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G*α-LOCALLY CLOSED SETS AND G*α-LOCALLY CLOSED FUNCTIONS

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ABSTRACT

*The purpose of this paper is to introduce the concepts of g*α- locally closed sets and g* α- locally closed functions. We investigate their basic properties. We also discuss their relationship with already existing concepts.*

INTRODUCTION

The notion of locally closed sets in topological space was introduced by Bourbaki [3]. Ganster and Reilly [6] further studied the properties of locally closed sets and defined the LC-continuity and LC-irresoluteness. In Literature many general topologists introduced the studies of locally closed sets. Balachandran *et al.* [1] introduced the concept of generalized locally closed sets and seven different notions of generalized continuities. In this paper we continue the study of generalizations locally closed sets and investigate the classes of G*α-Locally closed functions and study some of their properties.

PRELIMINARIES

Throughout this paper (X,τ)denotes a topological space with a topology τ on which no separation axioms are assumed unless otherwise mentioned.For a subset A of a space (X,τ),cl(A),int(A), A^c, P(X) denote the closure of A, the interior of A, the complement of A, the power set of X. We recall the following Definitions, Remarks, Corallary and Theorem which are prerequisite for this paper.

Definition 2.1: A subset A of a topological space (X,τ) is called
1. a semi-open set [9] if $A\subseteq cl(int(A))$ and a semiclosed set if $int(cl(A)\subseteq A$
2. an α-open set if [12] $A\subseteq int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A))\subseteq A$
3. a regular open set [16] if int(cl(A))=A and regular closed set if A= int(cl(A))

Definition 2.2: A subset A of a topological space (X,τ) is called
1. generalized closed (briefly g-closed) set [8] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open; the complement of g-closed set is g-open set.
2. regular generalised closed set (briefly rg-closed) [13] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and regular open in (X, τ); the complement of rg-closed set is rg-open set.
3. α- generalised closed set (briefly αg-closed) [10] if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is Open in (X, τ); the complement of αg-closed set is αg-open set.
4. g^-closed set [20] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi open in (X, τ); the
5. complement of g^-closed set is g^-open set.
6. g*-closed set [22] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is g*-open in (X,τ); the complement of g*-closed is g*-open set.
7. $g^{\#}$-closed set [18] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is αg-open in (X,τ); the complement of $g^{\#}$- closed set is $g^{\#}$-open set.
8. g*α closed set if $\alpha cl(A)\subseteq U$ whenever $A \subseteq U$ and U is g*open in X.

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Definition 2.3: A subset A of a topological space (X,τ) is called
1. locally closed (briefly lc) [6] if S= U∩F, where U is open and F is closed in (X,τ).
2. generalized locally closed (briefly glc) [1] if S= U∩F, where U is g-open and F is g-closed in (X,τ).
3. generalized locally semi-closed (briefly glsc) [7] if S= U∩F, where U is g-open and F is semi-closed in (X,τ).
4. locally semi-closed (briefly lsc) [7] if S= U∩F, where U is open and F is semi-closed in (X,τ).
5. α-locally closed (briefly αlc)[7] if S= U∩F, where U is α-open and F is α-closed in (X,τ).
6. g^-locally closed set(briefly g^lc)[21] if S= U∩F, where U is g^-open and F is g^-closed in (X,τ).
7. $g^{\#}$-locally closed (briefly $g^{\#}$lc) [19] if S= U∩F, where U is $g^{\#}$-open and F is $g^{\#}$-closed in (X,τ).
8. g*-locally closed (briefly g*lc) [22] if S= U∩F, where U is g*-open and F is g*-closed in (X,τ).

The class of all locally closed (resp. generalized locally closed, generalized locally semi-closed, locally semi-closed, g^-locally closed, $g^{\#}$-locally closed, g*-locally closed) sets in X is denoted by LC(X) (resp.GLC(X), GLSC(X), LSC(X), G^LC, GLC, G*LC)

Definition 2.4: [14] A space (X,τ) is called a ${}_{\alpha}T^{**}_{1/2}$ −space if every g*α-closed set is closed.

Recall that a subset A of a space (X,τ) is called dense if cl(A)=X

Definition 2.5: A subset A of a topological space (X,τ) is called
1. submaximal [5] if every dense subset is open.
2. g-submaximal [1] if every dense subset is g-open.
3. rg-submaximal [13] if every dense subset is rg-open.

Remark 2.6: For a topological space (X,τ), the following statements hold:
1. Every closed set is g*α-closed but not conversely [14]
2. Every g-closed set is g*α-closed but not conversely [14]
3. Every g*-closed set is g*α-closed but not conversely [14]
4. Every α-closed set is g*α-closed but not conversely [14]
5. A subset A of X is g*α-closed iff g*α-cl(A)=A [14]
6. A subset A of X is g*α-int A iff g*α-int(A)=A [14]
7. Every ${}_{\alpha}T^{**}_{1/2}$-space is a $T_{1/2}$ –space [14]

Corollary 2.7: If A is g*α-closed and F is closed, then A∩F is a g*α-closed set.

3. G*α-Locally closed Sets

We introduce the following definition.

Definition 3.1: A subset A of (X,τ) is called g*α-locally closed (briefly g*α-lc) if A =S∩G, where S is g*α-open and G is g*α-closed in (X,τ).

The class of all g*α-locally closed sets in X is denoted by G*α LC(X).

Proposition 3.2: Every g*α-closed (resp g*α-open) is g*α lc but not conversely.

Proof: It follows from Definition 3.1

Example 3.3: Let X = {1, 2, 3} with the topology τ ={∅, X, {1}, {1, 2},{1,3}}.Then the set {1} is g*α-lc but not g*α-closed and the set {2} is g*α-lc but not g*α-open in (X,τ).

Proposition 3.4: Every lc set is g*αlc but not conversely.

Proof: It follows from Remark 2.6(1)

Example 3.5: Let X ={1,2,3} with the topology τ ={∅, X,{2},{2,3}}. Then the set {1, 2} is g*α-lc but not lc.

Proposition 3.6: Every glc* set is g*αlc but not conversely

Proof: It follows from Remark 2.6(2)

Example 3.7: Let X ={1,2,3,4} with the topology τ ={∅, X,{1},{1,4},{1,2,4}}.Then the set {1,3} is g*α-lc but not glc* set in (X,τ)

Prop 3.8: Every αlc is g*αlc but not conversely

Proof: It follows from Remark 2.6(4)

Example 3.9: Let X ={1,2,3,4,5} with the topology τ ={∅, X,{1,2},{3,4}, {1,2,3,4}}. Then the set {2, 3, 4, 5} is g*α-lc but not αlc

Prop 3.10: Every lsc and glsc are g*α-lc but not conversely.

Proof: Proof follows from the Definition 2.3(3 &4)

Example 3.11: Let X ={1,2,3,4} with the topology τ ={∅, X,{1},{2},{1,2},{1,2,3}}. Then the set {1, 2, 4} is g*α-lc but not lsc and glsc

Prop 3.12: Every g*lc and $g^{\#}$lc are g*α-lc but not conversely.

Proof: Proof follows from Remark 2.6(3) and Definition 2.3(7)

Example 3.13: Let X ={1,2,3} with the topology τ ={∅, X,{2}}.Then the set {1,2} is g*α-lc but not g*lc and $g^{\#}$lc

Prop 3.14: Every g^lc is g*α-lc but not conversely.

Proof: Proof follows from Definition 2.3(6)

Example 3.15: Let X ={1,2,3,4} with the topology τ ={∅, X,{1},{1,2}, {1,3,4}}. Then the set {1, 2, 4} is g*α-lc but not g^lc

Theorem 3.16: If (X,τ) is a ${}_{\alpha}T^{**}_{1/2}$-space then G*αLC(X) = LC(X) = GlC(X)

Proof:

(i) Since every g*α-open set is open and g*α-closed set is closed in (X,τ),G*αLC(X) ⊆ LC(X).Also for any topological space (X, τ) LC(X) ⊆ G*α LC(X).Hence G*αLC(X) = LC(X)

(ii) For any topological space (X,τ) LC(X)⊆GLC(X).Also by Remark 2.6(7) and by hypothesis GlC(X)⊆LC(X) and hence GLC(X) = LC(X)

From (i) and (ii) G*αLC(X) = LC(X) = GlC(X)

Definition 3.18: A subset A of (X,τ) is called

(i) g*αlc* if A =S∩G, where S is g*α-open and G is closed in (X,τ).

(ii) g*αlc** if A =S∩G, where S is open and G is g*α-closed in (X,τ).

The class of all g*α-lc*(resp g*α-lc**)sets in topological spaces (X,τ) is denoted by G*αLC*(X) (resp G*αLC**(X))

Prop 3.19: Every lc set is g*αlc* and g*αlc** but not conversely

Proof: It follows from Definition 2.3(1) and Definition 3.18

Example 3.20: Let X ={1,2,3,4} with the topology τ ={∅, X,{2},{3,4},{2,3,4}}. Then the set {2, 3} is g*αlc* and g*αlc** but not lc.

Prop 3.21: Every g*αlc* set is g*αlc but not conversely.

Proof: It follows from Definition 3.18

Example 3.22: Let X = {1, 2, 3} with the topology τ ={∅, X, {1, 3}. Here the set {1, 2} is g*αlc but not g*αlc*

Prop 3.23: Every lsc is g*αlc* and g*αlc** but not conversely

Proof: It follows from Definition 3.18 and Definition 2.3(4)

Example 3.24: Let X ={1,2,3} with the topology τ ={∅, X, {2, 3}. Here the set {2} is g*αlc* and g*αlc** but not lsc

Prop 3.25: Every αlc is g*αlc* and g*αlc** but not conversely

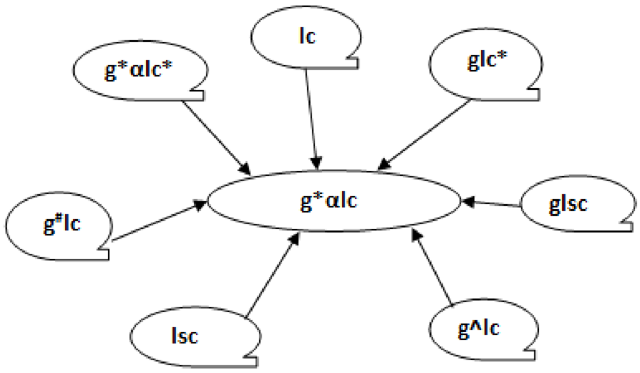
Proof: It follows from Definition 3.18 and Definition 2.3(5)

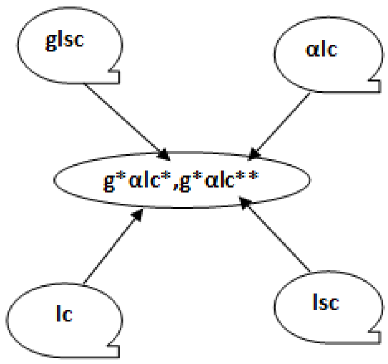
Example 3.26: Let X = {1, 2, 3} with the topology τ={∅, X, {1,2}}. Here the set {1} is g*αlc* and g*αlc** but not αlc

Prop 3.27: Every glsc is g*αlc* and g*αlc** but not conversely

Proof: It follows from Definition 3.18 and Definition 2.3(3)

Example 3.28: Let X ={1, 2, 3, 4} with the topology τ ={∅, X,{1},{1,4},{1,2,4}. Here the set {1, 3} is g*αlc* and g*αlc** but not glsc





Prop 3.29: If G*αO(X) = τ then G*αLC(X) = G*αLC*(X) =G*αLC**(X)

Proof: Since G*αO(X) = τ, (X,τ) is a $_{\alpha}\,T^{**}_{1/2}$-space and hence G*αLC(X) = G*αLC*(X) =G*αLC**(X)

Remark 3.30: The converse of the above proposition need not be true

Let X = {1, 2, 3} with the topology τ ={∅, X, {1}. Here G*αLC(X) = G*αLC*(X) = G*αLC**(X)

However G*αO(X) = {∅, X, {1}, {2}, {3}, {1, 2}, {1, 3}} $\neq \tau$

Theorem 3.31: Assume that G*αC(X) is closed under finite intersections. For a subset A of (X,τ) the following statements are equivalent.

1. A∈ G*αLC(X)
2. A=S∩g*αcl(A) for some g*α –open set S
3. g*α-cl(A)−A is g*α-closed
4. A∪ (g*α-cl(A)^c is g*α-open
5. A⊆g*α-int(A∪(g*α-cl(A))^c)

Proof:
(1)⇒ (2): Let A∈ G*αLC(X).Then A= S∩G where S is g* α-open and G is g* α-closed. Since A⊆ G, g*α-cl(A)⊆ G so S∩g* α-cl(A)⊆A. Also A⊆S and A⊆g*αcl(A) implies A⊆S∩g* α-cl(A) and therefore A=S∩g*α-cl(A)

(2)⇒(3): A=S∩g*αcl(A) implies g*α-cl(A)−A =g*αcl(A)∩S^c which is g*α-closed since S^c is g*α-closed and g*α-cl(A) is g*α closed.

(3)⇒(4): A∪(g*α-cl(A))^c = (g*α-cl(A) –A)^c and by assumption, (g*α-cl(A) –A)^c is g*α-open and So is A∪(g*α-cl(A))^c

(4)⇒(5): By assumption, A∪(g*α-cl(A))^c = g*α-int(A∪(g*α-cl(A))^c) and hence A⊆g*α-int(A∪(g*α-cl(A))^c)

(5)⇒(1): By assumption and since A⊆g*α-cl(A),A=g*α-int(A∪(g*α-cl(A))^c)∩g*α-cl(A). Therefore A∈ G*αLC(X).

Theorem 3.32: For a subset A of (X,τ) the following statements are equivalent

1. A∈ G*αLC*(X)
2. A=S∩cl(A) for some g*α –open set S
3. cl(A)−A is g*α-closed
4. A∪ (cl(A)^c is g*α-open

Proof:
(1)⇒ (2): Let A∈ G*αLC*(X). There exists an g*α-open set S and a closed set G such that A=S∩G.Since A⊆S and A⊆cl(A),A⊆S∩cl(A). Also, since cl(A)⊆ G,S∩cl(A)⊆S∩G =A. Therefore A= S∩cl(A)

(2)⇒ (1): Since S is g*α-open and cl(A) is a closed set A=S∩cl(A) ∈ G*αLC*(X)

(2)⇒ (3): since cl(A)−A =cl(A)∩S^c, cl(A) –A is g*αclosed by corollary 2.7

(3)⇒ (2): Let S =(cl(A))−A)^c. Then S is g*α-open in (X,τ) and A = S∩cl(A)

(3)⇒ (4): Let G = cl(A)−A. Then G^c = A∪ (cl(A))^c and A∪ (cl(A))^c is g*α-open.

(4)⇒ (3): Let S= A∪ (cl(A))^c. Then S^c is g*α-closed and S^c = cl(A)−A and so cl(A)−A is g*α-closed

Theorem 3.33: Let A be a subset of (X,τ). Then A∈ G*αLC**(X) iff A= S∩g*α-cl(A) for some open set S.

Proof: Let A∈ G*αLC**(X). Then A=S∩G where S is open and G is g*α-closed. Since A⊆G, g*α-cl(A)⊆G. We obtain A= A∩ g*α-cl(A) = S∩G∩ g*α-cl(A) = S∩ g*α-cl(A)

Corollary 3.34: Let A be a subset of (X,τ).If A∈ G*αLC**(X) then g*αcl(A))−A is g*α-closed and A∪(g*α-cl(A))^c is g*α-open

Proof: Let A∈ G*αLC**(X).Then by theorem 3.33, A= S∩ g*αcl(A) for some open set S and g*α-cl(A) −A = g*α-cl(A)∩S^c is g*α-closed in (X, τ).If G= g*α-cl(A) –A, then G^c = A∪(g*α-cl(A))^c and G^c is g*α-open and so is A∪(g*α-cl(A))^c

Prop 3.35:
1) If S ∈ LC(X,τ) then S ∈ G*αLC(X,τ), G*αLC*(X,τ), G*αLC**(X,τ)
2) If S ∈ GLC*(X,τ) then S ∈ G*αLC(X,τ), G*αLC*(X,τ), G*αLC**(X,τ)
3) If S ∈ G^LC(X,τ) then S ∈ G*αLC(X,τ), G*αLC**(X,τ)
4) If S ∈ $G^{\#}$ LC(X,τ) then S ∈ G*αLC(X,τ), G*αLC**(X,τ)
5) If S ∈ G*LC(X,τ) then S ∈ G*αLC(X,τ), G*αLC**(X,τ)

The proof is obvious from definitions 2.3, 3.1, 3.18

The converses of the prop 3.35 need not be true as seen from the following example

Example 3.36: Let X = {1,2,3} with the topology τ ={∅,X,{1},{1,3}}. G*αLC(X) = P(X)
G*αLC*(X)=P(X) G*αLC**(X) =P(X).LC ={∅,X,{1},{2},{3},{1,3},{2,3}}. Here A={1,2} and {1,2}∈ G*αLC, G*αLC*, G*αLC**,but {1,2}∉LC

Example 3.37: Let X = {1,2,3,4} with the topology τ ={∅,X,{1},{1,4},{1,2,4}}. G*αLC(X,τ) = P(X)
G*αLC*(X)=P(X), G*αLC**(X)=P(X). Let A={1,3}. Here {1,3}∈ G*αLC, G*αLC*, G*αLC** but {1, 3}∉ GLC*

Example 3.38: Let X = {1,2,3} with the topology τ ={∅,X,{2}}. G*αLC(X,τ) = P(X)
G*αLC**(X) = P(X). Let A={1,2}. Here {1,2}∈ G*αLC*, G*αLC** but {1,2}∉ G^LC,$G^{\#}$ LC

Example 3.39: Let X = {1,2,3} with the topology τ ={∅,X,{1}}. G*αLC(X,τ) = P(X)
G*αLC**(X) = P(X). Let A={2}. Here {1,2}∈ G*αLC*, G*αLC** but {1,2}∉ G*LC

4. g*α-dense sets and g*α-submaximal spaces

We introduce the following definition:

Definition 4.1: A subset A of a space (X,τ) is called g*α-dense if g*α-cl(A) = X

Example 4.2: Consider X = {1, 2, 3} with τ =={∅, X,{1},{1,3}}. Then the set {1, 3} is g*α-dense in (X,τ)

Prop 4.3: Every g*α-dense set is dense.

Proof: Let A be a g*α-dense set in (X,τ). Then g*α-cl(A) =X. since g*α-cl(A)⊆cl(A).We have cl(A) = X and so A is dense.

The converse of prop 4.3 need not be true as seen from the following example.

Example 4.4: Consider X ={1,2,3} with τ={∅, X, {1},{1,2}}.Then the set {1,3} is dense in (X,τ) but it is not g*α-dense in (X,τ)

Definition 4.5: A topological space (X,τ) is called g*α-submaximal if every dense subset in it is g*α-open in (X,τ)

Prop 4.6: Every submaximal space is g*α- submaximal

Proof: Let (X,τ) be a submaximal space and A be a dense subset of (X,τ). Then A is open.But every open set is g*α-open and so A is g*α-open. Therefore (X,τ) is g*α-submaximal.

The converse of prop 4.6 need not be true as seen from the following example.

Example 4.7: Consider X = {1, 2, 3} with τ = {∅,X}.Then G*αO(X) = P(X). We have every dense subset is g*α open and hence (X,τ) is g*α-submaximal. However the set {1,2} is dense in (X,τ) but it is not open in (X,τ).Therefore (X,τ) is not submaximal.

Prop 4.8: Every g- submaximal space is g*α-submaximal

Proof: Let (X,τ) be a g-submaximal space and A be a dense subset of (X,τ). Then A is g-open. But every g-open set is g*α-open and so A is g*α-open.Therefore (X,τ) is g*α-submaximal.

The converse of prop 4.8 need not be true as seen from the following example.

Example 4.9: Consider X ={1,2,3,4} with τ ={∅,X,{1},{2},{1,2},{1,2,3}}.Then G*αO(X) = {∅,X,{1},{2},{3},{1,2}, {1,3},{2,3},{1,2,3},{1,2,4}}. Every dense subset is g*α-open and hence (X,τ) is g*α-submaximal. However the set {1, 2, 4} is dense in (X,τ) but it is not g-open in (X,τ). Therefore (X,τ) is not g- submaximal.

Prop 4.10: Every g*α- submaximal space is rg-submaximal

Proof: Let (X,τ) be a g*α-submaximal space and A be a dense subset of (X,τ). Then A is g*α-open. But every g*α-open set is rg-open and so A is rg-open. Therefore (X,τ) is rg-submaximal.

The converse of prop 4.10 need not be true as seen from the following example

Example 4.11: Consider X ={1,2,3} with τ ={∅, X,{2,3}}.Then G*αO(X) = P(X). we have every dense subset is rg open and hence (X,τ) is rg-submaximal.But the set {1,3} is dense in (X,τ) but it is not g*α-open in (X,τ).Therefore (X,τ) is not g*α-submaximal.

$$\begin{tikzcd}
\textbf{g-submaximal} \arrow[r] & \textbf{g*}\boldsymbol{\alpha}\textbf{-submaximal} \arrow[r] & \textbf{rg-submaximal} \\
 & \textbf{submaximal} \arrow[u] &
\end{tikzcd}$$

Theorem 4.12: A of a space (X,τ) is g*α-submaximal iff P(X) = G*αLC*(X)

Proof: Necessity: Let A∈ P(X) and Let V = A∪$(cl(A))^c$. This implies that cl(V) =cl(A)∪$(cl(A))^c$= X. Hence cl(V)=X. Therefore V is a dense subset of X. Since (X,τ) is g*α-submaximal, V is g*α-open.
Thus A∪$(cl(A))^c$ is g*α-open and by theorem 3.32,we have A∈ G*αLC*(X)

Sufficiency: Let A be a dense subset of (X,τ). This implies A∪$(cl(A))^c$.=A∪X^c =A∪ ∅ =A. Now A∈ G*αLC*(X) implies that A= A∪$(cl(A))^c$ is g*α-open by theorem 3.32. Hence (X,τ) is g*α-submaximal.

Prop 4.13: Assume that G*O(X) forms a topology. For subsets A and B in (X,τ), the following are true.
1. 1.If A,B∈G*αLC(X) then A∩B∈G*αLC(X)
2. If A,B∈G*αLC*(X) then A∩B∈G*αLC*(X)
3. If A,B∈G*αLC**(X) then A∩B∈G*αLC**(X)
4. If A∈ G*αLC(X) and B is g*α-open (resp g*α-closed) then A∩B∈G*αLC(X)
5. If A∈ G*αLC*(X) and B is g*α-open (resp closed) then A∩B∈G*αLC*(X)
6. If A∈ G*αLC**(X) and B is g*α-closed (resp open) then A∩B∈G*αLC**(X)
7. If A∈ G*αLC*(X) and B is g*α-closed then A∩B∈G*αLC(X)
8. If A∈ G*αLC**(X) and B is g*α-open then A∩B∈G*αLC(X)
9. If A∈ G*αLC**(X) and B is G*αLC*(X) then A∩B∈G*αLC(X)

Proof: By Remark 2.6 and corollary 2.7 (1) to (8) hold (9) Let A = S∩G where S is open and G is g*α-closed and B=P∩Q where P is g*α- open and Q is closed. Then A∩B=(S∩P)∩(G∩Q) where S∩P is g*α- open and G∩Q is g*α-closedby corollary 2.7.Therefore A∩B∈G*αLC(X)

Remark 4.14: Union of two g*α- lc* sets need not be an g*α-lc* set. This can be proved by the following example.

Example 4.15: Consider X ={1,2,3} with τ ={∅, X,{1,2}}. We have G*αLC*(X) = {∅, X, {1}, {2}, {3}, {1, 2}}. Then the sets {2} and {3}are g*α-lc* sets but their union{2,3}∉ G*αLC*(X)

5. G*α-Locally closed functions and some of their properties

In this section, the concept of g*α-locally closed functions have been introduced and investigated the relation between g*α-locally closed functions and some other locally closed functions.

Definition 5.1: A function f: (X,τ)→(Y,σ) is called G*αLC-irresolute(resp G*αLC*-irresolute ,G*αLC**-irresolute) if f^{-1}(V)∈ G*αLC(X) (resp f^{-1}(V)∈ G*αLC*(X), f^{-1}(V)∈ G*αLC**(X)) for each V∈G*αLC(X) (resp V∈G*αLC*(Yσ), V∈G*αLC**(Y,σ))

Definition 5.2: A function f: (X,τ)→(Y,σ) is called G*αLC-continuous (resp G*αLC*-continuous, G*αLC**-continuous) if f^{-1}(V)∈ G*αLC(X,τ) (resp f^{-1}(V)∈ G*αLC*(X,τ), f^{-1}(V)∈ G*αLC**(X,τ)) for each open set V of (Y,σ)

Prop 5.3:

(i) If f is LC-continuous then it is G*αLC, G*αLC* and G*αLC**-continuous.
(ii) If f is GLC*-continuous then it is G*αLC, G*αLC* and G*αLC**-continuous.
(iii) If f is G^LC-continuous then it is G*αLC and G*αLC**-continuous.
(iv) If f is $G^{\#}$LC-continuous then it is G*αLC and G*αLC**-continuous.
(v) If f is G*LC-continuous then it is G*αLC and G*αLC**-continuous.

Proof:

(i) Let f: (X,τ)→(Y,σ) be LC-continuous.
To prove f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.
Let V be an open set of (Y,σ)
Since f: (X,τ)→(Y,σ) is LC-continuous, then f^{-1}(V) ∈ LC(X, τ) by prop 3.38(i) f^{-1}(V) ∈ G*αLC(X, τ) (resp G*αLC* and G*αLC**)
Therefore f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.

(ii) Let f: (X,τ)→(Y,σ) be GLC*-continuous
To prove f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.
Let V be an open set of (Y,σ)
Since f: (X,τ)→(Y,σ) is GLC-continuous,then f^{-1}(V) ∈ GLC*(X, τ) by prop 3.38(ii)
f^{-1}(V) ∈ G*αLC(X, τ)(resp G*αLC* and G*αLC**)
Therefore f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.

(iii) Let f: (X,τ)→(Y,σ) be $G^{\#}$ LC-continuous
To prove f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.
Let V be an open set of (Y,σ)
Since f: (X,τ)→(Y,σ) is G^LC-continuous, then f^{-1}(V) ∈ G^LC(X, τ) by prop 3.38(iii)
f^{-1}(V) ∈ G*αLC(X, τ) (resp G*αLC* and G*αLC**)
Therefore f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.

(iv) Let f: (X,τ)→(Y,σ) be $G^{\#}$LC-continuous
To prove f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.
Let V be an open set of (Y,σ)
Since f: (X,τ)→(Y,σ) is $G^{\#}$LC-continuous, then f^{-1}(V) ∈ $G^{\#}$LC(X, τ) by prop 3.38(iv)
f^{-1}(V) ∈ G*αLC(X, τ) (resp G*αLC* and G*αLC**)
Therefore f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.

(v) Let f: (X,τ)→(Y,σ) be G^{*}LC-continuous
To prove f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.
Let V be an open set of (Y,σ)
Since f: (X,τ)→(Y,σ) is G*LC-continuous, then f^{-1}(V) ∈ G^{*}LC(X, τ) by prop 3.38(v)
f^{-1}(V) ∈ G*αLC(X, τ) (resp G*αLC* and G*αLC**)
Therefore f: (X,τ)→(Y,σ) is G*αLC(resp G*αLC* and G*αLC**)-continuous.

Example 5.4: Let X = {1,2,3} =Y with τ={∅,X,{2,3}} and σ = {∅,Y,{2}}.Define f: (X,τ)→(Y,σ) by f(1) =1, f(2)=2 and f(3)=3.LC sets of (X,τ) are {{1},{2,3},∅,X}.G*αLC sets of (X,τ)=P(X).
G*αLC* sets of (X,τ)={∅,X,{1},{2},{3},{2,3}}. G*αLC**sets of (X,τ) = P(X).Here $f^{-1}(\emptyset)$ =∅ ϵG*αLC(X,τ) (resp G*αLC*(X,τ), G*αLC**(X,τ)), $f^{-1}(Y)$ =Y ϵG*αLC(X,τ)(resp G*αLC*(X,τ), G*αLC**(X,τ))for every open set V of (Y,σ).Hence f is G*αLC, G*αLC*and G*αLC**-continuous, Since {2} is an open set of (Y,σ).
But $f^{-1}(\{2\})$ ={2}∉ LC(X,τ)

Example 5.5: Let X = {1,2,3} =Y with τ={∅,X,{1},{1,3}} and σ = {∅,Y,{1,2}}.Define f: (X,τ)→(Y,σ) by f(1) =1, f(2)=2 and f(3)=3.GLC* sets of (X,τ) are {{1},{2},{3},{1,3},{2,3},∅,X}.G*αLC sets of (X,τ)=P(X).
G*αLC* sets of (X,τ)=P(X). G*αLC**sets of (X,τ)=P(X). Here $f^{-1}(\emptyset)$ =∅ ϵG*αLC(X,τ) (resp G*αLC*(X,τ), G*αLC**(X,τ)), $f^{-1}(Y)$ =Y ϵG*αLC(X,τ)(resp G*αLC*(X,τ), G*αLC**(X,τ))for every open set V of (Y,σ). Hence f is G*αLC, G*αLC*and G*αLC**-continuous,Since {1,2} is an open set of (Y,σ).But $f^{-1}(\{1,2\})$ ={1,2}∉ GLC*(X,τ)

Example 5.6: Let X = {1,2,3,4} =Y with τ={∅,X,{1},{1,2},{1,3,4}} and σ = {∅,Y,{1},{1,4},{1,2,4}}.
Define f: (X,τ)→(Y,σ) by f(1) =1,f(2)=2,f(3)=3 and f(4)=4.
G^LC sets of (X,τ)are{{1},{2},{1,2},{3,4},{1,3,4}, {2,3,4} ∅,X}. G*αLC sets of (X,τ)=P(X). G*αLC** sets of (X,τ)=P(X). G*αLC**sets of (X,τ)=P(X).Here $f^{-1}(\emptyset)$=∅ ϵG*αLC(X,τ) (resp G*αLC**(X,τ)), $f^{-1}(Y)$ =Y ϵG*αLC(X,τ) (resp G*αLC**(X,τ)) for every open set V of (Y,σ).Hence f is G*αLC and G*αLC**-continuous, Since {1,2,4} is an open set of (Y,σ). But $f^{-1}(\{1,2,4\})$ ={1,2,4}∉ G^LC(X,τ)

Example 5.7: Let X = {1, 2, 3} =Y with τ={∅,X,{1}} and σ = {∅,Y,{1},{1,3}}.Define f: (X,τ)→(Y,σ) by f(1) =1, f(2)=2 and f(3)=3.$G^{\#}$ LC sets of (X,τ) are {{1}, {2,3},∅, X}. G*αLC sets of (X,τ)=P(X).
G*αLC**sets of (X,τ)=P(X). Here $f^{-1}(\emptyset)$=∅ϵG*αLC(X,τ)(resp G*αLC**(X,τ)), $f^{-1}(Y)$ =Y ϵG*αLC(X,τ) (resp G*αLC**(X,τ)) for every open set V of (Y,σ). Hence f is G*αLC and G*αLC**-continuous, Since {1,3} is an open set of (Y,σ). But $f^{-1}(\{1,3\})$ ={1,3}∉ $G^{\#}$LC(X,τ)

Example 5.8: Let X = {1,2,3} =Y with τ={∅,X,{1}} and σ = {∅,Y,{2},{2,3}}.Define f: (X,τ)→(Y,σ) by f(1) =1, f(2)=2 and f(3)=3.G^{*}LC sets of (X,τ) are {{1},{2,3},∅,X}.G*αLC sets of (X,τ)=P(X).
G*αLC**sets of (X,τ)=P(X).Here $f^{-1}(\emptyset)$ =∅ ϵG*αLC(X,τ)(resp G*αLC**(X,τ)), $f^{-1}(Y)$=Y ϵG*αLC(X,τ) (resp G*αLC**(X,τ)) for every open set V of (Y,σ). Hence f is G*αLC and G*αLC**-continuous, Since {2} is an open set of (Y,σ). But $f^{-1}(\{2\})$ ={2}∉ G^{*}LC(X,τ)

Theorem 4.9: Let f: (X,τ)→(Y,σ) and f: (Y,σ)→(Z,η) be any two functions. Then
(i) g°f is G*αLC-irresolute if f and g are G*αLC-irresolute
(ii) g°f is G*αLC*-irresolute if f and g are G*αLC*-irresolute
(iii) g°f is G*αLC**-irresolute if f and g are G*αLC**-irresolute
(iv) g°f is G*αLC-continuous if f is G*αLC-continuous and g is continuous.
(v) g°f is G*αLC*-continuous if f is G*αLC*-continuous and g is continuous.
(vi) g°f is G*αLC**-continuous if f is G*αLC**-continuous and g is continuous.
(vii) g°f is G*αLC-continuous if f is G*αLC-irresolute and g is G*αLC-continuous
(viii) g°f is G*αLC*-continuous if f is G*αLC*-irresolute and g is G*αLC*-continuous
(ix) g°f is G*αLC**-continuous if f is G*αLC**-irresolute and g is G*αLC**-continuous

Proof:
(i) Given f: (X,τ)→(Y,σ) and g: (Y,σ)→(Z,η) are G*αLC-irresolute.
To prove g°f: (X,τ)→ (Z,η)is G*αLC-irresolute.
Let V∈G*αLC(z, η)
Since g: (Y,σ)→(Z,η) is G*αLC-irresolute,then g^{-1}(V) ∈ G*αLC(Y,σ)
Since f: (X,τ)→(Y,σ) is G*αLC-irresolute,then $f^{-1}(g^{-1}$(V))) ∈ G*αLC(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC(X,τ) for every v∈ G*αLC ∈ (Z,η)
Hence g°f: (X,τ)→(Z,η) is G*αLC-irresolute.

(ii) Given f: (X,τ)→(Y,σ) and g: (Y,σ)→(Z,η) are G*αLC*-irresolute.
To prove g°f: (X,τ)→ (Z,η)is G*αLC*-irresolute.
Let V∈G*αLC*(z, η)
Since g: (Y,σ)→(Z,η) is G*αLC*-irresolute, then g^{-1}(V) ∈ G*αLC*(Y,σ)
Since f: (X,τ)→(Y,σ) is G*αLC*-irresolute, then $f^{-1}(g^{-1}$(V))) ∈ G*αLC*(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC*(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC*(X,τ) for every v∈ G*αLC* ∈ (Z,η)
Hence g°f: (X,τ)→(Z,η) is G*αLC*-irresolute.

(iii) Given f: $(X,\tau)\to(Y,\sigma)$ and g: $(Y,\sigma)\to(Z,\eta)$ are G*αLC**-irresolute.
To prove g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC**-irresolute.
Let V∈G*αLC**(z,η)
Since g: $(Y,\sigma)\to(Z,\eta)$ is G*αLC**-irresolute,then g^{-1}(V) ∈ G*αLC**(Y,σ)
Since f: $(X,\tau)\to(Y,\sigma)$ is G*αLC**-irresolute,then $f^{-1}(g^{-1}(V))$) ∈ G*αLC**(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC**(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC**(X,τ) for every v∈ G*αLC** ∈ (Z,η)
Hence g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC**-irresolute

(iv) Given f: $(X,\tau)\to(Y,\sigma)$ is G*αLC-continuous and g: $(Y,\sigma)\to(Z,\eta)$ is continuous
To prove g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC-continuous
Let V be an open set of (Z,η)
Since g: $(Y,\sigma)\to(Z,\eta)$ is continuous, then g^{-1}(V) is an open set of (Y,σ)
Since f: $(X,\tau)\to(Y,\sigma)$ is G*αLC-continuous,then $f^{-1}(g^{-1}(V))$) ∈ G*αLC(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC(X,τ) for every open set V of (Z,η)
Hence g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC-continuous.

(v) Given f: $(X,\tau)\to(Y,\sigma)$ is G*αLC*-continuous and g: $(Y,\sigma)\to(Z,\eta)$ is continuous
To prove g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC*-continuous
Let V be an open set of (Z,η)
Since g: $(Y,\sigma)\to(Z,\eta)$ is continuous,then g^{-1}(V) is an open set of (Y,σ)
Since f: $(X,\tau)\to(Y,\sigma)$ is G*αLC*-continuous,then $f^{-1}(g^{-1}(V))$) ∈ G*αLC*(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC*(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC*(X,τ) for every open set V of (Z,η)
Hence g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC*-continuous.

(vi) Given f: $(X,\tau)\to(Y,\sigma)$ is G*αLC**-continuous and g: $(Y,\sigma)\to(Z,\eta)$ is continuous
To prove g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC**-continuous
Let V be an open set of (Z,η)
Since g: $(Y,\sigma)\to(Z,\eta)$ is continuous,then g^{-1}(V) is an open set of (Y,σ)
Since f: $(X,\tau)\to(Y,\sigma)$ is G*αLC**-continuous,then $f^{-1}(g^{-1}(V))$) ∈ G*αLC**(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC**(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC**(X,τ) for every open set V of (Z,η)
Hence g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC**-continuous

vii) Given f: $(X,\tau)\to(Y,\sigma)$ is G*αLC-irresolute and g: $(Y,\sigma)\to(Z,\eta)$ is G*αLC –continuous
To prove g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC-continuous
Let V be an open set of (Z,η)
Since g: $(Y,\sigma)\to(Z,\eta)$ is G*αLC- continuous,then g^{-1}(V)∈ G*αLC(Y,σ)
Since f: $(X,\tau)\to(Y,\sigma)$ is G*αLC-irresolute,then $f^{-1}(g^{-1}(V))$) ∈ G*αLC(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC(X,τ) for every open set V of (Z,η)
Hence g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC-continuous.

viii) Given f: $(X,\tau)\to(Y,\sigma)$ is G*αLC*-irresolute and g: $(Y,\sigma)\to(Z,\eta)$ is G*αLC* –continuous
To prove g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC*-continuous
Let V be an open set of (Z,η)
Since g: $(Y,\sigma)\to(Z,\eta)$ is G*αLC*- continuous,then g^{-1}(V)∈ G*αLC*(Y,σ)
Since f: $(X,\tau)\to(Y,\sigma)$ is G*αLC*-irresolute,then $f^{-1}(g^{-1}(V))$ ∈ G*αLC*(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC*(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC*(X,τ) for every open set V of (Z,η)
Hence g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC*-continuous.

ix) Given f: $(X,\tau)\to(Y,\sigma)$ is G*αLC**-irresolute and g: $(Y,\sigma)\to(Z,\eta)$ is G*αLC**–continuous
To prove g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC**-continuous
Let V be an open set of (Z,η)
Since g: $(Y,\sigma)\to(Z,\eta)$ is G*αLC**- continuous,then g^{-1}(V)∈ G*αLC**(Y,σ)
Since f: $(X,\tau)\to(Y,\sigma)$ is G*αLC**-irresolute,then $f^{-1}(g^{-1}(V))$ ∈ G*αLC**(X,τ)
(ie) $(g°f)^{-1}$(V) ∈ G*αLC**(X,τ)
Thus we get $(g°f)^{-1}$(V) ∈ G*αLC**(X,τ) for every open set V of (Z,η)
Hence g°f: $(X,\tau)\to(Z,\eta)$ is G*αLC**-continuous.

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