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CYCLIC PATH COVERS IN ZERO DIVISOR GRAPH

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ABSTRACT

Let R be a commutative ring and let $\Gamma(z_n)$ be the zero divisor graph of a commutative ring R, whose vertices are non-zero zero divisors of z_n , and such that the two vertices u,v are adjacent if n divides uv. In this paper, we have analyzed the maximum number of zero divisor graph $\Gamma(z_n)$, where n is a positive integer, by studying its properties and structure and thereby decomposing it into a finite number of paths and cycles, whose sum of the vertices in the cycle are equal where decomposition is a set of subgraphs $H_1,\ldots H_k$ that partition of the edges of G. That is for all i and j is $_{1\le i\le k} H_i = G$ and $E\ \ H_i \ \cap E\ \ H_j \ = \emptyset$.

***Keywords**: Commutative ring, Zero divisor graph, Decomposition, Cycles.*

1. INTRODUCTION

Let R be a commutative ring and let Z(R) be its set of zero-divisors. We associate a graph $\Gamma(R)$. to R with vertices $Z(R)^* = Z(R) - \{0\}$, the set of non-zero zero divisors of R and for distinct $x, y \in Z(R)^*$, the vertices x and y are adjacent if and only if $xy = 0$ [1, 2, 5, 6]. Throughout this paper, Consider the commutative ring R as z_n. and zero divisor graph $\Gamma(R)$ as $\Gamma(z_n)$. In this paper we are about to decompose [3, 4, 7, 8]. the zero divisor graph into paths and cycles whose sum of the vertices in the cycle are equal . Problems of this type are not only interesting in their own right, but also have potential applications in communication and switching networks. Sometimes it is desirable to decompose a communication or switching network into parts of certain specified types.

Definition 1.1: A graph G is decomposable into $H_1; H_2; : : : ;H_k$ if G has subgraphs $H_1; H_2; : : : ;H_k$ such that

1. each edge of G belongs to one of the H_i's for some i = 1; 2; : : : ; k; and
2. If $i \neq j$, then H_i and H_j have no edges in common.

Definition 1.2: Let G and H be two graphs. A graph G is decomposable into H's if each of the H_i's in the definition above is isomorphic to H.

2. DECOMPOSITION OF ZERO DIVISOR GRAPH

Theorem 2.1: For any $p > 3$, $\Gamma(z_{3p})$ can be decomposed into $(p-1)/2c_4$.

Proof: The proof is based on induction over p.

Case-(i): When $p = 5$,
The vertex set of $\Gamma(z_{15}) = \{3,5,6,9,10,12\}$. That is $|V(\Gamma(z_{15}))| = 6$. Let $u = 3$ and $v = 6$ then 15 does not divide uv. Then there is no path between u and v. Similarly, any two vertices which are multiple of 3 are non adjacent.

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Let $x = 5$ and $y = 15$ then 15 divides both ux and ux. Clearly, ux and uy are adjacent. Then $V(\Gamma(z_{15}))$ can be partitioned into two parts $v_1 = \{5,10\}$. and $v_2 = \{3,6,9,12\}$. Clearly, $\Gamma(z_{15})$ can be decomposed into two c_4. That is, {3, 5, 12, 10, 3} and {9, 5, 6, 10, 9}. Since, 5, 10 are the common points in both the cycles. This cycle is chosen in such a way that the total value of each cycle is $= 3 + 5 + 12 + 10 = 30 = 2(15) = 2(3p)$. Hence the number of $c_4 = 2 = (5-1)/2 = (p-1)/2$.

Case-(ii): When $p = 7$,
The vertex set of $\Gamma(z_{21}) = \{3,6,7,9,12,14,15,18\}$. That is $|V(\Gamma(z_{21}))| = 8$. Let $u = 3$ and $v = 12$ then 21 does not divide v . Clearly, uv are non adjacent. Let $u = 18$, $x = 7$ and $y = 14$, then 21 divides both ux and uy. That is 7, 14 are adjacent to all the remaining vertices. Then $V(\Gamma(z_{21}))$ can be partitioned into two parts $v_1 = \{7,14\}$ an $v_2 = \{3,6,9,12,15,18\}$. Clearly, $\Gamma(z_{21})$ can be decomposed into three c_4. That is cycles are {3, 7, 18, 14, 3}, {6, 7, 15, 14, 6} and {9, 7, 12, 14, 9}. Since, 7, 14 are the common points in all the cycles and these cycles are chosen in such a way that the total value of each cycle is 3 + 7 + 18 + 14 = 42 = 2(21) = 2(3p). The number of $c_4 = 3 = (7-1)/2 = (p-1)/2$. Hence, $\Gamma(z_{21})$ can be decomposed into $(p-1)/2c_4$, where $p = 7$.

Case-(iii): when $p > 7$,
In general, $\Gamma(z_{3p}) = \{3,6,\ldots,3\ p-1\ ,p,2p\}$. that is $|V(\Gamma((z_{3p}))| = p+1$. Using the above two cases $\Gamma(z_{3p})$ can be decomposed into $(p-1)/2c_4$. then the number of c_4. in $k_{2,p-1}$ is $(p-1)/2c_4$.

Theorem 2.2: For any $p > 5$, then $\Gamma(z_{5p})$ can be decomposed into $(p-1)/2c_8$ or $(p-1)c_4$.

Proof: The proof is based on induction over p.

Case-(i): When $p = 7$,
The vertex set of $\Gamma\ z_{35}\ =\ 5,7,10,14,15,20,21,25,28,30$. That is $V\ \Gamma\ z_{35}\ = 10$. Let $u = 5$ and $v = 20$ then 35 does not divide 100. Clearly u and v are non - adjacent. Let $u = 10$ and $X =\ 7,14,21,28$ then 35 divides ux for all $x \in X$. Clearly ux are adjacent. Then $V\ \Gamma\ z_{35}$ can be partitioned into two parts $V_1 =\ 7,14,21,28$ and $V_2 = \{5,10,15,20,25,30\}$. Clearly, $\Gamma\ z_{35}$ is $k_{4,6}$ and this $k_{4,6}$ can be decomposed into six c_4. That is, {5, 7, 30, 28, 5},{15, 7, 20, 28, 15}, {25, 7, 10, 28, 25}, {5, 14, 30, 21, 5}, {15, 14, 20, 21, 15}, {25, 14, 10, 21, 25}. Clearly, 7, 28 are the common points in three cycles and 17, 21 are the common points in the remaining three cycles. These cycles are chosen in such a way that the total value of each cycle is $= 5 + 7 + 30 + 28 = 70 = 2\ 35 = 2\ 7p$. Then number of $c_4 = 6 = \frac{7-1}{2} = \frac{p-1}{2}$. Similarly it can be decomposed into $\{5,7,30,28,5,14,30,21,5\}$, $\{15,7,20,28,15,14,20,21,15\}$, $\{25,7,10,28,25,14,10,21,25\}$ with the total value of each cycle is $= 5 + 7 + 30 + 28 + 5 + 14 + 30 + 21 = 140 = 4(35) = 4(5p)$. Then the number of C_8 is $3 = (7-1)/2 = (p-1)/2$. Therefore, Γ (Z35) can be decomposed into $(p-1)/2c_8$ or $(p-1)c_4$.

Case-(ii): When $p = 11$.
The vertex set of $\Gamma(z_{55}) = \{5,10,15..,50,11,..,44\}$. That is $|V(\Gamma(z_{35}))| = 14$. Let $u = 5$ and $v = 30$ then 55 does not divide 150. That is, u and v are non-adjacent. Let $u = 30$ and $X = \{11,22,33,44\}$ then 55 divides ux for all $x \in X$. Clearly ux are adjacent. Then $V(\Gamma(z_{55}))$ can be partitioned into two parts $V_1 = \{11, 22, 33, 44\}$ and $V_2 = \{5,10,15,20,25,30,35,40,45,50\}$. Clearly, $\Gamma(Z_{55})$ is $k_{4,10}$ and this $k_{4,10}$ can be decomposed into ten c_4. That is, {5, 11, 50, 44, 5}, {10, 11, 45, 44, 10}, {15, 11, 40, 44, 15}, {20, 11, 35, 44, 20}, {25, 11, 30, 44, 25}, {5, 22, 50, 33, 5}, {10, 22, 45, 33, 10}, {15, 22, 40, 33, 15}, {20, 22, 35, 33, 20},{25, 22, 30, 33, 25}. Clearly, 11, 44 are the common points in five cycles and 22, 33 are the common points in the remaining five cycles. These cycles are choosen in such a way that the total value of each cycle is $5 + 11 + 50 + 44 = 110 = 2(55) = 2(5p)$.Then number of $c_4 = 10 = (11-1)/2 = (p-1)/2$ or it can be decomposed into {5, 11, 50, 44, 5, 22, 50, 33, 5}, {10, 11, 45, 44, 10, 22, 45, 33, 10}, {15, 11, 40, 44, 15, 22, 40, 33, 15}, {25, 11, 30, 44, 25, 22, 30, 33, 25}, {20, 11, 35, 44, 20, 22, 35, 33, 20} with the total value of each cycle is $= 5 + 11 + 50 + 44 + 5 + 22 + 50 + 33 = 220 = 4(55) = 4(5p)$. Then the number of C_8 is $5 = (11-1)/2 = (p-1)/2$ Therefore, $\Gamma(Z_{55})$ can be decomposed into $(p-1)/2c_8$ or $(p-1)c_4$.

Case-(iii): When $p > 11$
In general, $\Gamma(z_{5p}) = \{5,10,\ldots,5(p-1),p,2p,3p,4p\}$. That is $|V(\Gamma(z_{5p}))| = p+3$. Using the above two cases the $V(\Gamma(z_{5p}))$can be partitioned into two parts $V_1 = \{p,2p,3p,4p\}$ and $V_2 = \{5,10,\ldots,5(p-1)\}$. Clearly, $\Gamma(z_{5p})$ is $k_{4,p-1}$ with the common points $\{p,2p,3p,4p\}$ which is adjacent to all the remaing vertices. Then the number of cycles in $k_{4,p-1}$ is $(p-1)/2\ c_8$ or $(p-1)c_4$.

Theorem 2.3: For any $p > 7$, $\Gamma(z_{7p})$ can be decomposed into $(3(p-1)/2)c_4$or $(p-1)c_6$

Proof: The proof is based on induction over p.

Case-(i): When $p = 11$,
The vertex set of $\Gamma(z\_77) = \{7, 14, 21, .., 70, 11, .., 66\}$. That is $|V(\Gamma(z_{77}))| = 16$. Let $u = 7$ and $v = 35$ then 77 does not divide 245. Clearly u and v are nonadjacent. Let $u = 21$ and $X = 11, 22, 33, 44, 55, 66$ then 77 divides ux for all $x \in X$. Clearly ux are adjacent. Then $V\ \Gamma\ z_{77}$ can be partitioned into two parts $V_1 = 11, 22, 33, 44, 55, 66$ and $V_2 = 7, 14, 21, 28, 35, 42, 49, 56, 63, 70$. Clearly, $\Gamma\ z_{77}$ is $k_{4,10}$ and this $k_{4,10}$ can be decomposed into fifteen c_4. That is,{7, 11, 70, 66, 7}, {14, 11, 63, 66, 14}, {21, 11, 56, 66, 21}, {28, 11, 49, 66, 28}, {35, 11, 42, 66, 35}, {7, 22, 70, 55, 7},{14, 22, 63, 55, 14}, {21, 22, 56, 55, 21},{28, 22, 49, 55, 28} ,{35, 22, 42, 55, 35}, {7, 33, 70, 44, 7}, {14, 33, 63, 44, 14}, {21,33, 56, 44, 21} {7, 33,70, 44,7} {14, 33, 63, 44, 14} {21, 33, 56, 44, 21}{28, 33, 49, 44, 28} and $\{35, 33, 42, 44, 35\}$. Clearly, V_1 are the common vertices in the cycles and these cycles are chosen in such a way that the total value of each cycle is $7 + 11 + 70 + 66 = 154 = 2(77) = 2(7p)$. Then number of $c_4 = 15 = 3(11 - 1)/2 = 3(p - 1)/2$ or $k_{4,10}$ can be decomposed into $10c_6$, That is, {7, 11, 70, 66, 7, 22, 70, 55, 7}, {14, 11, 63, 66, 14, 22, 63, 55, 14}, {21, 11, 56, 66, 21, 22, 56, 55, 21},{28, 11, 49, 66, 28,22, 49, 55, 28}{35, 11, 42, 66, 35,22, 42,55, 35}, {7, 11, 70, 66, 7, 33, 70, 44, 7}, {14, 11, 63, 66, 14, 33, 63, 44, 14}, {21, 11, 56, 66, 21, 22, 56, 55, 21}{28, 11, 49, 66, 28,33, 49, 44, 28} ,{35, 11, 42, 66, 35, 33, 42, 44, 35},.Then the number of c_6 is $10 = 11 - 1 = p - 1$ Therefore, $\Gamma(z_{77})$ can be decomposed into $(3(p-1)/2)c_4$ or $(p - 1)c_6$.

Case-(ii): When $p = 13$.
The vertex set of $\square(z_{91}) = \{7, 14, .., 84, 13, 26, ..78\}$ That is $|V(\Gamma(z_{91}))| = 18$. Let $u = 7$ and $v = 35$ then 91 does not divide 245. Clearly u and v are non-adjacent. Let $u = 21$ and $X = \{13, 26, 39, 52, 65, 78\}$ then 91 divides ux for all $x \in X$. Clearly ux are adjacent for all $x \in X$. Then $V\ \Gamma\ (z_{91})$ can be partitioned into two parts $V_1 = \{13, 26, 39, 52, 65, 78\}$ and $V_2 = \{7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84\}$. Clearly, $\Gamma(z_{91})$ is $k_{4,12}$ and this $k_{4,12}$ can be decomposed into eighteen c_4.

That is, {7, 13, 84, 78, 7}, {14,13,77,78,7}, {21,13,70,78,21}, {28,13,63,78,28}, {35,13,56,78,35},{42, 13, 49, 78, 42}, {7, 26, 84, 65, 7}, {14,26,77,65,7}, {21,26,70,65,21}, {28,26,63,65,28}, {35,26,56,65,35},{42, 26, 49, 65, 42},{7, 39, 84, 52, 7}, {14,32,77,52,7}, {21,39,70,52,21},{28,39,63,52,28}, {35,39,56,52,35},{42, 39, 49, 52, 42}. Clearly, V_1 are the common vertices in the cycles and these cycles are chosen in such a way that the total value of each cycle is $= 7 + 13 + 84 + 78 = 185 = 2(91) = 2(7p)$. Then number of $c_4 = 18 = 3(13 - 1)/2 = 3(p - 1)/2$ or $k_{4,12}$ can be decomposed into $12c_6$, That is, $\{7, 13, 84, 78, 7, 26, 84, 65, 7\}$, {14,13,77,78,7,26,77,65,7}, {21,13,70,78,21,26,70, 65, 21}, {28,13,63,78,28,26,63,65,28}, {35,13,56,78,35,39,56,52,35}, {42,13,49,78,42,39,49,52} $\{7, 13, 84, 78, 7, 39, 84, 52, 7\}$, {14,13,77,78,7,39,77,52,7}, {21,13,70,78,21,39,70,52,21}, {28,13,63,78,28,39,63,52,28},{35,13,56,78,35,39, 56,52,35} ,{42,13,49,78,42,39,49,52,42}. Then the number of c_6 is $12 = 13 - 1 = p - 1$ Therefore, $\Gamma(z_{91})$ can be decomposed into $(3(p-1)/2)c_4$ or $(p - 1)c_6$

Case-(iii): When $p > 13$.
In general, $\Gamma(z_{7p}) = \{7, 14, \dots, 7(p-1), p, 2p, .., 6p\}$. That is $|V(\Gamma(z_{7p})| = p + 5$. Using the above two cases the $\Gamma(z_{7p})$ can be partitioned into two parts $V_1 = \{p, 2p, 3p, 4p, 5p, 6p\}$ and $V_2 = \{77, 14, \dots, 7(p - 1)\}$. Clearly, $\Gamma(z_{7p})$ is $k_{6,p-1}$ with the common points $\{p, 2p, 3p, 4p, 5p, 6p\}$ which is adjacent to all the remaining vertices. Then the number of cycles in $k_{6,p-1}$ is $(3(p-1)/2)c_4$ or $(p - 1)c_6$

Theorem 2.4: FOR any distinct prime p and q, $\Gamma(z_{pq})$ can be decomposable into $(q - 1)$ $Cp-1$, where $q > p$.

Proof: The vertex set of $\Gamma(z_{pq})$ is $\{p, 2p, 3p, ..(p-1)p, q, 2q, ..(p-1)q\}$. That is $|V(\Gamma(z_{pq}))| = p + q - 2$. Using the above theorem $\Gamma(z_{pq})$ is $k_{p-1,q-1}$ for $p > 3$, with the common points $\{q, 2q, ..(p - 1)q\}$ which is adjacent to all the remaining vertices. Therefore the number of cycles in $k_{p-1,q-1}$ is $(q - 1)c_{p-1}$.

Theorem 2.5: For any prime $p > 4$, $\Gamma(z_{4p})$ can be decomposable into $k_{1,2(p-1)}$ and $k_{2,p-1}$ or $(p-1)/2c_4$.

Proof: The proof is based on induction over p.

Case-(i): When $p = 5$,
The vertex set of $\Gamma(z_{20}) = \{2, 4, 6, ..18, 5, 10, 15\}$. That is $|V(\Gamma(z_{20}))| = 11$. Let u = 4 and v = 10 then 20 divides uv. Clearly u and v are adjacent. Let $X = \{5, 15\}$ and $v = 10$ then 20 divides vxfor all $x \in X$. Clearly ux are non-adjacent. Then first $\Gamma(z_{20})$is decomposed into $k_{1,8}$. Consider $u = 4$ and $v = 5$ then 20 divides uv. That is u and v are adjacent. Then the remaining vertices can be partitioned into two parts $V_1 = \{4, 8, 12, 16\}$ and V2 = {5, 15} which are adjacent to each other. Secondly, $\Gamma(z_{20})$is further decomposed into $k_{2,4}$ Futher it can be decomposed into two cycles of c_4, that is {4, 5, 16, 15, 4} and {8, 5, 12, 15, 8}. These cycles are chosen in such a way that the total value of each cycle is $40 = 2(20) = 2(5p)$. Finally, this implies $\Gamma(z_{20})$can be completely decomposed into $k_{1,8}$ and $k_{2,4}$ or $2c_4$.

Case-(ii): When p = 7.
The vertex set of $\Gamma(z_{28}) = \{2, 4, 6, ..26, 7, 14, 21\}$. That is $|V(\Gamma(z_{28})| = 15$ Let $u = 6$ and $v = 14$ then 28 divides uv. Clearly u and v are adjacent. Let $X = \{7, 21\}$ and $v = 14$ then 24 divides vx. Clearly ux are non-adjacent for all $x \in X$. Then first $\Gamma(z_{28})$is decomposed into $k_{1,12}$. Consider $u = 8$ and $v = 7$ then 24 divides uv. That is u and v are adjacent. Then the remaining vertices can be partitioned into two parts $V_1 = \{4, 8, 12, 16, 20, 24\}$ and $V_2= \{7, 21\}$ which are adjacent to each other. Secondly, $\Gamma(z_{28})$ is further decomposed into $k_{2,6}$ which is further decomposed into three cycles of c_4, that is {4, 7, 24, 14, 4} ,{8, 7, 20, 14, 8} and {12, 7, 16, 14, 12}. These cycles are chosen in such a way that the total value of each cycle is $56 = 2(28) = 2(4p)$. Finally, this implies $\Gamma\ z_{28}$ can be completely decomposed into $k_{1,12}$ and $k_{2,6}$ or $3c_4$.

Case-(iii): When $p > 7$.
In general, $V((z_{4p})) = \{2, 4, 6 \dots 2(p-1), p, 2p, 3p\}$. That is$|V(\Gamma(z_{4p})| = 2p + 1$. Using the above cases $\Gamma(z_{4p})$can be decomposed into $k_{1,2(p-1)}$ and $k_{2,p-1}$ which can be further decomposed into $(p-1)/2$ cycles of length 4 that is $(p-1)/2\ c_4$.

Theorem 2.6: For any prime $p > 6$, $\Gamma(z_{6p})$can be decomposed into $k_{1,3(p-1)}$, $k_{2,p-1}$ and $k_{2,2(p-1)}$ or $3(p-1)/2\ c_4$.

Proof: The proof is based on induction over p.

Case-(i): When $p = 7$.
The vertex set of $\Gamma(z_{42}) = \{2, 3, 4, ...40, 7, 14, 21, 28, 30\}$. That is $|V(\Gamma(z_{42})| = 29$ Let u = 21 and let X be the set consisting of multiple of 2 other than multiple of 3. Then ux is divided by the 42.That is ux is adjacent. Then, $\Gamma(z_{42})$ is first decomposed into $k_{1,20}$ From the remaining vertices consider the set $V_1 = \{14, 28\}$ and $V_2 = \{3,9,\dots,39,6,12,\dots 36\}$. Clearly any vertex of V_1 is adjacent to any vertex of V_2. Then, $\Gamma(z_{42})$ is decomposed into $k_{2,12}$. Further it can be decomposed into six cycles of c_4. That is {14,3,28,39,14}, {14,9,28,33,14}, {14,15,28,27,14}, {14,6,28,36,14}, {14,12,28,30,14},{14,18,28,24,14}. These cycles are chosen in such a way that the sum of the vertices is 98 .Finally the remaining vertices can be partitioned into $V_3 = \{7, 35\}$ and $V_4 = \{6,12,\dots 36\}$. Clearly any vertex of V_3 is adjacent to any vertex of V_4 . Then, $\Gamma(z_{42})$ is decomposed into $k_{2,6}$. Further it can be decomposed into three cycles of c_4. That is {7,6,35,36,7},{7,12,35,30,7},{7,18,35,24,7}. These vertices is chosen in such a way that the sum of the vertices is 91. Finally, this implies $\Gamma\ z_{42}$ can be completely decomposed into $k_{1,20}$, $6c_4$ and $3c_4$. That is $k_{1,20}$, $9c_4$.

Case-(ii): When $p = 11$.
The vertex set of $\Gamma(z_{77}) = \{2, 3, 4, ...60, 11,22,33,44,55\}$. That is $|V(\Gamma(z_{77})| = 45$. Let u = 33 and let X be the set consisting of multiple of 2 other than multiple of 3. Then ux is divided by the 77. That is ux is adjacent.Then , $\Gamma(z_{42})$ is first decomposed into $k_{1,32}$. From the remaining vertices consider the set $V_1=\{22, 44\}$ and $V_2=\{3, 9,\dots,63,6,12,\dots 60\}$. Clearly any vertex of V_1 is adjacent to any vertex of V_2. Then, $\Gamma(z_{42})$ is decomposed into $k_{2,20}$. Further it can be decomposed into ten cycles of c_4. That is {22,3,44,63,22}, {22,9,44,57,22}, {22,15,44,51,22}, {22,21,44,45,22}, {22,27,44,39,22},{22,6,44,60,22},{22,12,44,54,22},{22,18,44,48,22},{22,24,44,42,22},{22,30,44,46,22}.These cycles are chosen in such a way that the sum of the vertices is $154 = 2(77) = 2(6p)$.Finally the remaining vertices can be partitioned into $V_3 = \{11, 55\}$ and $V_4 = \{6,12,\dots 60\}$. Clearly any vertex of V_3 is adjacent to any vertex of V_4 . Then , $\Gamma(z_{42})$ is decomposed into $k_{2,6}$. Further it can be decomposed into three cycles of c_4. That is {11,6,55,60,11}, {11,12,55,54,11},{11,18,55,48,11},{11,24,55,42,11},{11,30,55,36,11}. These vertices are chosen in such a way that the sum of the vertices is 143. Finally, this implies $\Gamma\ z_{42}$ can be completely decomposed into $k_{1,32}$, $10c_4$ and $5c_4$. That is $k_{1,32}$, $15c_4$.

Case-(iii): When $p > 11$.
In general, $V((z_{6p})) = \{2, 4, 6 \dots 2(3p-1), 3, 6 \dots .3(2p-1), p, 2p, 3p, 4p, 5p\}$. That is $|V(\Gamma(z_{4p})| = 4p + 1$. Using the above cases $\Gamma(z_{6p})$can be decomposed into $k_{1,3p-1}$ $k_{2,2(p-1)}$ and $k_{2,p-1}$ which can be further decomposed into $(p-1)/2$ cycles and $(p-1)$ cycles of length 4 that is $\frac{p-1}{2} + p - 1$ of c_4 . That is $3(p-1)/2\ c_4$.

Theorem 2.7: For any prime p, $\Gamma\ z_{p^2}$ is not decomposable into Hamilton cycles.

Proof: The vertex set of $\Gamma\ z_{p^2}$ is $\{p, 2p, 3p, .. (p-1)p\}$. Clearly p is adjacent to all the vertices in $V(\Gamma(z_{p^2}))$. Also note that any two vertices in $\Gamma(z_{p^2})$ is adjacent and hence $\Gamma(z_{p^2})$is a complete graph. Clearly each vertices of $\Gamma(z_{p^2})$has degree $p - 2$. Let q be the number of edges of $\Gamma(z_{p^2})$ then $q = (p-1)(p-2)/2$. Suppose $\Gamma(z_{p^2})$is decomposable into Hamilton cycles C_{p-1}. Since each C_{p-1} has $p-1$ edges. So the number of such cycles in the decomposition must be $q(p-1) = (p-1)(p-2)/2(p-1) = (p-2)/2 = p/2 - 1$, which is not an integer for any prime p. Hence this is impossible so $\Gamma(z_{p^2})$is not decomposable into Hamilton cycles.

Theorem 2.8: For any prime $p > 2$, $\Gamma(z_{p^2})$is decomposable into $(p-1)/2$ Hamilton paths P_{p-1}

Proof: Using the above theorem, $\Gamma(z_{p^2})$is a complete graph, where p is any prime and $|V(\Gamma(z_{p^2})| = p-1$. Since, for any $n \geq 1$, k_{2n+1} is decomposable into Hamilton cycles. Consider $\Gamma(z_{7^2})$, the vertices set of $\Gamma(z_{7^2})$, is {7, 14, 21, 28, 35, 42} that is $\{p, 2p, 3p, .. (p-1)p\}$ where $p = 7$. Label the vertices x_1, x_2, x_3, x_4, x_5, x_6 and form the Hamilton path as follows

Path 1: $x_1, x_5, x_3, x_6, x_4, x_2$ (P_6)
Path 2: $x_3, x_1, x_2, x_6, x_5, x_4$ (P_6)
Path 3: $x_6, x_1, x_4, x_3, x_2, x_5$ (P_6)

Hence $\Gamma(z_{p^2})$ is decomposable into $(p-1)/2$ Hamilton path P_{p-1}

For example in $\Gamma(z_{5^2})$, the number of vertices are $\{5, 10, 15, 20\}$.

For $p = 5$, $p - 1/2 = 5 - 1/2 = 4/2 = 2$ Hamilton path P_4

3. CONCLUSION

In the zero divisor graph $\Gamma\ z_n$ where n is any positive integer, the number of cycles of c_4 that has been decomposed for $n = 3p, 4p, 5p, 6p, 7p$ forms an inequality $o(D(\Gamma(z_{3p}))) \leq o(D(\Gamma(z_{4p}))) < o(D(\Gamma(z_{5p}))) < o(D(\Gamma(z_{6p}))) \leq o(D(\Gamma(z_{7p})))$ where $o(D(\Gamma(z_n)))$ is known as the number of cycles of c_4, while the complete graph $\Gamma(z_{p^2})$ is only decomposable into Hamilton path P_{p-1} but not has Hamilton cycles.

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