rightarrow (\mathcal{F}(S, \tau)^{(S,\tau)}, \lozenge)$ is defined by $\big(\delta(m)(s)(t)\big) = d(s, m)(t),\ s, t \in S, m \in M$.
(ii) For a morphism $(f, \propto): ((S, \tau), (M,*), d) \rightarrow ((R, \gamma), (N,\times), h)$ in SIM,$\prod(f, \propto) \coloneqq (f, \propto)$.
  From the property (ii) © in definition IV.3, It follows, that $\delta(m): (S, \tau) \rightarrow \mathcal{F}(S, \tau) = (S, \tau), \sigma_{(S,\tau)})$ is a morphism in Set(Q), and $\delta(1_M)(s)(x) = d(s, 1_M)(x) = \tau(s, x) = \eta_{(s,\tau)}(s)(x)$. Since $\delta(m * n)(s)(t) = d(x, m * n)(t) = \vee_{x \in S} d(s, m)(x) \otimes \mathrm{d(x, n)(t)} = (\delta(\mathrm{n})\, \lozenge \delta(m)(s)(t), \delta(m)$ is a monoid homomorphism.
  For $\prod\big((R, \gamma), (N,\times), h\big) = \big((R, \gamma), (N,\times), \omega\big)$, we have
  $F(f)(\delta(m)(s)(r) = (\eta_{(R,\gamma)} of)\ \lozenge 1_{F(S,\tau)}(\delta(m)(s)(r) = \vee_{x \in S} \delta(m)(s)(x) \otimes \gamma(\mathrm{f(x), r} = \vee_{x \in S} h\big(f(s), \propto(m)\big)(r)$, and it follow that $\prod(f, \propto)$ is a morphism in $Aut_{Set(Q)}|\boldsymbol{F}|$. Hence, $\prod$ *is* a functor and it could be verified easily that $\prod$ *is* an isomorphism.

4. Automata in spaces with fuzzy partitions: Spaces with fuzzy partitions represent new ground category for some mathematical constructions and applications. These structures generalize classical sets and also sets with similarity relations and it is natural to define automata theory in these structures.

Definition IV.4: A system $((S,A),(M,*),d)$ is called a (Q – valued) automaton in a space with a fuzzy partition, if
1) $(S,A) \in |\mathbf{Space\ FP}|$ is a space of automata states, $A = \{A_\lambda : \lambda \in \wedge\}$,
2) $(M,*)$ is a monoid of inputs,
3) $d: S \times M \to R(S,A)$ is a map, such that

$i)$ $d(s,1_M) = \delta_{S,A}$, where $\delta_{S,A}$ is the characteristic similarity relation of (S,A),

$ii)$ For each $s,t \in S, m,n \in M, d(s,m*n)(t) = \bigvee_{x\in S} d(s,m)(x) \otimes d(x,n)(t)$,

$iii)$ For each $s,x \in S, s' \in core(A_\lambda), m \in M, d(s,m)(x) \otimes A_\lambda(s) \leq d(s',m)(x)$.

From the condition (iii) it follows

iii) For each $\lambda \in \wedge, s, s' \in core(A_\lambda), d(s,m) = d(s'm)$.

By **Automaton Space FP** we denote the department with Q – valued automata in spaces with fuzzy partitions as objects and with morphisms.

$(f,u,\propto): ((S,A),(M,*),d) \to ((R,\mathcal{B}),(N,\times),h)$ defined by

$i)$ $(f,u):(S,A) \to (R,\mathcal{B})$ a morphism in the department **Space FP,**

ii) $\propto: (M,*) \to (N,\times)$ a monoid homomorphism,

iii) For each $s \in S, r \in R, m \in M, h(f(s), \propto(m))(r) = \bigvee_{x\in S} d(s,m)(x) \otimes \delta_{\mathcal{R},\mathcal{B}}(f(x), r)$.

Then the following proposition holds.

Proposition IV.4: Let $R = (\mathcal{R}, \quad, v)$ be the monad (in twin form) in the category SpaceFP, Then the categories **Automaton Space FP** and $Aut_{\boldsymbol{SpaceFP}}$ [R] are isomorphic. In this section we introduce an extended version of a monad morphism and introduce a department of monads Monad. It is prove that for any morphism of monads **R** → **S** in departments C, D respectively, there exists a functor $\boldsymbol{Aut_C[R] \to Aut_D[R]}$**.** This construction allows us to show that there exist functions among various departments of fuzzy automata. A notion of a monad morphism was introduced in various papers. We need a modified version of that morphism, which will allow to define in a tangible way a morphism between corresponding Kleisli departments. Once Recall a definition of Klesili Department $\mathbf{C_T}$**,** defined by a monad (in dual form) $T = (T, \Diamond, \eta)$be a monad (in twin form) in C. Then the Kleisli department $\mathbf{C_T}$ **of T** is defined by

1) $|\mathbf{C_T}| = |C|$,

2) For any objects $a,b \in |C|, Hom_{\mathbf{C_T}}(a,b) = Hom_K(a,Tb)$**.** Morphisms in $\mathbf{C_T}$ are denoted by $a \rightsquigarrow b$**.**

3) A composition of morphisms $f: a \rightsquigarrow b, g: b \rightsquigarrow c\ is\ defined\ by\ g\Diamond$f.

Let **T** = (T, ◊, η) and $\mathbf{R} = (\mathcal{R}, \quad, \mu)$ be monad (in twin form) in departments **C** and **D,** respectively and let natural transformation. Then we can construct maps

$\forall a,b \in |K|, \widetilde{\emptyset}_{a,b} : Hom_{\mathbf{C_T}}(a,b) \to Hom_{DR}(Ha,Hb), defined\ by\ \widetilde{\emptyset}_{a,b}(f) = \widetilde{\emptyset}_b o\ H(f)$. Then $\widetilde{\emptyset}$, defined by

$\forall a,b \in |K|, f: a \rightsquigarrow b, \widetilde{\emptyset}(a) = Ha, \widetilde{\emptyset}(f) = \widetilde{\emptyset}_{a,b}(f)$, is a factor $\mathbf{C_T} \to \mathbf{D_R}$ of Kleisli departments, if and only if for each morphisms $f: a \rightsquigarrow b, g: b \rightsquigarrow c$ in $\mathbf{C_T}$, the following equalities hold:

$\widetilde{\emptyset}_c o\ H(g \Diamond f) = (\widetilde{\emptyset}_c o\ H(g)) \square (\widetilde{\emptyset}_c o\ H(f))$ and $\widetilde{\emptyset}_c o(H.\eta) = \mu.H$.

Definition IV.6: The department **Monad** is defined by

1) Objects are monads (in twin form) in departments,

2) For monads T = (T, ◊, η) and $\mathbf{R} = (\mathcal{R}, \quad, \mu)$ (in twin form) in departments **C** and **D,** respectively, $\emptyset$: **T** → **R** is a morphism in **Monad,** if

a) There exists a function H: **C** → **D** ,

b) $\emptyset : H\ o\ T \to R\ o\ H$ is a natural transformation,

c) $\widetilde{\emptyset}$: $\mathbf{C_T} \to \mathbf{D_R}$ is a functor between corresponding Kleisli departments.

3) Let $\emptyset$: **T** → **R** and Ψ: **R** → **S** be morphisms in **Monad** (in departments **C,D** AND **E** respectively), with respect to functors H : **C** → **D** and H : **D** → **E**, respectively. Then the composition Ψ ● $\emptyset$:= H o G$\emptyset$.

In the following proposition we show a relationship between morphisms in the department Monad and functors of departments of automata in monads. By Aut we denote the department with objects $Aut_C[T]$, where C are deprtments and T are monads (in twin form) in C and with functors between departments as morphisms.

Proposition IV.5: A functor ∧: Monad→ Aut.

Proposition IV.6: In the department Monad, there exist the following morphisms:

1) **P** → **Z** with respect to the identity functor 1_{set}.

2) **Z** → **F** with respect to the identity functor **Set** ↪ **Set(Q),**

3) **F** → **R**, with respect to the functor I: **Set(Q)** → **SpaceFP.**

Corollary IV. 1:The following diagram of functors between departments of fuzzy type automata commutes.

$$\begin{array}{ccc} Aut_{set}[P] & \overset{\cong}{\to} & ND \\ \downarrow & & \downarrow \\ Aut_{set}[Z] & \overset{\cong}{\to} & FUZ \\ \downarrow & & \downarrow \\ Aut_{set(Q)}[F] & \overset{\cong}{\to} & SIM \\ \downarrow & & \downarrow \\ Aut_{SpaceFP}[R] & \overset{\cong}{\to} & AUTSP \end{array}$$

CONCLUSION

The purpose of this section is to show how the abstract pattern recognition problems developed can be used to examine the faithfulness of a device F which decodes message transmitted across a noisy channel. We first give a quick review of the results for the sake of completeness. We have indicated that using topological and fuzzy topological ideas in the study of intuitionist fuzzy machines/automata, introduced may be beneficial. Although we have confined here ourselves by only showing, e.g., that intuitionistic fuzzy sub-systems and intuitionistic strong fuzzy subsystems have meaningful topological interpretations. We developed fuzzy automata monads have some results on, what may be called, separated, connected, strongly connected and retrievable intuitionistic fuzzy automata monads.

Work Cited:

1. S. Shelke and S. Apte " A fuzzy-based classification scheme for unconstrained handwritten Devanagari character recognition ", 2015 International Conference on communication, Information & Computing Technology (ICC ICC), 2015.
2. E. Lughofer, E. Weigl, W. Heidl, C. Eitzinger, and T. Radauer, "Integrating new classes on the fly in evolving fuzzy classifier designs and their application in visual inspection." Applications of soft Computing. Vol. 35, pp:558-582,2015.
3. F. Abu Zaid, E. Gradel, M. Grohe, and W. Pakusa. Choiceless Polynomial Time on Structures with small Abelian color classes. In Mathematical Foundations of Computer Science 2014, pages 50-62. Springer, 2014.
4. A. Dawar, The Nature and power of Fixed –point Logic with Counting. ACM SIGLOG News, pages 8-21, 2015.
5. Y.L. He, X-Z. Wang, and J. Z. Huang, "Fuzzy nonlinear regression analysis using a random weight network," Information Sciences, Vol. 364, pp. 222-240, 2016.
6. R. Fierimonte, M. Barbato, A. Rosato, and M. Panella, "Distributed learning of random weights fuzzy neural networks," Proc. Of IEEE Int. Conf. on Fuzzy Systems (FUZZ-IEEE 2016), 2016.
7. P. Tufekei, "Prediction of a full load electrical power output of a baseload operated combined cycle power plant using machine learning methods," Int. J. of Electrical Power & Energy Systems, Vol.60, pp,126-140, 2014.
8. W. Pakusa, Linear Equation Systems and the Search for a Logical Characterisation of Polynomial Time. Ph.D. thesis, RWTH Aachen University 2016.
9. F. Meng and X. Chen, "A new method for a triangular fuzzy compare wise judgment matrix process based on consistency analysis," International Journal of Fuzzy Systems, pp,1-20, 2016.
10. S. Kubler, J. Robert, W. Derigent, A. Voisin, and Y. Le Traon, "A state-of-the-art survey & testbed of fuzzy ahp (fashp) applications," Expert systems with Applications, Vol.65, pp.398-422, 2016.

Source of support: Nil, Conflict of interest: None Declared.