DOUBLE PATH - UNIONS OF E-CORDIAL GRAPHS

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ABSTRACT

Path unions are obtained by attaching a single copy each of given graph to the vertices of a path $P_m$. It is denoted by $P_m(G)$. A double path unions is obtained by attaching two copies of given graph to each vertex of path $P_m$. It is denoted by $P_m(2-G)$. We show that $P_m(2-C_3)$, $P_m(2-C_4)$, $P_m(2$-kite) are families of E- cordial graphs.

Key words: path, path union, labeling, cordial, etc.

Subject Classification: 05C78.

1. INTRODUCTION

In 1997 Yilmaz and Cahit introduced weaker version of edge graceful labeling E-cordial labeling. [4]. Let $G$ be a $(p, q)$ graph. $f:E \rightarrow \{0, 1\}$ Define $f$ on $V$ by $f(v) = \sum (f(vu))(vu) \in E(G)) (\mod 2)$. The function $f$ is called as E-cordial labeling if $|v(f)-v(f)| \leq 1$ and $|ef(0)-ef(1)| \leq 1$ where $v(i)$ is the number of vertices labeled with $i =0, 1$. And $e_f(i)$ is the number of edges labeled with $i = 0, 1$. We follow the convention that $v_f(0,1) = (a, b)$ for $v_f(0)=a$ and $v_f(1)=b$ further $e_f(0,1) = (x, y)$ for $e_f(0)=x$ and $e_f(1)=y$. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit prove that trees $T_n$ are E-cordial iff for $n$ not congruent to 2(mod 4), $K_n$ are E-cordial iff $n$ not congruent to 2(mod 4), $F_n$ are E-cordial iff for $n$ not congruent to 1(mod 4) etc. Yilmaz and Cahit observe that A graph on $n$ vertices can not be E-cordial if $n$ is congruent to 2 (mod 4). One should refer Dynamic survey of graph labeling by Joe Gallian [2] for more results on E-cordial graphs.

In this paper we consider path unions on two copies of same graph. It is called as double path union. It is obtained by attaching two copies of a graph at each vertex of path $P_m$. Note that this graph has $|E(G)| = m-1 + m(\|E(G)\|)$ and $|V(G)| = m$. This pathunion is denoted by $P_m(2-G)$. We discuss e- cordiality for $G = C_3, C_4$, kite [3].

The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary [3] and Dynamic survey of graph labeling by Joe Gallian [2].

2. THEOREMS PROVED

2.1 pathunion of Double $C_3$ (i.e. $G = P_m(2-C_3)$ is E- Cordial iff $m$ is odd number.

Proof: We define $G$ as $V(G) = \{v_1, v_2, \ldots, v_m\} \cup \{u_{i1}, u_{i2}, u_{i3}, u_{i4}\}$ and $E(G) = \{e_i=(v_{i+1}) / I = 1,2,\ldots,m\} \cup \{e_{i1}=(v_{i}u_{i1}), e_{i2}=(u_{i1}u_{i2}), e_{i3}=(u_{i2}v_i), e_{i4}=(v_{i}e_{i3}), e_{i5}=(u_{i3}u_{i4}), e_{i6}=(u_{i4}v_i)\}$.

To obtain a desired E-cordial function $f$: $E(G) \rightarrow \{0, 1\}$. We show labeled copies of two units as shown in figure 2.2 and 2.3 below. We use to connect them too obtain a path- union on larger $m$.

Figure 2.1: $v_f(0,1) = (3,3), e_f(0,1) = (3,3)$ $P_m(2-C_3)$ E- cordial.

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Figure-2.2: Unit A \( v_f(0,1) = (1,4), e_f(0,1) = (3,3) \)

Figure-2.3: Unit B \( v_f(0,1) = (5,6), e_f(0,1) = (7,7) \) The label of \( v_3 \) is 1. Here \( v_1 \) from C will be attached.

Figure-2.4: Unit C \( v_f(0,1) = (5,6), e_f(0,1) = (7,7) \) The label of \( v_3 \) is 0 where \( v_1 \) from type B will be attached.

Unit B is attached at unit A with vertex \( v_1 \) to obtain \( P_3(2-C_3) \). To obtain \( P_5(2-C_3) \) unit B is attached at point \( v_3 \) of unit B.

Then sequence of unit C, B, C, B… is followed. We get \( P_m(2-C_3) \) for \( m=2x+1, x=0,1,2,… \)

To obtain a path union on \( P_{m+2} \) we first obtain a path union on \( P_m \). (m is odd number) The process is recursive. The label numbers observed are \( e_f(0,1) = (3 +7x,3+7x) \) for edges for all \( m \) and \( v_f(0,1) = (10k+3,10k+2) \) for \( m>3 \) and \( m \equiv 1(\mod 4) \), \( k = \frac{m-1}{4} \).

If \( m \equiv 3(\mod 4) \), \( k = \frac{m-3}{4} \) and \( v_f(0,1) = (10k+7,10k+8) \) (\( m \geq 3 \))

2.2 pathunion of Double \( C_4 \) (i.e. \( G = P_m(2-C_4) \) is E- Cordial iff \( m \) is not congruent to 2 (mod 4). We below give different types of structure used to form \( P_m(2-C_4) \). They are of type C, A, D and B.

Figure-2.5: Type C \( v_f(0,1) = (4,4), e_f(0,1) = (5,4) \); label of w
Vertex v of one structure is identified with vertex w of suitable other copy to obtain a path of more length. We below give a scheme how to use different types of structures above to obtain a path-union of bigger length. To obtain a path union on $P_{m+1}$ we first obtain a path union on $P_m$. The process is recursive.

<table>
<thead>
<tr>
<th>m</th>
<th>New type used</th>
<th>Resultant $V_f(0,1)$</th>
<th>Resultant $e_f(0,1)$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>(3,4)</td>
<td>(4,4)</td>
<td>E-cordial</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>(8,6)</td>
<td>(9,8)</td>
<td>Not E-cordial</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>(11,10)</td>
<td>(13,13)</td>
<td>E-cordial</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>(14,14)</td>
<td>(17,18)</td>
<td>E-cordial</td>
</tr>
<tr>
<td>5</td>
<td>D</td>
<td>(17,18)</td>
<td>(22,22)</td>
<td>E-cordial</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>(22,20)</td>
<td>(27,26)</td>
<td>Not E-cordial</td>
</tr>
</tbody>
</table>

Further sequence of B, B, D, C is repeated…

$V_f(0,1) = (14x,14x)$ and $e_f(0,1) = (13+18x,13+18x)$, $e_f(0,1) = (17+18x,17+18x)$ for $m = 4x$.

$V_f(0,1) = (3+14x,4+14x)$, $e_f(0,1) = (4+18x,4+18x)$ for $m = 4x +1$ and for $m > 1$

$V_f(0,1) = (8+14x,6+14x)$ and $e_f(0,1) = (9+18x,9+18x)$ for $m = 4x+2$

$V_f(0,1) = (11+14x,10+14x)$ and $e_f(0,1) = (13+18x,13+18x)$ for $m = 4x+3$

Thus G is E – cordial graph.

2.3 Path union of double kite given by $P_m(2\text{-kite})$ is E-cordial.

**Proof:** We define The graph as follows : $V(G) = \{v_1, v_2, ..., v_m\} \cup \{u_{i,1}, u_{i,2}, u_{i,3} / I = 1, 2, ..., m-1\} \cup \{u_{i,4}, u_{i,5}, u_{i,6}\}$, $E(G) = \{e_f=(v_i v_{i+1}) / i = 1, 2, ..., m-1\} \cup \{c_{i,1}=(v_i u_{i,1}), c_{i,2}=(u_{i,1} u_{i,2}), c_{i,3}=(u_{i,2} u_{i,3}), c_{i,4}=(u_{i,3} v_i), c_{i,5}=(u_{i,1} u_{i,3}) / i = 1, 2, ..., m-1\} \cup \{c_{i,6}=(v_i u_{i,4}), c_{i,7}=(u_{i,4} u_{i,5}), c_{i,8}=(u_{i,5} u_{i,6}), c_{i,9}=(u_{i,6} v_i), c_{i,10}=(u_{i,1} u_{i,6}) / i = 1, 2, ..., m-1\}$. To define E-cordial labeling we construct different units related to $P_1(2\text{-kite})$. We give a scheme to connect these units to obtain path of given length m.
Figure-2.10: P$_1$(2-kite). Labeled copy. v$_f$(0,1) = (4,3), e$_f$(0,1) = (5,5)

Figure-2.11: Labeled copy. e$_f$(0,1) = (4,4), ef(0,1) = (5,6)

Figure-2.12: Labeled copy. e$_f$(0,1) = (4,4), ef(0,1) = (6,5)

Figure-2.13: Labeled copy. e$_f$(0,1) = (4,4), ef(0,1) = (6,5)

Figure-2.14: P$_2$(2-kite). Labeled copy. e$_f$(0,1) = (6,8), ef(0,1) = (11,10). NOT E-cordial
The vertex v on type B, D, C is used to identify with vertex w to obtain a path union of larger length. We below give a scheme how to use different types of structures above to obtain a path-union of bigger length. To obtain a path union on $P_{m+1}$ we first obtain a path union on $P_m$. The process is recursive.

$m \equiv 0 \pmod{4}$ $v_f(0,1) = (14x, 14x)$, $e_f(0,1) = (21+22x, 22+22x)$ where $m = 4x$

$m \equiv 1 \pmod{4}$ $v_f(0,1) = (14x+3, 14x+4)$, $e_f(0,1) = (5+22x, 5+22x)$ where $m = 4x +1$

$m \equiv 2 \pmod{4}$ $v_f(0,1) = (14x+6, 14x+8)$, $e_f(0,1) = (11+22x, 10+22x)$ where $m = 4x +2$

$m \equiv 3 \pmod{4}$ $v_f(0,1) = (14x+11, 14x+10)$, $e_f(0,1) = (16+22x, 16+22x)$ where $m = 4x +3$

CONCLUSIONS

We have established e-cordiality of double path union. By double path union we mean two copies of graph G are attached at every vertex of path $P_m$. It is necessary to investigate the concept for more graphs before coming across any conclusion.

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