

Γ -SEMI SUB NEAR-FIELD SPACES OF A Γ -NEAR-FIELD SPACE OVER NEAR-FIELD PART I

Smt. THURUMELLA MADHAVI LATHA¹

Research Scholar, Junior Lecturer, Department of Mathematics,
APSWREIS Tadepalli, Guntur District, Amaravathi, Andhra Pradesh, INDIA.

Dr T V PRADEEP KUMAR²

Assistant Professor of Mathematics,
A N U College of Engineering & Technology
Department of Mathematics, Acharya Nagarjuna University
Nambur, Nagarjuna Nagar 522 510. Guntur District. Andhra Pradesh, INDIA.

Dr N V NAGENDRAM^{*3}

Professor of Mathematics,
Kakinada Institute of Technology & Science (K.I.T.S.)
Department of Humanities & Science (Mathematics)
Tirupathi (Vill.) Peddapuram (M), Divili 533 433
East Godavari District. Andhra Pradesh, INDIA.

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ABSTRACT

In this paper we introduce the semi sub near-field spaces in Γ -near- field space over a near-field (PART I), and we three Smt. Thurumella Madhavi Latha, Dr. T V Pradeep Kumar and Dr. N V Nagendram together investigate the related properties of generalization of a semi sub near-field spaces in Γ -near-field space over a near-field.

Keywords: Γ -near-field space; Γ -Semi sub near-field space of Γ -near-field space; Semi near-field space of Γ -near-field space.

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SECTION 1: INTRODUCTION

In this paper, Part I consisting two important sections we introduce the semi sub near-field spaces in Γ -near- field space over a near-field, and we three Smt. Thurumella Madhavi Latha, Dr. T V Pradeep Kumar and Dr. N V Nagendram together investigate the related properties of generalization of a semi sub near-field spaces in Γ -near- field space over a near-field.

As a generalization of a semi sub near-field spaces in Γ -near- field space over a near-field, introduced the notion of semi sub near-field spaces in Γ -near- field space over a near-field, extended many classical notions of semi sub near-field spaces in Γ -near- field space over a near-field. In this chapter we develop the algebraic theory of semi sub near-field spaces in Γ -near- field space over a near-field.

The notion of a semi sub near-field spaces in Γ -near- field space over a near-field is introduced and some examples are given. Further the terms; commutative semi sub near-field spaces in Γ -near- field space, quasi commutative semi sub near-field spaces in Γ -near- field space, normal semi sub near-field spaces in Γ -near- field space, left pseudo commutative semi sub near-field spaces in Γ -near- field space, right pseudo commutative semi sub near-field spaces in Γ -near-field space are introduced. It is proved that (1) if S is a commutative semi sub near-field spaces in Γ -near- field space then S is a quasi commutative semi sub near-field spaces in Γ -near- field space, (2) if S is a quasi commutative semi sub near-field spaces in Γ -near- field space then S is a normal semi sub near-field spaces in Γ -near- field space, (3) if S is a commutative semi sub near-field spaces in Γ -near- field space, then S is both a left pseudo commutative and a right pseudo commutative semi sub near-field spaces in Γ -near- field space over a near-field. Further the terms; left identity, right identity, identity, left zero, right zero, zero of a semi sub near-field spaces in Γ -near- field space over a near-field are introduced. It is proved that if a is a left identity and b is a right identity of a semi sub near- field spaces

in Γ -near- field space, then $a = b$. It is also proved that any Γ -semi sub near- field spaces in Γ -near-field space has at most one identity. It is proved that if a is a left zero and b is a right zero of a semi sub near-field spaces in Γ -near- field space, then $a = b$ and also it is proved that any Γ -semi sub near-field spaces in Γ -near-field space over a near-field has at most one zero element.

SECTION 2: PRELIMINARIES

In section 2, the terms; Γ -semi sub near-field space, Γ -semi sub near- field space generated by a sub near-field space, cyclic Γ -semi sub near- field space of a Γ -semi near-field space and cyclic Γ -semi near-field space are introduced. It is proved that (1) the non-empty intersection of any two Γ -semi sub near-field spaces of a Γ -semi near-field space T is a Γ -semi near-field space of T , (2) the nonempty intersection of any family of Γ -semi sub near-field spaces of a Γ -semi near-field space T is a Γ -semi near-field space of T . It is also proved that if B is a non-empty subset of a Γ -semi near-field space T , then the Γ -semi sub near-field space of T generated by B is the Inter-section of all Γ -semi sub near-field spaces of T containing B .

2.1. Γ -Semi Sub Near-Field Space Of Γ - Near-Field Space Over A Near-Field.

In this section, the notion of a Γ -semi sub near-field space is introduced and some examples are given. Further the terms commutative Γ -semi sub near-field space, quasi commutative Γ -semi sub near-field space, normal Γ -semi sub near-field space, left pseudo commutative Γ -semi sub near-field space, right pseudo commutative Γ -semi sub near-field space are introduced. It is proved that (1) if S is a commutative Γ -semi sub near-field space then S is a quasi commutative Γ -semi sub near-field space, (2) if S is a quasi commutative Γ -semi sub near-field space then S is a normal Γ -semi sub near-field space, (3) if S is a commutative Γ -semi sub near-field space, then S is both a left pseudo commutative and a right pseudo commutative Γ -semi sub near-field space. Further the terms; left identity, right identity, identity, left zero, right zero, zero of a Γ -semi sub near-field space are introduced.

It is proved that if a is a left identity and b is a right identity of a Γ -semi sub near-field space S , then $a = b$. It is also proved that any Γ -semi sub near-field space S has at most one identity. It is proved that if a is a left zero and b is a right zero of a Γ -semi sub near-field space S , then $a = b$ and it is also proved that any Γ -semi sub near-field space S has at most one zero element.

We now introduce the notion of a Γ -semi sub near-field space of Γ -near-field space over a near-field.

Definition 2.1: Γ -semi sub near-field space. Let S and Γ be two non-empty sets. Then S is called a Γ -semi sub near-field space if there exist a mapping from $S \times \Gamma \times S \rightarrow S$ which maps $(a, \alpha, b) \rightarrow a\alpha b$ satisfying the condition: $(a\gamma b)\mu c = a\gamma(b\mu c)$ for all $a, b, c \in S$ and $\gamma, \mu \in \Gamma$.

Note 2.2: Let S be a Γ -semi sub near-field space. If M and L are two sub near field spaces of S , we shall denote the set $\{a\gamma b : a \in M, b \in L \text{ and } \gamma \in \Gamma\}$ by $M\Gamma L$.

In the following some examples of Γ -semi sub near-field spaces are given.

Example 2.3: Let S be the set of all non-positive integers and Γ be the set of all non-positive even integers. If $a\alpha b$ denote as usual multiplication of integers for $a, b \in S$ and $\alpha \in \Gamma$, then S is a Γ -semi sub near-field space.

Example 2.4: Let Q be the set of rational numbers and $\Gamma = K$ be the set of natural numbers. Define a mapping from $Q \times \Gamma \times Q$ to Q by $a\alpha b = \text{usual product of } a, \alpha, b$; for $a, b \in Q, \alpha \in \Gamma$. Then Q is a Γ -semi sub near-field space.

Example: 2.5: Let $S = \{5n + 4 : n \text{ is a positive integer}\}$ and $\Gamma = \{5n + 1 : n \text{ is a positive integer}\}$. Then S is a Γ -semi sub near-field space with the operation defined by $a\alpha b = a + \alpha + b$ where $a, b \in S, \alpha \in \Gamma$ and $+$ is the usual addition of integers.

Example 2.6: Let S be the set of all integers of the form $4n+1$ where n is an integer and Γ denote the set of all integers of the form $4n+3$. If $a\gamma b$ is $a + \gamma + b$, for all $a, b \in S$ and $\gamma \in \Gamma$, then S is a Γ -semi sub near-field space.

Example 2.7: Let $S = \{\{\phi\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$ and $\Gamma = \{\{\phi\}, \{a\}, \{a, b, c\}\}$. If for all $A, C \in S$ and $B \in \Gamma$, $ABC = A \cap B \cap C$, then S is a Γ -semi sub near-field space.

Example 2.8: Let $S = \{\phi\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$ and $\Gamma = \{\{a, b, c\}\}$. If for all $A, C \in S$ and $B \in \Gamma$, $ABC = A \cap B \cap C$, then S is a Γ -semi sub near-field space.

Example 2.9: Let S be the set of all 2×3 matrices over Q , the set of rational numbers and Γ be the set of all 3×2 matrices over Q . Define $A\alpha B$ = usual matrix product of A, α, B ; for all $A, B \in S$ and for all $\alpha \in \Gamma$. Then S is a Γ -semi sub near-field space. Note that S is not a semi sub near-field space.

Example 2.10: Let $S = \{-i, 0, i\}$ and $\Gamma = S$. Then S is a Γ -semigroup under the multiplication of complex numbers, while S is not a semigroup under multiplication of complex numbers.

Example 2.11: Let S be a Γ -semi sub near-field space and α a fixed element in Γ . We define $ab = a\alpha b$ for all $a, b \in S$. We can show that $(S, .)$ is a Γ - semi sub near-field space and we denote this Γ -semi sub near-field space by $S\alpha$.

Example 2.12: Let S be a semi sub near-field space and Γ be a mapping and also a non-empty sub near-field space. Define a mapping from $S \times \Gamma \times S \rightarrow S$ as $a\alpha b = ab$, for all $a, b \in S$ and $\alpha \in \Gamma$. Then S is a Γ -semi sub near-field space.

Verification: Let $a, b, c \in S$ and $\alpha, \beta \in \Gamma$. Then $(a\alpha b)\beta c = (ab)\beta c = (ab)c = a(bc) = a\alpha(bc) = a\alpha(b\beta c)$. Therefore S is a Γ -semi sub near-field space.

Note 2.13: Every semi sub near-field space can be considered to be a Γ -semi sub near-field space. Thus the class of all Γ -semi sub near-field spaces includes the class of all Γ -semi sub near-field spaces.

Example 2.14: Free Γ -Semi sub near-field space. Let X and Γ be two non-empty Γ -semi sub near-field spaces. A sequence of elements $\alpha_1\alpha_2\alpha_3 \dots \alpha_{n-1}\alpha_n$ where $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n \in X$ and $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n \in \Gamma$ is called a word over the alphabet X relative to Γ . The set S of all words with the operation defined from $S \times \Gamma \times S$ to S as $(a_1\alpha_1a_2\alpha_2 \dots a_{n-1}\alpha_{n-1}a_n) \gamma (b_1\beta_1b_2\beta_2 \dots b_{m-1}\beta_{m-1}b_m) = a_1\alpha_1a_2\alpha_2 \dots a_{n-1}\alpha_{n-1}a_n \gamma b_1\beta_1b_2\beta_2 \dots b_{m-1}\beta_{m-1}b_m$ is a Γ -semi sub near-field space. This Γ -semi sub near-field space is called free Γ -semi sub near-field space over the alphabet X relative to Γ .

In the following we introduce the notion of a commutative Γ -semi sub near-field space.

Definition 2.15: Commutative Γ -semi sub near-field space. A Γ -semi sub near-field space S is said to be commutative provided $a\gamma b = b\gamma a$ for all $a, b \in S$ and $\gamma \in \Gamma$.

Note 2.16: If S is a commutative Γ -semi sub near-field space then $a\Gamma b = b\Gamma a$ for all $a, b \in S$.

Note 2.17: Let S be a Γ -semi sub near-field space and $a, b \in S$ and $\alpha \in \Gamma$. Then $a\alpha a\alpha b$ is denoted by $(a\alpha)^2b$ and consequently $a\alpha a\alpha a\alpha \dots (n \text{ terms})b$ is denoted by $(a\alpha)^nb$.

In the following we introduce a quasi commutative Γ -semi sub near-field space.

Definition 2.18: Quasi commutative sub near-field space. A Γ -semi sub near-field space S is said to be quasi commutative provided for each $a, b \in S$, there exists a natural number n such that $a\gamma b \gamma (b\gamma)^n a \quad \forall \gamma \in \Gamma$.

Note 2.19: If a Γ -semi sub near-field space S is quasi commutative then for each $a, b \in S$, there exists a natural number n such that, $a\Gamma b = (b\Gamma)^na$.

Theorem 2.20: If S is a commutative Γ -semi sub near-field space then S is a quasi commutative Γ -semi sub near-field space.

Proof: Suppose that S is a commutative Γ -semi sub near-field space. Let $a, b \in S$. Now $a\alpha b = b\alpha a \Rightarrow a\alpha b = (b\alpha)^1a$. Therefore S is a quasi commutative Γ -semi sub near-field space.

In the following we introduce the notion of a normal Γ -semi sub near-field space.

Definition 2.21: Normal Γ -semi sub near-field space. A Γ -semi sub near-field space S is said to be normal Γ -semi sub near-field space provided $a\alpha S = S\alpha a \quad \forall \alpha \in \Gamma$ and $\forall a \in S$.

Note 2.22: If a Γ -semi sub near-field space S is normal Γ -semi sub near-field space then $a\Gamma S = S\Gamma a$ for all $a \in S$.

Theorem 2.23: If S is a quasi commutative Γ -semi sub near-field space then S is a normal Γ -semi sub near-field space.

Proof: Let S be a commutative Γ -semi sub near-field space. By theorem 2.20, S is a quasi commutative Γ -semi sub near-field space. By theorem 2.23, S is a normal Γ -semi sub near-field space. Therefore every commutative Γ -semi sub near-field space is a normal Γ -semi sub near-field space.

In the following we are introducing left pseudo commutative Γ -semi sub near-field space and right pseudo commutative Γ -semi sub near-field space.

Definition 2.23: Left pseudo Commutative Γ -semi sub near-field space. A Γ -semi sub near-field space S is said to be left pseudo commutative provided $a\Gamma b\Gamma c = b\Gamma a\Gamma c$ for all $a, b, c \in S$.

Definition 2.24: Pseudo Commutative Γ -semi sub near-field space. A Γ -semi sub near-field space S is said to be right pseudo commutative provided $a\Gamma b\Gamma c = a\Gamma c\Gamma b$ for all $a, b, c \in S$.

Theorem 2.25: If S is a commutative Γ -semigroup, then S is both a left pseudo commutative Γ -semi sub near-field space and a right pseudo commutative Γ -semi sub near-field space.

Proof: Suppose that S is commutative Γ -semi sub near-field space. Then $a\Gamma b\Gamma c = (a\Gamma b)\Gamma c = (b\Gamma a)\Gamma c = b\Gamma a\Gamma c$. Therefore S is a left pseudo commutative Γ -semi sub near-field space. Again $a\Gamma b\Gamma c = a\Gamma (b\Gamma c) = a\Gamma (c\Gamma b) = a\Gamma c\Gamma b$. Therefore S is a right pseudo commutative Γ -semi sub near-field space. This completes the proof of the theorem.

Note 2.26: The converse of the above theorem is not true. i.e., if S is a left and right pseudo commutative Γ -semi sub near-field space then S need not be a commutative Γ -semi sub near-field space.

Example 2.27: Let $S = \{a, b, c\}$ and $\Gamma = \{x, y, z\}$. Define a binary operation ‘.’ In S as shown in the following table:

	<i>. a b c</i>
<i>a a a a</i>	
<i>b a a a</i>	
<i>c a b c</i>	

Define a mapping $S \times \Gamma \times S \rightarrow S$ by $a\alpha b = ab$ for all $a, b \in S$ and $\alpha \in \Gamma$. It is easy to see that S is a Γ -semi sub near-field space. Now S is a left and right pseudo commutative Γ -semi sub near-field space. But S is not a commutative Γ -semi sub near-field space.

In the following we are introducing left identity, right identity and identity of a Γ -semi sub near-field space of Γ -semi sub near-field space.

Definition 2.28: Left identity. An element ‘ α ’ of a Γ -semi sub near-field space S is said to be a left identity of S provided $a\alpha s = s$ for all $s \in S$ and $\alpha \in \Gamma$.

Definition 2.29: Right identity. An element ‘ α ’ of a Γ -semi sub near-field space S is said to be a right identity of S provided $s\alpha a = s$ for all $s \in S$ and $\alpha \in \Gamma$.

Definition 2.30: Identity. An element ‘ α ’ of a Γ -semi sub near-field space S is said to be a two sided identity or an identity provided it is both a left identity and a right identity of S .

Theorem 2.31: If a is a left identity and b is a right identity of a Γ -semi sub near-field space S , then $a = b$.

Proof: Since a is a left identity of S , $a\alpha s = s$ for all $s \in S$ and $\alpha \in \Gamma$ and hence $a\alpha b = b$ for all $\alpha \in \Gamma$. Since b is a right identity of S , $s\alpha b = s$ for all $s \in S$ and $\alpha \in \Gamma$ and hence $a\alpha b = a$ for all $\alpha \in \Gamma$. Now $a = a\alpha b = b$.

Theorem 2.32: Any Γ -semi sub near-field space S has at most one identity.

Proof: Let a, b be two identity elements of the Γ -semi sub near-field space S . Now a can be considered as a left identity and b can be considered as a right identity of S . By theorem 2.31, $a = b$. Then S has at most one identity. This completes the proof of the theorem.

Note 2.33: The identity (if exists) of a Γ -semi sub near-field space is usually denoted by 1.

Definition 2.34: Γ -monoid sub near-field space. A Γ -semi sub near-field space S with identity is called a Γ -monoid sub near-field space.

In the following we are introducing left zero, right zero and zero of a Γ -semi sub near-field space.

Definition 2.35: left zero sub near-field space. An element a of a Γ -semi sub near-field space S is said to be a left zero sub near-field space of S provided $a\alpha s = a$ for all $s \in S$ and $\alpha \in \Gamma$.

Definition 2.36: right zero sub near-field space. An element a of a Γ -semi sub near-field space S is said to be a right zero sub near-field space of S provided $s\alpha a = a$ for all $s \in S$ and $\alpha \in \Gamma$.

Definition 2.37: Two sided zero sub near-field space or zero sub near-field space. An element a of a Γ -semi sub near-field space S is said to be a two sided zero sub near-field space or zero sub near-field space provided it is both a left zero and a right zero of S .

We are now introducing left zero Γ -semi sub near-field space, right zero Γ -semi sub near-field space and zero Γ -semi sub near-field space.

Definition 2.38: left zero Γ -semi sub near-field space. A Γ -semi sub near-field space in which every element is a left zero is called a left zero Γ -semi sub near-field space.

Definition 2.39: right zero Γ -semi sub near-field space. A Γ -semi sub near-field space in which every element is a right zero is called a right zero Γ -semi sub near-field space.

Definition 2.40: zero Γ -semi sub near-field space or a null Γ -semi sub near-field space. A Γ -semi sub near-field space with 0 in which the product of any two elements equals to 0 is called a zero Γ -semi sub near-field space or a null Γ -semi sub near-field space.

Theorem 2.41: If a is a left zero and b is a right zero of a Γ -semi sub near-field space S , then $a = b$.

Proof: Since a is a left zero of S , $\alpha s = a$ for all $s \in S$, $\alpha \in \Gamma$ and so $\alpha b = a$ for all $\alpha \in \Gamma$. Since b is a right zero of S , $s\alpha b = b$ for all $s \in S$ and $\alpha \in \Gamma$ and hence $\alpha b = b$ for all $\alpha \in \Gamma$. Now $a = \alpha b = b$.

Theorem 2.42: Any Γ -semi sub near-field space S has at most one zero element (i.e. zero Γ -semi sub near-field space or a null Γ -semi sub near-field space).

Proof: Let a, b be two zeros of the Γ -semi sub near-field space S . Now a can be considered as a left zero and b can be considered as a right zero. By theorem 2.41, $a = b$. Thus S has at most one zero.

Note 2.43: The zero (if exists) of a Γ -semi sub near-field space is usually denoted by 0 .

Notation 2.44: Let S be a Γ -semi sub near-field space. If S has an identity, let $S^1 = S$ and if S does not have an identity, let S^1 be the Γ -semi sub near-field space S with an identity adjoined usually denoted by the symbol 1 . Similarly if S has a zero, let $S_0 = S$ and if S does not have a zero, let S_0 be the Γ -semi sub near-field space S with zero adjoined usually denoted by the symbol 0 .

SECTION 3: MAIN RESULTS

In this section, Following terms left Γ -sub near-field space, right Γ -sub near-field space, Γ -sub near-field space, proper Γ -sub near-field space, trivial Γ -sub near-field space, maximal left Γ -sub near-field space, maximal right Γ -sub near-field space, maximal Γ -sub near-field space, Γ -sub near-field space generated by a sub near field space B , principal left Γ -sub near-field space, principal right Γ -sub near-field space, principal Γ -sub near-field space of a Γ -sub near-field space over a near-field are introduced. Also left duo Γ -semi sub near-field space, right duo Γ -semi sub near-field space, duo Γ -semi sub near-field space, left simple Γ -semi sub near-field space, right simple Γ -semi sub near-field space and simple Γ -semi sub near-field space are introduced. It is proved that (1) the nonempty intersection of two left Γ -sub near-field spaces of a Γ -semi sub near-field space T is a left Γ -sub near-field space of T , (2) the nonempty intersection of any family of left Γ -sub near-field spaces of a Γ -semi sub near-field space T is a left Γ -sub near-field space of T , (3) the union of two left Γ -sub near-field spaces of a Γ -semi sub near-field space T is a left Γ -sub near-field space of T and (4) the union of any family of left Γ -sub near-field spaces of a Γ -semi sub near-field space T is a left Γ -sub near-field space of T . It is also proved that (1) the nonempty intersection of two right Γ -sub near-field spaces of a Γ -semi sub near-field space T is a right Γ -sub near-field space of T , (2) the nonempty intersection of any family of right Γ -sub near-field spaces of a Γ -semi sub near-field space T is a right Γ -sub near-field space of T , (3) the union of two right Γ -sub near-field spaces of a Γ -semi sub near-field space T is a right Γ -sub near-field space of T and (4) the union of any family of right Γ -sub near-field spaces of a Γ -semi sub near-field space T is a right Γ -sub near-field space of T . Further it is proved that (1) the nonempty intersection of two Γ -sub near-field spaces of a Γ -semi sub near-field space T is a Γ -sub near-field space of T , (2) the nonempty intersection of any family of Γ -sub near-field spaces of a Γ -semi sub near-field space T is a Γ -sub near-field space of T , (3) the union of two Γ -sub near-field spaces of a Γ -semi sub near-field space T is a Γ -sub near-field space of T and (4) the union of any family of Γ -sub near-field spaces of a Γ -semi sub near-field space T is a Γ -sub near-field space of T .

It is proved that if T is a Γ -semi sub near-field space and $a \in T$ then (i) $L(a) = a \cup T \Gamma a$, (ii) $N(a) = a \cup a \Gamma S$, (iii) $J(a) = a \cup a \Gamma T \cup T \Gamma a \cup T \Gamma a \Gamma T$. It is proved that a Γ - semi sub near-field space T is a duo Γ - semi sub near-field space if and only if $x \Gamma S^1 = S^1 \Gamma x$ for all $x \in T$. Further it is also proved that every normal Γ - semi sub near-field space is a duo Γ -semi sub near-field space. It is proved that (1) a Γ –semi sub near-field space T is a left simple Γ –semi sub near-field space if and only if $T \Gamma a = T$ for all $a \in T$, (2) a Γ –semi sub near-field space T is a right simple Γ –semi sub near-field space if and only if $a \Gamma T = T$ for all $a \in T$, (3) a Γ –semi sub near-field space T is a simple Γ –semi sub near-field space if and only if $T \Gamma a \Gamma T = T$ for all $a \in T$.

3.1. Γ -semi sub normal near-field space of Γ -semi sub near-field space over near-field

The term Γ -semi sub normal near-field space plays a special role in the theory of Γ -semi sub near-field spaces. In this section, the terms; Γ -semi sub normal near-field space, Γ - semi sub normal near-field space generated by a sub near-field space, cyclic Γ - semi sub normal near-field space of a Γ -semi sub near-field space and cyclic Γ -semi sub near-field space are introduced. It is proved that (1) the nonempty intersection of two Γ - semi sub normal near-field spaces of a Γ -semi sub near-field space S is a Γ - semi sub normal near-field space of S , (2) the non-empty intersection of any family of Γ - semi sub normal near-field space of a Γ -semi sub near-field space S is a Γ - semi sub normal near-field space of S . It is also proved that if A is a nonempty Γ - semi sub near-field space of a Γ -semi sub near-field space S , then the Γ -semi sub normal near-field space of S generated by A is the intersection of all Γ - semi sub normal near-field spaces of S containing A .

Definition 3.2: Let S be a Γ -semi sub normal near-field space. A non-empty subset T of S is said to be a Γ - semi sub normal near-field space of S if $a \gamma b \in T$, for all $a, b \in T$ and $\gamma \in \Gamma$.

Note 3.3: A non-empty Γ -semi sub near-field space T of a Γ -semi sub normal near-field space S is a Γ -semi sub normal near-field space of S iff $T \Gamma T \subseteq T$.

Example 3.4: Let $S = [0, 1]$ and $\Gamma = \{1/n: n \text{ is a positive integer}\}$. Then S is a Γ -semi sub near-field space under the usual multiplication. Let $T = [0, 1/2]$. Now T is a non-empty Γ -semi sub near-field space of S and $a \gamma b \in T$, for all $a, b \in T$ and $\gamma \in \Gamma$. Then T is a Γ -semi sub near-field space of S .

Theorem 3.5: The nonempty intersection of two Γ -semi sub near-field spaces of a Γ -semi sub near-field space S is a Γ -semi sub near-field space of S .

Proof: Let T_1, T_2 be two Γ -semi sub near-field spaces of S . Let $a, b \in T_1 \cap T_2$ and $\gamma \in \Gamma$.

$a, b \in T_1 \cap T_2 \Rightarrow a, b \in T_1$ and $a, b \in T_2$

$a, b \in T_1, \gamma \in \Gamma, T_1$ is a Γ -semi sub near-field space of $S \Rightarrow a \gamma b \in T_1$.

$a, b \in T_2, \gamma \in \Gamma, T_2$ is a Γ -semi sub near-field space of $S \Rightarrow a \gamma b \in T_2$.

$a \gamma b \in T_1, a \gamma b \in T_2 \Rightarrow a \gamma b \in T_1 \cap T_2$. Therefore $T_1 \cap T_2$ is a Γ -semi sub near-field space of S . This completes the proof of the theorem.

Theorem 3.6: The nonempty intersection of any family of Γ -semi sub near-field spaces of a Γ -semi sub near-field space S is a Γ -semi sub near-field space of S .

Proof: Let $\{T_\alpha\}_{\alpha \in \Delta}$ be a family of Γ -semi sub near-field spaces of S and let $T = \bigcap_{\alpha \in \Delta} T_\alpha$.

Let $a, b \in T$ and $\gamma \in \Gamma$.

$a, b \in T \Rightarrow a, b \in \bigcap_{\alpha \in \Delta} T_\alpha$

$\Rightarrow a, b \in T_\alpha$ for all $\alpha \in \Delta$.

$a, b \in T_\alpha, \gamma \in \Gamma, T_\alpha$ is a Γ -semi sub near-field space of $S \Rightarrow a \gamma b \in T_\alpha$.

$a \gamma b \in T_\alpha$ for all $\alpha \in \Delta \Rightarrow a \gamma b \in T$

$\forall \alpha \in \Delta \Rightarrow a \gamma b \in T$.

Therefore T is a Γ -semi sub near-field space of S . This completes the proof of the theorem.

In the following we are introducing a Γ -semi sub near-field space which is generated by a sub near-field space and a cyclic Γ -semi sub near-field space of Γ -semi near-field space over a near-field. This completes the proof of the theorem.

Definition 3.7: Γ -semi sub near-field space of S generated by A . Let S be a Γ -semi sub near-field space and A be a non-empty sub near-field space of S . The smallest Γ -semi sub near-field space of S containing A is called a Γ -semi sub near-field space of S generated by A . It is denoted by $\langle A \rangle$.

Theorem 3.8: Let S be a Γ -semi sub near-field space and A be a nonempty sub near-field space of S . Then $\langle A \rangle = \{a_1 \alpha_1 a_2 \alpha_2 \dots a_{n-1} \alpha_{n-1} a_n : n \in \mathbb{N}, a_1, a_2, \dots, a_n \in A, \alpha_1, \alpha_2, \dots, \alpha_{n-1} \in \Gamma\}$.

Proof: Let $T = \{a_1 \alpha_1 a_2 \alpha_2 \dots a_{n-1} \alpha_{n-1} a_n : n \in \mathbb{N}, a_1, a_2, \dots, a_n \in A, \alpha_1, \alpha_2, \dots, \alpha_{n-1} \in \Gamma\}$.
 Let $a, b \in T$ and $\gamma \in \Gamma$.

$a \in T \Rightarrow a = a_1 \alpha_1 a_2 \alpha_2 \dots a_{m-1} \alpha_{m-1} a_m$ where $a_1, a_2, \dots, a_m \in A, \alpha_1, \alpha_2, \dots, \alpha_{m-1} \in \Gamma$.

$b \in T \Rightarrow b = b_1 \beta_1 b_2 \beta_2 \dots b_{n-1} \beta_{n-1} b_n$ where $b_1, b_2, \dots, b_n \in A, \beta_1, \beta_2, \dots, \beta_{n-1} \in \Gamma$.

Now $a\gamma b = (a_1 \alpha_1 a_2 \alpha_2 \dots a_{m-1} \alpha_{m-1} a_m) \gamma (b_1 \beta_1 b_2 \beta_2 \dots b_{n-1} \beta_{n-1} b_n) \in T$.

Therefore T is a Γ - semi sub near-field space of S .

Let K be a Γ - semi sub near-field space of S such that $A \subseteq K$.

Let $a \in T$. Then $a = a_1 \alpha_1 a_2 \alpha_2 \dots a_{n-1} \alpha_{n-1} a_n$ where $a_1, a_2, \dots, a_n \in A, \alpha_1, \alpha_2, \dots, \alpha_{n-1} \in \Gamma$
 $a_1, a_2, \dots, a_n \in A, A \subseteq K \Rightarrow a_1, a_2, \dots, a_n \in K$.

$a_1, a_2, \dots, a_n \in K, \alpha_1, \alpha_2, \dots, \alpha_{n-1} \in \Gamma, K$ is a Γ - semi sub near-field space of sub near-field space over near-field.

$\Rightarrow a_1 \alpha_1 a_2 \alpha_2 \dots a_{n-1} \alpha_{n-1} a_n \in K \Rightarrow a \in K$. Therefore, $T \subseteq K$.

So T is the smallest Γ - semi sub near-field space S of sub near-field space over near-field containing A . Hence $\langle A \rangle = T$.

Theorem 3.9: Let S be a Γ - semi sub near-field space and A be a non-empty sub near-field space of S . Then $\langle A \rangle =$ The intersection of all Γ - semi sub near-field spaces of S containing A .

Proof: Let Δ be the set of all Γ - semi sub near-field spaces of S containing A .

Since S is a Γ - semi sub near-field space of S containing $A, S \in \Delta$. So $\Delta \neq \emptyset$.

Let $T^* = \bigcap_{T \in \Delta} T$. Since $A \subseteq T$ for all $T \in \Delta, A \subseteq T^*$.

By theorem 3.4, T^* is a Γ - semi sub near-field space of S .

Since $T^* \subseteq T$ for all $T \in \Delta, T^*$ is the smallest Γ - semi sub near-field space of S containing A . Therefore $T^* = \langle A \rangle$. This completes the proof of the theorem.

Definition 3.10: cyclic Γ -semi sub near-field space. Let S be a Γ -semi sub near-field space of Γ -sub near-field space over a near-field space. A Γ -semi sub near-field space T of S is said to be cyclic Γ -semi sub near-field space of S if T is generated by a single element sub near-field space of S .

Note 3.11: Let T be a Γ -semi sub near-field space of Γ -semi sub near-field space S . Then T is cyclic iff

$$T = \bigcup_{n \in \mathbb{N}} (a\Gamma)^{n-1} a, \text{ for some } a \in S.$$

Definition 3.12: cyclic Γ - semi sub near-field space. A Γ -semi sub near-field space S is said to be a cyclic Γ - semi sub near-field space if S is a cyclic Γ - semi sub near-field space of S itself.

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**Corresponding Author(s): Smt. T. Madhavi Latha¹, Dr. T V Pradeep Kumar² and Dr. N. V. Nagendram³
 Professor of Mathematics, Kakinada Institute of Technology & Science, Tirupathi (v), Peddapuram(M),
 Divili - 533 433, East Godavari District, Andhra Pradesh. India.**

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