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Pairwise $s^{**}gO$ - compact spaces in bitopological spaces

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ABSTRACT

*Balachandran [1] introduced the notion of GO-compactness by involving g-open sets. Quite recently, Caldas et al. in investigated this class of compactness and characterized several of its properties. In this paper we introduced a new type of compact spaces called pairwise s**gO - compact spaces and study its properties.*

Keywords: *pairwise s**gO - compact, pairwise pre s**g - closed.*

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1. INTRODUCTION

The notions of compactness is useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness. The productivity and fruitfulness of these notions of compactness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness have been introduced and investigated. Balachandran, Sundaram and Maki [1] introduced a class of compact space called GO-compact space and GO-connected space using g-open cover.

In 1995, sg - compact spaces were introduced by Caldas [3]. According to him, a topological space (X, τ) is called *sg - compact* if every cover of X by sg - open sets has a finite sub cover. Devi, Balachandran and Maki *[11]* defined the same concept and they used the term *SGO* - compactness. Recently, the notions of pairwise *S*GO* - compact spaces were introduced by K.Kannan [8] in bitopological spaces in 2009. In this section we define and study the concept of pairwise *s**gO* - compact spaces in bitopological spaces.

The main focus of this paper is to introduce a new type of compact spaces called pairwise *s**gO* - compact spaces and study its properties.

2. PRELIMINARIES

Definition 2.1 [8]: A bitopological space (X, τ_1, τ_2) is ***pairwise S*GO - compact*** if every pairwise s*g - open cover of X has a finite sub cover .

Definition 2.2: A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is ***pairwise pre semi - closed*** if $f : (X, \tau_1) \to (Y, \sigma_1)$ and $f : (X, \tau_2) \to (Y, \sigma_2)$ are pre semi closed .

Definition 2.3: (X, τ) is called an ***s - normal space*** if given two disjoint closed sets A and B in X , there exist disjoint semi open neighbourhoods U and V of A and B respectively .

Definition 2.4 [6]: A bitopological space (X, τ_1, τ_2) is said to be pairwise compact in the sense of Fletcher, Hoyle and Patty [FHP] (to be abbreviated as **FHP compact**) if every pairwise open cover μ of X has a finite subcover.

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3. PAIRWISE *s**gO* - COMPACT

In the year 2013, *s**g* - closed sets were introduced by K.Kannan [9]. In this section we define and study the concept of pairwise *s**gO* - compact spaces in bitopological spaces.

Definition 3.1: A nonempty collection $\varsigma = \{A_i , i \in I$; *an index set* $\}$ is called a ***pairwise*** s**g **- *open cover*** of a bi space (X, τ_1, τ_2). If $X = \cup A_i$ and $\varsigma \subseteq \tau_1$ - *s**gO* - $(X, \tau_1, \tau_2) \cup \tau_2$ - *s**gO*(X, τ_1, τ_2) and ς contains atleast one member of τ_1 - *s**gO* - (X, τ_1, τ_2) and one member of τ_2 - *s**gO* - (X, τ_1, τ_2) .

Definition 3.2: A bitopological space (X, τ_1, τ_2) is ***pairwise*** *s**gO* **- *compact*** if every pairwise s**g - open cover of X has a finite subcover.

Definition 3.3: A set A of a bitopological space (X, τ_1, τ_2) is ***pairwise*** *s**gO* **- *compact relative to X*** if every pairwise *s**g* - open cover of B by has a finite subcover as a subspace**.**

Example 3.1: Let $\mathcal{C} = \{G_\alpha : \alpha \in \Lambda \}$ be a pairwise *s**g* - open covering for X so that each G_α is a pairwise *s**g* - open set and $X = \cup\{G_\alpha : \alpha \in \Lambda\}$. G_α^c is the complement of G_α is a finite set by definition of cofinite topology . Therefore, $G_\alpha^c = \{x_1, x_2, \dots ,x_n\}$ i.e. a finite set . Now each element of G_α^c is also an element of X whose cover is $\mathcal{C}$ and hence each member of G_α^c is contained in one or other member of G_α . At the most for each $x_i \in G_\alpha^c$ ∃ a set G_{α_i} in $\mathcal{C}$ such that $x_i \in G_{\alpha_i}$. Hence $G_{\alpha_0^c} \subset G_{\alpha_1} \cup G_{\alpha_2} \cup \dots \cup G_{\alpha_n}$. Above relation shows that the finite collection $\mathcal{C}^*$={ $G_{\alpha_0}, G_{\alpha_1}, \dots, G_{\alpha_n}$} is a finite pairwise *s**g* - open covering for X & hence (X, τ_1, τ_2) is pairwise *s**gO* - compact .

Theorem 3.1: If (X, τ_1) and (X, τ_2) are Hausdorff and (X, τ_1, τ_2) is pairwise *s**gO* - compact then $\tau_1 = \tau_2$.

Proof: Let (X, τ_1) and (X, τ_2) be Hausdorff and (X, τ_1, τ_2) is pairwise *s**gO* - compact. Since every *s**gO* - compact space is pairwise compact we have (X, τ_1) and (X, τ_2) are Hausdorff and (X, τ_1, τ_2) is pairwise compact. Let F be τ_1 - closed in X. Then F^c is τ_1 - open in X. Let $\varsigma = \{A_i , i \in I$; *an index set*$\}$ be the τ_2 - open cover for X. Therefore, $\varsigma \cup F^c$ is the pairwise open cover for X . Since X is pairwise compact, $X = F^c \cup A_1 \cup \dots\dots\dots \cup A_n$. Hence $F = A_1 \cup \dots\dots\dots \cup A_n$. Hence F is τ_2 - compact. Since (X, τ_2) is Hausdorff we have F is τ_2 - closed. Similarly, every τ_2 - closed set is τ_1 - closed. Therefore, $\tau_1 = \tau_2$.

Theorem 3.2: If Y is τ_i - *s**g* closed subset of a pairwise *s**gO* - compact space (X, τ_1, τ_2) then Y is τ_j - *s**gO* - compact .

Proof: Let X be a pairwise *s**gO* - compact space. Let $\varsigma = \{A_i, i \in I$; *an index set*$\}$ be the τ_j - *s**g* open cover of Y. Since Y is τ_1 - *s**g* closed subset, Y^c is τ_i - *s**g* open. *Also* $\varsigma \cup A^c = Y^c \cup \{ A_i , i \in I$; *an index set* $\}$ be a pairwise s**g - open cover of X. Since X is pairwise *s**gO* - compact, $X = Y^c \cup A_1 \cup \dots\dots\dots \cup A_n$. Hence $Y = A_1 \cup \dots\dots \cup A_n$. Therefore, Y is τ_j - *s**gO* - compact.

Since every τ_i - *s**g* - closed set is τ_j - closed. We have the following

Theorem 3.3: If Y is τ_i - closed subset of a pairwise *s**gO* - compact space (X, τ_1, τ_2) then Y is τ_j - *s**gO* - compact.

Theorem 3.4: Pairwise *s**g* - continuous image of a pairwise *s**gO* - compact space is pairwise *s**gO* - compact.

Proof: Let (X, τ_1, τ_2) be a pairwise *s**gO* - compact. Let $f : (X , \tau_1 , \tau_2) \xrightarrow{onto} (X^*, \tau_1^*, \tau_2^*)$ be a pairwise *s**g* - continuous. Let $\{G_i\}$ be a pairwise *s**g* - open cover of X^*. $\Rightarrow \{ f^{-1} (G_i) \}$ is pairwise *s**g* - open cover of X. ⇒ ∃ a finite sub cover of X [because X is pairwise *s**gO* - compact] $\{ f^{-1}(G_1), f^{-1}(G_2), \dots , f^{-1}(G_n)\}$. $\Rightarrow G_1, \dots\dots\dots ,G_n$ is a subcover of G_i . ⇒ X^* is pairwise *s**gO* - compact .

Definition 3.4: A bitopological space (X, τ_1, τ_2) is said to be *pairwise s**g - Hausdorff* if for each pair of distinct points x and y of X, there exist $U \in \tau_1$ - *s**gO* and $V \in \tau_j$ - *s**gO* such that $x \in U, y \in V$ and $U \cap V = \phi$.

Definition 3.5: A map $f : X \to Y$ is called $\tau_1\tau_2$ - *s**g continuous* if the inverse image of each $\sigma_1\sigma_2$ - *s**g* closed in Y is τ_2 - closed in X.

Remark 3.1: Pairwise *s**gO* - compact space & pairwise *s**g* - T_2 - space which is not pairwise *s**g* - connected. The following example supports our claim.

Example 3.2: Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$. Let $\varsigma = \{\{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Since (X, τ_1, τ_2) is pairwise $s^{**}gO$ - compact we have X is finite. Since X is pairwise $s^{**}g$ - T_2, $\exists$ a τ_1 - $s^{**}g$ open set $U = \{a\}$ & τ_2 - $s^{**}g$ open set $V = \{b, c\}$ such that $a \in U$, $b \in V$ & $U \cap V = \phi$. $\Rightarrow X = U \cup V$ with $U \cap V = \phi$. Hence X is pairwise $s^{**}g$ - disconnected.

Remark 3.2: A pairwise $s^{**}gO$ - compact subset of a bitopological space X is need not be τ_2 - $s^{**}g$ closed. The following example supports or claim.

Example 3.3: Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a, c\}\}$ and $\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$. Let $\varsigma = \{\{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Let $A = \{a, c\}$. Now $A \subset \{a\} \cup \{b, c\}$. Hence by definition A is pairwise $s^{**}gO$ - compact set. But A is not τ_2 - $s^{**}g$ closed its complement $\{b\}$ is not τ_2 - $s^{**}g$ open.

Remark 3.3: A pairwise $s^{**}gO$ - compact space which is not pairwise $s^{**}g$ - Hausdorff.

Definition 3.6: A function $f : (X, \tau) \to (Y, \sigma)$ is ***pre*** $s^{**}gO$ ***- closed*** if $f(U)$ is $s^{**}g$ - closed in X for every $s^{**}g$ - closed set in Y.

Definition 3.7: A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is ***pairwise pre*** s**g ***- closed*** if $f : (X, \tau_1) \to (Y, \sigma_1)$ and $f : (X, \tau_2) \to (Y, \sigma_2)$ are pre $s^{**}g$ - closed .

Definition 3.8: A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is ***pairwise*** s**g ***- continuous*** if the inverse image of each σ_2 - closed set in Y is $\tau_1\tau_2$ - $s^{**}g$ - closed set in X.

Theorem 3.5: Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be pairwise $s^{**}g$ - continuous, bijective and pairwise pre $s^{**}g$ - closed. Then the image of a pairwise $s^{**}gO$ - compact space under f is pairwise $s^{**}gO$ - compact.

Proof: Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be pairwise $s^{**}g$ - continuous, bijective and pairwise pre $s^{**}g$ - closed. Let X be a pairwise $s^{**}gO$ - compact. Let $\varsigma = \{A_i, i \in I \text{ ; an index set}\}$ be the τ_2 - $s^{**}g$ open cover of Y. Then $Y = \cup A_i$ and $\varsigma \subseteq \sigma_1$ - $s^{**}gO$ - $(X, \tau_1, \tau_2) \cup \sigma_2$ - $s^{**}gO$ - (X, τ_1, τ_2) and ς contains atleast one member of σ_1 - $s^{**}gO(X, \tau_1, \tau_2)$ and one member of σ_2 - $s^{**}gO$ - (X, τ_1, τ_2). Therefore, $X = f^{-1}(\cup A_i) = \cup f^{-1}(A_i)$ and $f^{-1}(\varsigma) \subseteq \tau_1$ - $s^{**}gO$ - $(X, \tau_1, \tau_2) \cup \tau_2$ - $s^{**}gO$ - (X, τ_1, τ_2) and $f^{-1}(\varsigma)$ contains atleast one member of τ_1 - $s^{**}gO$ - (X, τ_1, τ_2) and one member of τ_2 - $s^{**}gO$ - (X, τ_1, τ_2). Therefore, $f^{-1}(\varsigma)$ is the pairwise $s^{**}g$ - open cover of X. Since X is pairwise s**gO - compact, we have $X = f^{-1}(\cup A_i)$, $i = 1$ to n. $\Rightarrow Y = f(x) = \cup A_i$, $i = 1$ to n . Hence ς has the finite subcover. Therefore, Y is the pairwise compact.

Theorem 3.6: If (X, τ_1, τ_2) is pairwise $s^{**}gO$ - compact space then both (X, τ_1) and (X, τ_2) are $s^{**}gO$ - compact.

Proof: To prove (X, τ_1) is $s^{**}gO$ - compact space we must prove for any $s^{**}g$ - open cover of X has a finite sub cover. Let $\{U_i\}$, $i \in \Lambda$ is a $s^{**}g$ - open cover of X, implies $f^{-1}(U_i)$, $i \in \Lambda$ is a $s^{**}g$ - open cover of X, so (X, τ_1) is $s^{**}gO$ - compact. And by the same way we prove (X, τ_2) is $s^{**}gO$ - compact.

Theorem 3.7: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a pairwise $s^{**}g$ - continuous and X is pairwise $s^{**}g$ - connected , then Y is pairwise $s^{**}g$ - connected.

Proof: Suppose that Y is not pairwise $s^{**}g$ - connected. Let $Y = A \cup B$ where A and B are disjoint non-empty σ_i - $s^{**}g$ open & σ_j - $s^{**}g$ open sets in Y. Since f is pairwise $s^{**}g$ - continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty τ_i - open and τ_j - open sets in X. This contradicts the fact that X is $s^{**}g$ -connected. Hence Y is connected.

Proposition 3.8: If A and B are two pairwise $s^{**}gO$ - compact subsets of a bitopological space (X, τ_1, τ_2) then $A \cup B$ is pairwise $s^{**}gO$ - compact subset of X.

Proof: Given A and B are two pairwise $s^{**}gO$ - compact subsets of a bitopological space (X, τ_1, τ_2). We shall prove that $A \cup B$ is pairwise $s^{**}gO$ - compact subset of X, We have to prove that for any pairwise $s^{**}g$ - open cover of $A \cup B$ has a finite sub cover .

Let $\{U_i \ / \ i \in \Lambda \}$ be any pairwise $s^{**}g$ - open cover of $A \cup B$. Then $A \cup B \subseteq \{\cup U_i \ / \ i \in \Lambda\}$ and therefore $A \subseteq \bigcup U_i$ and B $\subseteq \bigcup U_i$, which implies that $\{\bigcup U_i \ / \ i \in \Lambda\}$ is an pairwise $s^{**}g$ - open cover of A and B, where i, j = 1, 2 and i $\neq$ j. But A and B are pairwise $s^{**}gO$ - compact subsets. Therefore there exist $i_1, i_2, \ldots, i_n \in \Lambda$ and

$t_1, t_2, \ldots, t_n \in \Lambda$ such that $\{U_{i_1}, U_{i_2}, \ldots, U_{i_n}\}$ and $\{U_{t_1}, U_{t_2}, \ldots, U_{t_n}\}$ is a finite sub cover of A and B respectively, then $\{U_{i_1}, U_{i_2}, \ldots, U_{i_n}\} \cup \{U_{t_1}, U_{t_2}, \ldots, U_{t_n}\}$ is a finite sub cover of $A \cup B$. Therefore $A \cup B$ is an pairwise *s**gO* - compact subset of X.

Theorem 3.9: Every pairwise *s**gO* - compact subset of a pairwise *s**g* - Hausdorff space is pairwise *s**g* - closed.

Proof: Suppose that A be a pairwise *s**gO* - compact subset of a pairwise *s**g* - Hausdorff space X. Since X is pairwise *s**g* - Hausdorff, the subspace A is pairwise *s**g* - Hausdorff. By hypothesis, A is a pairwise *s**gO* - compact. Hence A is pairwise compact. Let $x \in X - A$. For every $a \in A$ we have $a \neq x$. But X is pairwise *s**g* - Hausdorff. Hence there exist τ_i - *s**g* open nbds U_α of a and a τ_j - *s**g* open nbds V_α of x such that $U_\alpha \cap V_\alpha = \phi$ … (1), where $i, j = 1, 2$ and $i \neq j$. But then the collection $\varsigma = \{U_a : a \in A\}$ is an pairwise *s**g* - open cover of A. But A is pairwise compact. Hence ς has a finite sub collection $\{U_{a_1}, U_{a_2}, \ldots, U_{a_n}\}$ covering A. Put U= $U_{a_1} \cup U_{a_2} \cup \ldots \cup U_{a_n}$. Then U is an τ_i - *s**g* open set with $A \subset U$. Consider the corresponding τ_j - *s**g* open sets $V_{a_1}, V_{a_2}, \ldots, V_{a_n}$. Write $V = V_{a_1} \cap V_{a_2} \cap \ldots \cap V_{a_n}$. Then V is an τ_j - *s**g* open set with $x \in V$. By virtue of (1), $U \cap V = \phi$. $\Rightarrow x \in U \subset X - V \subset X - A \Rightarrow x \in U \subset X - A \Rightarrow X - A$ is τ_i - s**g open $\Rightarrow$ A is τ_i - *s**g* closed. Similarly, A is τ_j - *s**g* closed. Hence A is pairwise *s**g* - closed.

Theorem 3.10: A pairwise *s**g* - closed subset of pairwise *s**gO* - compact space is pairwise *s**gO* - compact relative to X .

Proof: Let A be a pairwise *s**g* - closed subset of a pairwise *s**gO* - compact space X . Then A^c is pairwise *s**g* - open in X. Let S be a cover of A by pairwise *s**g* - open sets in X. Then, $\{S, A^c\}$ is a pairwise *s**g* - open cover of X. Since X is pairwise *s**gO* - compact, it has a finite subcover, say $\{G_1, G_2, \ldots, G_n\}$. If this subcover contains A^C, we discard it. Otherwise leave the subcover as it is. Thus, we have obtained a finite pairwise *s**g* - open subcover of A and so A is pairwise *s**gO* - compact relative to X.

Theorem 3.11: Suppose that A is a pairwise *s**gO* - compact subset of a pairwise *s**g* - Hausdorff space X. Let $x \in X - A$. Then there exist disjoint τ_1 - *s**g* open neighborhood U of A and τ_2 - *s**g* open neighborhood V of X respectively.

Proof: By hypothesis, X is pairwise *s**g* - Hausdorff. Let $a \in A$ arbitrarily. Then there exist disjoint τ_1 - *s**g* open neighborhoods U_a of A and τ_2 - *s**g* open neighborhoods V_x of X respectively. The collection $\mathcal{C} = \{U_a : a \in A\}$ is a pairwise *s**g* - open cover of A. But A is pairwise *s**gO* - compact. Accordingly, this collection $\mathcal{C}$ has a finite sub cover $\{U_{a_1}, U_{a_2}, \ldots, U_{a_n}\}$. Let $U = U_{a_1} \cup \ldots \cup U_{a_n}$. Put $V = V_{a_1} \cap \ldots \cap V_{a_n}$. Then $A \subset U$ and $x \in V$. Also U is τ_1 - *s**g* open and V is τ_2 - *s**g* open. Since $U_{a_i} \cap V_{a_i} = \phi \, for \, 1 \le i \le n$. We obtain that $U \cap V = \phi$. We have proved the result.

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