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A NOTE ON SINGULAR MULTIPARAMETER MATRIX EIGENVALUE PROBLEM

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ABSTRACT

In this paper, a note on singular Multiparameter matrix eigenvalue problems is discussed. To find solutions of singular Multiparameter problem, the original problem has been reformulated into another system based on linear combination of certain operator determinants. The new system has been by applying Kronecker Product Method adopted by Atkinson for Right Definite problem and it is proved that only eigenvectors can be evaluated by this approach. MATLAB program is used for numerical calculations.

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1. INTRODUCTION

The abstract settings of Multiparameter Matrix Eigenvalue Problems (MMEP) to be studied is

$$\left(A_i - \sum_{j=1}^{k} \lambda_j B_{ij}\right) x_i = 0, \quad i = 1, 2, \dots\dots, k \tag{1.1}$$

where the problem is to find k-tuple of values $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4 \dots\dots\dots\dots \lambda_k) \epsilon\, C^k$ for non-zero vector x_i. The operators $A_{i,} B_{ij}$ are self-adjoint, bounded and linear that act on separable Hilbert Spaces H_i, $x_i \in H_i$. The k-tuple $\lambda\, \epsilon\, C^k$ is called an eigenvalue and the decomposible tensor product $x = x_1 \otimes x_2 \otimes x_3 \dots\dots\dots . \otimes x_k$ is the corresponding (right) eigenvector. Similarly left eigenvector can also be defined.

MMEPs arise in desperate scientific domains, particularly in mathematical physics when the method of separation of variables technique is used to solve boundary value problems. Here we present the fundamental notions regarding the theory of Multiparameter problem in Hilbert space adopted by Atkinson [4], [5] as follows:

First we consider the linear transformation $\mathrm{B}_{\mathrm{ij}}^{+}$ on H that are induced by B_{ij} and are defined by

$$\mathrm{B}_{\mathrm{ij}}^{+}(x_1 \otimes x_2 \dots\dots\dots . \otimes x_k) = x_1 \otimes x_2 \otimes \dots\dots \otimes x_{i-1} \otimes \mathrm{B}_{\mathrm{ij}} x_i \otimes x_{i+1} \otimes \dots . \otimes x_k$$

On the decomposable tensor $x_1 \otimes x_2 \dots\dots\dots . \otimes x_k$ where $x_i \in H_i$, extended to H by linearity.

We may define the operator determinants,

$$\Delta_0 = \begin{vmatrix} \mathrm{B}_{11}^{+} & .. & \mathrm{B}_{1,\mathrm{k}}^{+} \\ \vdots & \vdots & \vdots \\ \mathrm{B}_{\mathrm{k}1}^{+} & \dots & \mathrm{B}_{\mathrm{k,k}}^{+} \end{vmatrix} \tag{1.2}$$

and

$$\Delta_{\mathrm{i}} = \begin{vmatrix} \mathrm{B}_{11}^{+} & \dots & \mathrm{B}_{1,\mathrm{i}-1}^{+} & \mathrm{A}_1^{+} & \mathrm{B}_{1,\mathrm{i}+1}^{+} & \dots & \mathrm{B}_{1\mathrm{k}}^{+} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathrm{B}_{\mathrm{k}1}^{+} & \dots & \mathrm{B}_{\mathrm{k,i}-1}^{+} & \mathrm{A}_{\mathrm{k}}^{+} & \mathrm{B}_{\mathrm{k,i}+1}^{+} & \dots & \mathrm{B}_{\mathrm{kk}}^{+} \end{vmatrix} \tag{1.3}$$

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Generally, the problem (1.1) is considered as nonsingular i.e when Δ_0 is positive definite and such a problem is called Right Definite [5]. Atkinson proved [4] that a nonsingular system of the form (1.1) can be transformed into a system of Generalized Eigenvalue Problem of the form

$$\Delta_i x = \lambda_i \Delta_0 x \tag{1.4}$$

In this case the matrices $\Delta_0^{-1}\Delta_i$ (i = 1; 2; : : : ; k) commute and all the eigenvalues of (1.1) are coincide with the eigenvalues of (1.4). Several Numerical Methods are available to treat the Right Definite case, but they are particularly in two-parameter case, for reference [3], [6], [7], [8].

MMEP is called singular [2] if Δ_0 is singular. MMEP obtained by a linearization of the Polynomial eigenvalue problem [9] is singular. If the operator determinant Δ_0 is singular, then infinitely many λ_i satisfies the associated systems of generalized eigenvalue problems of the form (1.4) of the corresponding MMEPs, which makes it difficult to compute appropriate eigenvalues $(\lambda_1, \lambda_2, \dots \dots \dots \lambda_k)$. There are limited numerical tools to treat the singular case in the existing literature. In [10], Muhic etc. all., showed that finite regular eigenvalues of (1.1) are related to the finite regular eigenvalues of (1.4). If all eigenvalues of (1.1) are algebraically simple, they agree with the finite regular eigenvalues of the singular matrix pencils. In [10] numerical algorithm is presented which based on the staircase algorithm developed by Van Dooren, to compute the common regular part of (1.4) and extract the finite regular eigenvalues. The algorithm returns matrices Q and P with orthonormal columns such that the finite regular eigenvalues are the eigenvalues of the following generalized eigenvalue problems

$$\widetilde{\Delta_i} x = \lambda_i \widetilde{\Delta_0} x \tag{1.4}$$

where $\widetilde{\Delta_i} = P^* \Delta_i Q$ and then $\widetilde{\Delta_0}$ is non singular. But when order of the matrix is very large, this approach is not an efficient one, as that in nonsingular case, due to the computational complexity. This approach is still open for more than two parameter problems. In [1], Muhic etc. all., proved that a singular two-parameter eigenvalue problem can be solved by computing the common regular eigenvalues of the associated system of two singular generalized eigenvalue problems. Another algorithm may be found in [9], where the Jacobi-Davidson type methods presented in [6], [7] has been extended to the regular singular MMEP.

In our present study we consider the singular case, where the original problem will be reformulated into another system based on linear combination of certain operator determinants. Kronecker product method will be applied on the new system to find the solutions.

2. AN APPROACH FOR SINGULAR CASE

Suppose the linear combination

$$\Delta = \alpha_0 \Delta_0 + \alpha_1 \Delta_1 + \dots \dots \dots \dots + \alpha_k \Delta_k \tag{2.1}$$

is non singular. Let

$$\lambda = \alpha_0 \lambda_0 + \alpha_1 \lambda_1 \dots + \dots \dots \dots \dots + \alpha_k \lambda_k \neq 0 \tag{2.2}$$

Theorem 1: Under the assumptions of (2.1) and (2.2), Multiparameter system (1.4) reduces to

$$(\Delta_i - \lambda^{-1} \lambda_i \Delta) x = 0 \tag{2.3}$$

Proof: Pre multiplying above equations of (1.4) respectively by $\alpha_1, \alpha_2, \dots \dots \dots, \alpha_k$ and then adding all we get

$$\alpha_1 \Delta_1 x + \alpha_2 \Delta_2 x + \dots \dots \dots \dots + \alpha_k \Delta_k x = \alpha_1 \lambda_1 \Delta_0 x + \alpha_2 \lambda_2 \Delta_0 x \dots \dots \dots \dots + \alpha_k \lambda_k \Delta_0 x$$

$$\Rightarrow (\alpha_0 \Delta_0 + \alpha_1 \Delta_1 + \alpha_2 \Delta_2 + \dots \dots \dots + \alpha_k \Delta_k) x = (\alpha_0 + \alpha_1 \lambda_1 + \alpha_2 \lambda_2 + \dots \dots \dots + \alpha_k \lambda_k) \Delta_0 x$$

$$\Rightarrow \Delta x = \lambda \Delta_0 x$$

$$\Rightarrow \Delta_0 x = \lambda^{-1} \Delta x$$

Substituting in equation (1.4), we ge $(\Delta_i - \lambda^{-1} \lambda_i \Delta) x = 0$. Hence the theorem.

Equation (2.3) can be rewritten as:

$$(\Delta_i - k_i \Delta) x = 0 \tag{2.4}$$

Equations of (2.4) are of the similar form as that of the equations (1.4). Since Δ is nonsingular by our assumption (2.1), and hence Kronecker Product Method can be applied to solve these. Here the eigenvectors $x = x_1 \otimes x_2 \otimes x_3 \dots \dots \dots . \otimes x_k$ obtained from system of equations (2.4) will also be the eigenvector of the system (1.4) for any choice of $\alpha_i, i = 1,2, \dots \dots, k$. Thus this approach provides us a technique to find the eigenvectors of Multiparameter problems for singular case.

3. MODEL PROBLEM, RESULTS AND DISCUSSIONS

For numerical illustration we consider following singular three-parameter problems

$$\begin{pmatrix}-1&0&0\\0&1&0\\0&0&1\end{pmatrix}x_1=\left\{\lambda_1\begin{pmatrix}1&2&3\\1&0&1\\3&4&7\end{pmatrix}+\lambda_2\begin{pmatrix}1&1&1\\0&0&0\\0&0&0\end{pmatrix}+\lambda_3\begin{pmatrix}0&0&0\\0&0&0\\2&2&2\end{pmatrix}\right\}x_1 \tag{3.1}$$

$$\begin{pmatrix}6&0&0\\0&7&0\\0&0&8\end{pmatrix}x_2=\left\{\lambda_1\begin{pmatrix}2&0&0\\0&3&0\\0&0&4\end{pmatrix}+\lambda_2\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}+\lambda_3\begin{pmatrix}6&0&0\\0&2&0\\0&0&-2\end{pmatrix}\right\}x_2 \tag{3.2}$$

$$\begin{pmatrix}2&0&0\\0&3&0\\0&0&4\end{pmatrix}x_3=\left\{\lambda_1\begin{pmatrix}5&0&0\\0&3&0\\0&0&4\end{pmatrix}+\lambda_2\begin{pmatrix}7&0&0\\0&8&0\\0&0&1\end{pmatrix}+\lambda_3\begin{pmatrix}2&0&0\\0&3&0\\0&0&1\end{pmatrix}\right\}x_3 \tag{3.3}$$

The Multiparameter systems represented by the equations (3.1), (3.2) and (3.3) are singular because for this system the operator determinant, $|\Delta_0|$ vanishes. Two different choices for $\alpha_i, i=0,1,2,3$ are considered for computational works and the results are presented in **Table 1** and **Table 2**. α_i are selected in such a way that the corresponding Δ becomes nonsingular. Table 1 contains the results, when α_0 = 3, α_1 = 4, α_3 = 5, α_4 = 3. Similarly, Table 2 contains the results when α_0 = -1, α_1 = 7, α_3 = -3, α_4 = 4. Kronecker Product Method has been applied and accordingly k_1, k_2 and k_3 are evaluated similar to that of Right definite case. From the tables we see that eigenvectors for both the cases are same, but (k_1, k_2, k_3) are different and hence eigen 3-tuples are also different. Thus this approach allows us only to find the eigenvectors of singular case.

Table-1: Case I, when $\alpha_1 = 3,\ \alpha_2 = 4,\ \alpha_3 = 5,\ and\ \alpha_4 = 3$.

(k_1,k_2,k_3)	Eigenvectors
$(0.1414,-0.0875,0.1218)$	$(-0.7826\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 - 0.1492\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.6044\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$
$(-0.0335,0.0208,0.1773)$	$(-0.3843\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.2566\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 - 0.8868\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$
$(-0.7973,0.4936,0.4198)$	(0.3371 0 0 0 0 0 0 0 0 0.8026 0 0 0 0 0 0 0 0 -0.4921 0 0 0 0 0 0 0 0)^T
$(0.4382,-0.1095,-0.1072)$	(0 -0.6024 0 0 0 0 0 0 0 0 -0.5774 0 0 0 0 0 0 0 0 0.5511 0 0 0 0 0 0)^T
$(0.9020,-0.4548,-0.3872)$	(0 0 0 0 -0.6810 0 0 0 0 0 0 0 -0.5107 0 0 0 0 0 0 0 0 0.5248 0 0 0 0 0)^T
$(0.5854,-0.2908,-0.2043)$	(0 0 0 0 0 0 0.7176 0 0 0 0 0 0 0 0 0.4491 0 0 0 0 0 0 0 0 -0.5323 0 0)^T
$(0.1408,-0.0649,0.1035)$	(0 0 -0.7822 0 0 0 0 0 0 0 0 -0.1714 0 0 0 0 0 0 0 0 0.5990 0 0 0 0 0 0)^T
$(0.1481,-0.0370,0.0741)$	(0 -0.7761 0 0 0 0 0 0 0 0 -0.2238 0 0 0 0 0 0 0 0 0.5895 0 0 0 0 0 0 0)^T
$(0.2394,-0.0503,-0.0036)$	(0 0 0 0 -0.7290 0 0 0 0 0 0 0 0 -0.3863 0 0 0 0 0 0 0 0 0.5651 0 0 0 0)^T
$(0.2103,-0.0443,0.0173)$	(0 0 0 0 0 0 0 0 0.7475 0 0 0 0 0 0 0 0 0.3412 0 0 0 0 0 0 0 0 -0.5699 0)^T
$(0.0068, 0.1328, 0.0503)$	(0 0 0.6318 0 0 0 0 0 0 0 0 -0.0185 0 0 0 0 0 0 0 0 -0.7749 0 0 0 0 0 0)^T
$(-0.0395,0.0099,0.1914)$	(0 0.4739 0 0 0 0 0 0 0 0 -0.2897 0 0 0 0 0 0 0 0 0.8316 0 0 0 0 0 0 0)^T
$(-0.0310,-0.1285,0.5554)$	(0 0 0 0 -0.4817 0 0 0 0 0 0 0 0 0.7492 0 0 0 0 0 0 0 0 -0.4547 0 0 0 0)^T
$(-0.0945,0.1886,0.0353)$	(0 0 0.4845 0 0 0 0 0 0 0 0 -0.8512 0 0 0 0 0 0 0 0 0.2018 0 0 0 0 0 0)^T
$(-0.0881,-0.1517,0.5982)$	(0 0 0 0 0 0 0 0.4608 0 0 0 0 0 0 0 0 -0.7633 0 0 0 0 0 0 0 0 0.4528 0)^T
$(-0.0683,-0.0430,0.4085)$	(0 0 0 -0.4558 0 0 0 0 0 0 0 0 0.7410 0 0 0 0 0 0 0 0 -0.4931 0 0 0 0 0)^T
$(-0.0590,-0.0572,0.4359)$	(0 0 0 0 0 0 -0.4361 0 0 0 0 0 0 0 0 0.7570 0 0 0 0 0 0 0 0 -0.4866 0 0)^T
$(-0.0634,0.1547,0.1395)$	(0 0 0 0 0 -0.3032 0 0 0 0 0 0 0 0 0.9505 0 0 0 0 0 0 0 0 0.0679 0 0 0)^T
$(-0.0260,0.0856,0.1839)$	(0 0 0 0 0 -0.1340 0 0 0 0 0 0 0 0 0.5853 0 0 0 0 0 0 0 0 -0.7996 0 0 0)^T
$(-0.0311,0.0622,0.2293)$	(0 0 0 0 0 0 0 0 0.2808 0 0 0 0 0 0 0 0 -0.6988 0 0 0 0 0 0 0 0 0.6579)^T
$(0.0574,-0.1007,0.3167)$	(0 0 0 0 0 0 0.7384 0 0 0 0 0 0 0 0 0.0345 0 0 0 0 0 0 0 0 -0.6735 0 0)^T
$(0.0573,-0.0994,0.3152)$	(0 0 0 0 0 0 0 -0.7384 0 0 0 0 0 0 0 0 -0.0346 0 0 0 0 0 0 0 0 0.6734 0)^T
$(0.0609,-0.0978,0.3025)$	(0 0 0 0.7410 0 0 0 0 0 0 0 0 0.0382 0 0 0 0 0 0 0 0 -0.6704 0 0 0 0 0)^T
$(0.0607,-0.0958,0.3003)$	(0 0 0 0 0 0.7411 0 0 0 0 0 0 0 0 0.0384 0 0 0 0 0 0 0 0 -0.6703 0 0 0)^T
$(0.0602,-0.0923,0.2964)$	(0 0 0 0 -0.7413 0 0 0 0 0 0 0 0 -0.0388 0 0 0 0 0 0 0 0 0.6701 0 0 0 0)^T
$(0.0598,-0.1196,0.3384)$	(0 0 0 0 0 0 0 0 -0.7375 0 0 0 0 0 0 0 0 -0.0329 0 0 0 0 0 0 0 0 0.6745)^T
$(-0.0833, 0.1667, 0.1667)$	$(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$

Table-2: Case II, when $\alpha_1 = -1,\ \alpha_2 = 7,\ \alpha_3 = -3,\ and\ \alpha_4 = 4$.

(k_1,k_2,k_3)	Eigenvectors
$(0.0900,-0.0557,0.0775)$	$(-0.7826\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 - 0.1492\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.6044\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$
$(-0.1363,0.0844,0.7206)$	$(-0.3843\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.2566\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 - 0.8868\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$
$(0.1440,-0.0891,-0.0758)$	(0.3371 0 0 0 0 0 0 0 0 0.8026 0 0 0 0 0 0 0 0 -0.4921 0 0 0 0 0 0 0 0)^T
$(0.1496,-0.0374,-0.0366)$	(0 -0.6024 0 0 0 0 0 0 0 0 -0.5774 0 0 0 0 0 0 0 0 0.5511 0 0 0 0 0 0)^T
$(0.1541,-0.0777,-0.0662)$	(0 0 0 0 -0.6810 0 0 0 0 0 0 0 -0.5107 0 0 0 0 0 0 0 0 0.5248 0 0 0 0 0)^T
$(0.1497,-0.0743,-0.0522)$	(0 0 0 0 0 0 0.7176 0 0 0 0 0 0 0 0 0.4491 0 0 0 0 0 0 0 0 -0.5323 0 0)^T
$(0.0975,-0.0450,0.0717)$	(0 0 -0.7822 0 0 0 0 0 0 0 0 -0.1714 0 0 0 0 0 0 0 0 0.5990 0 0 0 0 0 0)^T
$(0.1121,-0.0280,0.0561)$	(0 -0.7761 0 0 0 0 0 0 0 0 -0.2238 0 0 0 0 0 0 0 0 0.5895 0 0 0 0 0 0 0)^T
$(0.1399,-0.0294,-0.0021)$	(0 0 0 0 -0.7290 0 0 0 0 0 0 0 0 -0.3863 0 0 0 0 0 0 0 0 0.5651 0 0 0 0)^T

$(0.1344, -0.0283, 0.0111)$	(0 0 0 0 0 0 0 0 0.7475 0 0 0 0 0 0 0 0.3412 0 0 0 0 0 0 0 0 -0.5699 0)^T
$(-0.0336, -0.6568, -0.248)$	(0 0 0.6318 0 0 0 0 0 0 0 0 -0.0185 0 0 0 0 0 0 0 0 -0.7749 0 0 0 0 0 0)^T
$(-0.1408, 0.0352, 0.6815)$	(0 0.4739 0 0 0 0 0 0 0 0 -0.2897 0 0 0 0 0 0 0 0 0.8316 0 0 0 0 0 0 0)^T
$(-0.0517, -0.0703, 0.3040)$	(0 0 0 0 -0.4817 0 0 0 0 0 0 0 0 0.7492 0 0 0 0 0 0 0 0 -0.4547 0 0 0 0)^T
$(0.0465, -0.2830, -0.0530)$	(0 0 0.4845 0 0 0 0 0 0 0 0 -0.8512 0 0 0 0 0 0 0 0 0.2018 0 0 0 0 0 0)^T
$(-0.0415, -0.0714, 0.2814)$	(0 0 0 0 0 0 0 0.4608 0 0 0 0 0 0 0 0 -0.7633 0 0 0 0 0 0 0 0 0.4528 0)^T
$(-0.0570, -0.0359, 0.3412)$	(0 0 0 -0.4558 0 0 0 0 0 0 0 0 0.7410 0 0 0 0 0 0 0 0 -0.4931 0 0 0 0 0)^T
$(-0.0455, -0.0410, 0.3127)$	(0 0 0 0 0 0 -0.4361 0 0 0 0 0 0 0 0 0.7570 0 0 0 0 0 0 0 0 -0.4866 0 0)^T
$(0.1769, -0.4640, -0.4185)$	(0 0 0 0 0 -0.3032 0 0 0 0 0 0 0 0 0.9505 0 0 0 0 0 0 0 0 0.0679 0 0 0)^T
$(-0.1016, 0.3350, 0.7196)$	(0 0 0 0 0 -0.1340 0 0 0 0 0 0 0 0 0.5853 0 0 0 0 0 0 0 0 -0.7996 0 0 0)^T
$(-0.0660, 0.1320, 0.4867)$	(0 0 0 0 0 0 0 0 0 0.2808 0 0 0 0 0 0 0 0 -0.6988 0 0 0 0 0 0 0 0 0.6579)^T
$(0.0309, -0.0540, 0.1700)$	(0 0 0 0 0 0 0.7384 0 0 0 0 0 0 0 0 0.0345 0 0 0 0 0 0 0 0 -0.6735 0 0)^T
$(0.0308, -0.0537, 0.1702)$	(0 0 0 0 0 0 0 -0.7384 0 0 0 0 0 0 0 0 -0.0346 0 0 0 0 0 0 0 0 0.6734 0)^T
$(0.0335, -0.0538, 0.1664)$	(0 0 0 0.7410 0 0 0 0 0 0 0 0 0.0382 0 0 0 0 0 0 0 0 -0.6704 0 0 0 0 0)^T
$(0.0337, -0.0532, 0.1667)$	(0 0 0 0 0 0.7411 0 0 0 0 0 0 0 0 0.0384 0 0 0 0 0 0 0 0 -0.6703 0 0 0)^T
$(0.0340, -0.0520, 0.1671)$	(0 0 0 0 -0.7413 0 0 0 0 0 0 0 0 -0.0388 0 0 0 0 0 0 0 0 0.6701 0 0 0 0)^T
$(0.0297, -0.0593, 0.1678)$	(0 0 0 0 0 0 0 0 -0.7375 0 0 0 0 0 0 0 0 -0.0329 0 0 0 0 0 0 0 0 0.6745)^T
$(0.2000, -0.4000, -0.4000)$	$(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$

4. CONCLUSIONS

The approach discussed above can be applied only to find the eigenvectors of singular problems. Again this approach is suitable for the matrices of small order as if the matrices A_i and B_{ij} are of dimension $n \times n$, then dimension of the corresponding Kronecker system increases to $n^2 \times n^2$. This increase in complexity of dimension demands the development of other numerical techniques for solving Multiparameter problems without Kronecker Product of matrices and such numerical techniques are available for Right Definite problems. Thus it is necessary to develop iterative schemes of singular problem for k>2. Hence this may be considered for future prospects of Multiparameter singular problems, and it will conduit new avenues for future research in this area.

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