

## OPERATION ON EDGE REGULAR INTUITIONISTIC FUZZY GRAPHS

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### **ABSTRACT**

**In this paper, degree of an edge in cartesian product, composition and alpha product of two intuitionistic fuzzy graphs are defined. Edge regular property of cartesian product, composition and alpha product of two intuitionistic fuzzy graphs are discussed.**

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### **1. INTRODUCTION**

Some operations on fuzzy graphs were introduced by John N. Modernson and Peng [4], R. Parvathi , M.G. Karunambigai and K.T. Atanassov[10] introduced operations on intuitionistic fuzzy graphs. Operations on Regular intuitionistic fuzzy graphs were studied by Ismail Mohaideen and *et al.*, [3]. M.G. Karunambigai et.al.,[5] introduced the concept of edge regular intuitionistic fuzzy graph. A. Nagoor Gani and B. Fathima Kani [6] introduced alpha product in fuzzy graphs. S. Ravi Narayanan and S. Murugesan[12] discussed the properties of edge irregular intuitionistic fuzzy graphs. S. Ravi Narayanan and S. Murugesan[13] introduced pseudo degree of a vertex, pseudo total degree of a vertex in an intuuitionistic fuzzy graphs. These motivates us to discuss some properties of cartesian product, composition and alpha product of edge regular intuitionistic fuzzy graphs.

Throughout this paper, the Graph  $G_1$  takes the membership value  $((\mu_1, \gamma_1), (\mu_2, \gamma_2))$  and the Graph  $G_2$  takes the membership value  $((\mu'_1, \gamma'_1), (\mu'_2, \gamma'_2))$

### **2. PRELIMINARIES**

In this section we see the definitions related to fuzzy graphs and intuitionistic fuzzy graphs

**Definition 2.1:** [1] An intuitionistic fuzzy graph with an underlying set  $V$  is defined to be a pair  $G = (V, E)$  where

- (i)  $V = \{v_1, v_2, v_3, \dots, v_n\}$  such that  $\mu_i: V \rightarrow [0, 1]$  and  $\gamma_i: V \rightarrow [0, 1]$  denote the degree of membership and non membership of the element  $v_i \in V$ , ( $i = 1, 2, 3, \dots, n$ ) , such that  $0 \leq \mu_i(v_i) + \gamma_i(v_i) \leq 1$
- (ii)  $E \subseteq V \times V$  where  $\mu_2: V \times V \rightarrow [0, 1]$  and  $\gamma_2: V \times V \rightarrow [0, 1]$  are such that  
 $\mu_2(v_i, v_j) \leq \min\{\mu_i(v_i), \mu_j(v_j)\}$  and  $\gamma_2(v_i, v_j) \leq \max\{\gamma_i(v_i), \gamma_j(v_j)\}$  and  $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$  , for every  $(v_i, v_j) \in E$ , ( $i, j = 1, 2, \dots, n$ ).

**Definition 2.2:** [4] Let  $G: (A,B)$  be an intuitionistic fuzzy graph on  $G^*(V,E)$  .Then the degree of a vertex  $v_i \in G$  is defined by  $d(v_i) = (d\mu_i(v_i), d\gamma_i(v_i))$ ,where  $d\mu_i(v_i) = \sum \mu_2(v_i, v_j)$  and  $d\gamma_i(v_i) = \sum \gamma_2(v_i, v_j)$  , for  $(v_i, v_j) \in E$  and  $\mu_2(v_i, v_j) = 0$  and  $\gamma_2(v_i, v_j) = 0$  for  $(v_i, v_j) \notin E$ .

**Definition 2.3:** [3] Let  $G: (A,B)$  be an intuitionistic fuzzy graph and let  $e_{ij} \in B$  be an edge of  $G$ . Then the degree of an edge  $e_{ij}$  is defined as  $d_\mu(e_{ij}) = d_\mu(v_i) + d_\mu(v_j) - 2\mu_2(e_{ij})$  and  $d_\gamma(e_{ij}) = d_\gamma(v_i) + d_\gamma(v_j) - 2\gamma_2(e_{ij})$  and the edge degree of  $G$  is  $d(e_{ij}) = (d_\mu(e_{ij}), d_\gamma(e_{ij}))$  .

**Definition 2.4:** [3] Let  $G: (A,B)$  be an intuitionistic fuzzy graph and let  $e_{ij} \in B$  be an edge of  $G$ .  $G$  is said to be edge regular intuitionistic fuzzy graph if all the edges of  $G$  have the same degree.

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**Definition 2.5:** [5] Let  $G_1 : (A_1, B_1)$  and  $G_2 : (A_2, B_2)$  where  $A_1 = (\mu_1, \gamma_1), B_1 = (\mu_2, \gamma_2), A_2 = (\mu'_1, \gamma'_1), B_2 = (\mu'_2, \gamma'_2)$  be two intuitionistic fuzzy graph on  $G^* : (V, E)$ . Then the cartesian product of  $G_1$  and  $G_2$  is defined as  $G = G_1 \times G_2 : (V, E)$  where  $V = V_1 \times V_2$  and  $E = \{(u_1, u_2)(v_1, v_2) : u_1 = v_1 \& u_2 v_2 \in E_2 \text{ or } u_2 = v_2 \& u_1 v_1 \in E_1\}$  with  $((\mu_1 \times \mu'_1)(\gamma_1 \times \gamma'_1))(u_1, u_2) = \{\min(\mu_1(u_1), \mu'_1(u_2)), \max(\gamma_1(u_1), \gamma'_1(u_2))\}$ , for every  $(u_1, u_2) \in E$

$$(\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(v_1, v_2)) = \begin{cases} \{\min(\mu_1(u_1), \mu'_2(u_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, v_2))\} & u_1 = v_1, u_2 v_2 \in E_2 \\ \{\min(\mu'_1(u_2), \mu_2(u_1, v_1)), \max(\gamma'_1(u_2), \gamma_2(u_1, v_1))\} & u_2 = v_2, u_1 v_1 \in E_1 \\ 0 & \text{otherwise} \end{cases}$$

**Definition 2.5:** [5] Let  $G_1 : (A_1, B_1)$  and  $G_2 : (A_2, B_2)$  where  $A_1 = (\mu_1, \gamma_1), B_1 = (\mu_2, \gamma_2), A_2 = (\mu'_1, \gamma'_1), B_2 = (\mu'_2, \gamma'_2)$  be two intuitionistic fuzzy graph on  $G^* : (V, E)$ . Then the composition of two intuitionistic fuzzy graphs  $G_1$  and  $G_2$  is defined as  $G = G_1 \circ G_2 : (V, E)$  where  $V = V_1 \times V_2$  and  $E = \{(u_1, u_2)(v_1, v_2) : u_1 = v_1 \& u_2 v_2 \in E_2 \text{ or } u_2 = v_2 \& u_1 v_1 \in E_1\}$  with  $((\mu_1 \times \mu'_1)(\gamma_1 \times \gamma'_1))(u_1, u_2) = \{\min(\mu_1(u_1), \mu'_1(u_2)), \max(\gamma_1(u_1), \gamma'_1(u_2))\}$ , for every  $(u_1, u_2) \in E$

$$(\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(v_1, v_2)) = \begin{cases} \{\min(\mu_1(u_1), \mu'_2(u_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, v_2))\} & u_1 = v_1, u_2 v_2 \in E_2 \\ \{\min(\mu'_1(u_2), \mu_2(u_1, v_1)), \max(\gamma'_1(u_2), \gamma_2(u_1, v_1))\} & u_2 = v_2, u_1 v_1 \in E_1 \\ \{\min(\mu'_1(u_2), \mu'_2(v_2), \mu_2(u_1, v_1)), \max(\gamma'_1(u_2), \gamma'_2(v_2), \gamma_2(u_1, v_1))\} & u_2 \neq v_2, u_1 v_1 \in E_1 \end{cases}$$

**Definition 2.5:** [5] Let  $G_1 : (\sigma_1, \mu_1)$  and  $G_2 : (\sigma_2, \mu_2)$  be two fuzzy graphs. Then the alpha product of two fuzzy graphs  $G_1$  and  $G_2$  is denoted by  $G_1 \times_{\alpha} G_2$  and defined as  $(\sigma_1 \times_{\alpha} \sigma_2)(u_1, u_2) = \sigma_1(u_1) \Lambda \sigma_2(u_2)$

$$(\mu_1 \times_{\alpha} \mu_2)(u_1, u_2)(v_1, v_2) = \begin{cases} \sigma_1(u_1) \Lambda \mu_2(u_2, v_2) & u_1 = v_1, u_2 v_2 \in E_2 \\ \sigma_2(u_2) \Lambda \mu_1(u_1, v_1) & u_2 = v_2, u_1 v_1 \in E_1 \\ \mu_1(u_1, v_1) \Lambda \sigma_2(u_2) \Lambda \sigma_2(v_2) & u_1 v_1 \in E_1, u_2 v_2 \notin E_2 \\ \mu_2(u_2, v_2) \Lambda \sigma_1(u_1) \Lambda \sigma_1(v_1) & u_1 v_1 \notin E_1, u_2 v_2 \in E_2 \end{cases}$$

## 2. EDGE REGULAR PROPERTY OF CARTESIAN PRODUCT OF INTUITIONISTIC FUZZY GRAPHS

In this section, degree of an edge in cartesian product of two intuitionistic fuzzy graphs is defined and we see about the edge regular property of cartesian product of two intuitionistic fuzzy graph.

### Degree of an edge

By definition for any  $((u_1, u_2), (v_1, v_2)) \in E$ , we have

$$\begin{aligned} d_{G_1 \times G_2}((u_1, u_2), (v_1, v_2)) &= \sum_{(w_1, w_2) \neq (v_1, v_2)} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(w_1, w_2)) \\ &\quad + \sum_{(w_1, w_2) \neq (u_1, u_2)} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((w_1, w_2)(v_1, v_2)) \end{aligned}$$

If  $u_1 = v_1, (u_2, v_2) \in E_2$

$$\begin{aligned} d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) &= \sum_{(w_1, w_2) \neq (v_1, u_2)} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(w_1, w_2)) \\ &\quad + \sum_{(w_1, w_2) \neq (u_1, u_2)} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((w_1, w_2)(v_1, u_2)) \\ &= \sum_{u_1=w_1, w_2 \neq v_2} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(u_1, w_2)) \\ &\quad + \sum_{w_1 \neq v_1, u_2=w_2} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(w_1, u_2)) \\ &\quad + \sum_{w_1=u_1, w_2 \neq u_2} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, w_2)(u_1, v_2)) \\ &\quad + \sum_{w_1 \neq u_1, w_2=v_2} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((w_1, v_2)(u_1, v_2)) \\ &= \sum_{u_1=w_1, w_2 \neq v_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{w_1 \neq v_1, u_2=w_2} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\ &\quad + \sum_{w_1=u_1, w_2 \neq u_2} \{\min(\mu_1(u_1), \mu'_2(w_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(w_2, v_2))\} \\ &\quad + \sum_{w_1 \neq u_1, w_2=v_2} \{\min(\mu_2(w_1, u_1), \mu'_1(v_2)), \max(\gamma_2(w_1, u_1), \gamma'_1(v_2))\} \\ &= \sum_{w_2 \neq v_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{w_1 \in V_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\ &\quad + \sum_{w_2 \neq u_2} \{\min(\mu_1(u_1), \mu'_2(w_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(w_2, v_2))\} \\ &\quad + \sum_{w_1 \in V_1} \{\min(\mu_2(w_1, u_1), \mu'_1(v_2)), \max(\gamma_2(w_1, u_1), \gamma'_1(v_2))\} \end{aligned}$$

If  $u_2 = v_2$ ,  $(u_1, v_1) \in E_1$

$$\begin{aligned}
 d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) &= \sum_{(w_1, w_2) \neq (v_1, u_2)} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(w_1, w_2)) \\
 &\quad + \sum_{(w_1, w_2) \neq (u_1, u_2)} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((w_1, w_2)(v_1, u_2)) \\
 &= \sum_{u_1=w_1} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(u_1, w_2)) \\
 &\quad + \sum_{w_1 \neq v_1, u_2=w_2} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((u_1, u_2)(w_1, u_2)) \\
 &\quad + \sum_{w_1=v_1} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((w_1, u_2)(v_1, u_2)) \\
 &\quad + \sum_{w_1 \neq u_1, w_2=u_2} (\mu_2 \times \mu'_2)(\gamma_2 \times \gamma'_2)((w_1, u_2)(v_1, u_2)) \\
 &= \sum_{u_1=w_1} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\
 &\quad + \sum_{w_1 \neq v_1, u_2=w_2} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\
 &\quad + \sum_{u_1=w_1} \{\min(\mu_1(v_1), \mu'_2(w_2, u_2)), \max(\gamma_1(v_1), \gamma'_2(w_2, u_2))\} \\
 &\quad + \sum_{w_1 \neq u_1, u_2=w_2} \{\min(\mu_2(w_1, v_1), \mu'_1(u_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(u_2))\} \\
 &= \sum_{w_2 \in V_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\
 &\quad + \sum_{w_1 \neq v_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\
 &\quad + \sum_{w_2 \in V_2} \{\min(\mu_1(v_1), \mu'_2(w_2, u_2)), \max(\gamma_1(v_1), \gamma'_2(w_2, u_2))\} \\
 &\quad + \sum_{w_1 \neq u_1} \{\min(\mu_2(w_1, v_1), \mu'_1(u_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(u_2))\}
 \end{aligned}$$

**Theorem 2.1** Let  $G_1$  and  $G_2$  be two intuitionistic fuzzy graph such that  $\mu_1 \geq \mu'_2$ ,  $\gamma_1 \leq \gamma'_2$ ,  $\mu'_1 \geq \mu_2$ ,  $\gamma'_1 \leq \gamma_2$

1. If  $((u_1, u_2), (u_1, v_2)) \in E$ , then

$$d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2, v_2)$$

2. If  $((u_1, u_2), (v_1, u_2)) \in E$ , then

$$d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) = d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2)$$

**Proof:**

1. For any  $((u_1, u_2), (u_1, v_2)) \in E$ , we have

$$\begin{aligned}
 d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) &= \sum_{w_2 \neq v_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\
 &\quad + \sum_{w_1 \in V_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\
 &\quad + \sum_{w_2 \neq u_2} \{\min(\mu_1(u_1), \mu'_2(w_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(w_2, v_2))\} \\
 &\quad + \sum_{w_1 \in V_1} \{\min(\mu_2(w_1, u_1), \mu'_1(v_2)), \max(\gamma_2(w_1, u_1), \gamma'_1(v_2))\} \\
 &= \sum_{w_2 \neq v_2} (\mu'_2(u_2, w_2), \gamma'_2(u_2, w_2)) + \sum_{w_1 \in V_1} (\mu_2(u_1, w_1), \gamma_2(u_1, w_1)) \\
 &\quad + \sum_{w_2 \neq u_2} (\mu'_2(w_2, v_2), \gamma'_2(w_2, v_2)) + \sum_{w_1 \in V_1} (\mu_2(w_1, u_1), \gamma_2(w_1, u_1))
 \end{aligned}$$

$$d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2, v_2)$$

2. For any  $((u_1, u_2), (v_1, u_2)) \in E$ , we have

$$\begin{aligned}
 d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) &= \sum_{w_2 \in V_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\
 &\quad + \sum_{w_1 \neq v_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\
 &\quad + \sum_{w_2 \in V_2} \{\min(\mu_1(v_1), \mu'_2(w_2, u_2)), \max(\gamma_1(v_1), \gamma'_2(w_2, u_2))\} \\
 &\quad + \sum_{w_1 \neq u_1} \{\min(\mu_2(w_1, v_1), \mu'_1(u_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(u_2))\} \\
 &= \sum_{w_2 \in V_2} (\mu'_2(u_2, w_2), \gamma'_2(u_2, w_2)) + \sum_{w_1 \neq v_1} (\mu_2(u_1, w_1), \gamma_2(u_1, w_1)) \\
 &\quad + \sum_{w_2 \in V_2} (\mu'_2(w_2, u_2), \gamma'_2(w_2, u_2)) + \sum_{w_1 \neq u_1} (\mu_2(w_1, v_1), \gamma_2(w_1, v_1))
 \end{aligned}$$

$$d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) = d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2)$$

**Theorem 2.2:** Let  $G_1$  and  $G_2$  be two intuitionistic fuzzy graph such that  $\mu_1 \geq \mu'_2$ ,  $\gamma_1 \leq \gamma'_2$ ,  $\mu'_1 \geq \mu_2$ ,  $\gamma'_1 \leq \gamma_2$ ,  $\mu_1(u) = c_1$ ,  $\gamma_1(u) = c_2$ , for all  $u \in V$ . Then

1. For  $((u_1, u_2), (u_1, v_2)) \in E$ , then

$$d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) = 2d_{G_1}(u_1) + (c_1, c_2) (d^*_{G_2}(u_2) + d^*_{G_2}(v_2) - 2)$$

2. For  $((u_1, u_2), (v_1, u_2)) \in E$ , then

$$d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) = d_{G_1}(u_1, v_1) + 2(c_1, c_2) (d^*_{G_2}(u_2))$$

**Proof:** 1. For any  $((u_1, u_2), (u_1, v_2)) \in E$ , we have

$$\begin{aligned}
 d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) &= \sum_{w_2 \neq v_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\
 &\quad + \sum_{w_1 \in V_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\
 &\quad + \sum_{w_2 \neq u_2} \{\min(\mu_1(u_1), \mu'_2(w_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(w_2, v_2))\} \\
 &\quad + \sum_{w_1 \in V_1} \{\min(\mu_2(w_1, u_1), \mu'_1(v_2)), \max(\gamma_2(w_1, u_1), \gamma'_1(v_2))\} \\
 &= \sum_{w_2 \neq v_2} ((\mu_1(u_1), \gamma_1(u_1)) + \sum_{w_1 \neq v_1} (\mu_2(u_1, w_1), \gamma_2(u_1, w_1)))
 \end{aligned}$$

$$\begin{aligned}
 & + \sum_{w_2 \neq u_2} ((\mu_1(u_1), \gamma_1(u_1)) + \sum_{w_1 \neq u_1} (\mu_2(w_1, u_1), \gamma_2(w_1, u_1))) \\
 & = \sum_{w_2 \neq v_2} (c_1, c_2) + \sum_{w_1 \in V_1} (\mu_2(u_1, w_1), \gamma_2(u_1, w_1)) \\
 & + \sum_{w_2 \neq u_2} (c_1, c_2) + \sum_{w_1 \in V_1} (\mu_2(w_1, u_1), \gamma_2(w_1, u_1)) \\
 & = (c_1, c_2) (d_{G_2}^*(u_2) - 1) + d_{G_1}(u_1) + (c_1, c_2) ((d_{G_2}^*(v_2) - 1) + d_{G_1}(u_1))
 \end{aligned}$$

$$d_{G_1 \times G_2}((u_1, u_2), (v_1, v_2)) = 2d_{G_1}(u_1) + (c_1, c_2) (d_{G_2}^*(u_2) + d_{G_2}^*(v_2) - 2)$$

2. For any  $((u_1, u_2), (v_1, u_2)) \in E$ , we have

$$\begin{aligned}
 d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) &= \sum_{w_2 \in V_2} \{ \min(\mu_1(u_1), \mu_2'(u_2, w_2)), \max(\gamma_1(u_1), \gamma_2'(u_2, w_2)) \} \\
 &+ \sum_{w_1 \neq v_1} \{ \min(\mu_2(u_1, w_1), \mu_1'(u_2)), \max(\gamma_2(u_1, w_1), \gamma_1'(u_2)) \} \\
 &+ \sum_{w_2 \in V_2} \{ \min(\mu_1(v_1), \mu_2'(w_2, u_2)), \max(\gamma_1(v_1), \gamma_2'(w_2, u_2)) \} \\
 &+ \sum_{w_1 \neq u_1} \{ \min(\mu_2(w_1, v_1), \mu_1'(u_2)), \max(\gamma_2(w_1, v_1), \gamma_1'(u_2)) \} \\
 &= \sum_{w_2 \in V_2} ((\mu_1(u_1), \gamma_1(u_1)) + \sum_{w_1 \neq v_1} (\mu_2(u_1, w_1), \gamma_2(u_1, w_1))) \\
 &+ \sum_{w_2 \in V_2} ((\mu_1(u_1), \gamma_1(u_1)) + \sum_{w_1 \neq u_1} (\mu_2(w_1, v_1), \gamma_2(w_1, v_1))) \\
 &= \sum_{w_2 \in V_2} (c_1, c_2) + d_{G_1}(u_1, v_1) + \sum_{w_2 \in V_2} (c_1, c_2) \\
 &= d_{G_1}(u_1, v_1) + 2(c_1, c_2) (d_{G_2}^*(u_2))
 \end{aligned}$$

$$d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) = d_{G_1}(u_1, v_1) + 2(c_1, c_2) (d_{G_2}^*(u_2))$$

**Remark 2.3:** If  $G_1$  and  $G_2$  are two edge regular intuitionistic fuzzy graph, then  $G_1 \times G_2$  need not be edge regular intuitionistic fuzzy graph.

**Remark 2.4:** If  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph, then  $G_1$  and  $G_2$  need not be edge regular intuitionistic fuzzy graph.

**Theorem 2.5:** Let  $G_1 : (A_1, B_1)$  and  $G_2 : (A_2, B_2)$  be two regular intuitionistic fuzzy graph of same degree on  $G_1^* : (V, E)$  and  $G_2^* : (V, E)$  such that  $\mu_1 \geq \mu_2'$ ,  $\gamma_1 \leq \gamma_2'$ . Then  $G_1$  and  $G_2$  are edge regular intuitionistic fuzzy graph of same degree if and only if  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph.

**Proof:** Let  $G_1$  and  $G_2$  be two regular intuitionistic fuzzy graph of degree  $(c_1, c_2)$ . Then

$$d_{G_1}(u_1) = d_{G_2}(u_2) = (c_1, c_2), \text{ for all } u_1 \in V_1, u_2 \in V_2.$$

Assume that  $G_1$  and  $G_2$  are  $(k_1, k_2)$ -edge regular intuitionistic fuzzy graph. Then

$$d_{G_1}(u_1, v_1) = d_{G_2}(u_2, v_2) = (k_1, k_2), \text{ for all } (u_1, v_1) \in E_1, (u_2, v_2) \in E_2.$$

By theorem 2.1 we have for any  $((u_1, u_2), (v_1, v_2)) \in E$ ,

When  $u_1 = v_1, (u_2, v_2) \in E_2$

$$d_{G_1 \times G_2}((u_1, u_2), (v_1, v_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2, v_2) = 2(c_1, c_2) + (k_1, k_2)$$

When  $u_2 = v_2, (u_1, v_1) \in E_1$

$$d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) = 2d_{G_2}(u_2) + d_{G_1}(u_1, v_1) = 2(c_1, c_2) + (k_1, k_2)$$

Hence  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph.

Conversely suppose  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph.

Let  $(u_1, v_1), (w_1, x_1) \in E_1$  be two edges of  $G_1$  and let  $u_2 \in V_2$ . Then  $((u_1, u_2), (v_1, u_2)), ((w_1, u_2), (x_1, u_2)) \in E$

$$\begin{aligned}
 \text{Also } d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) &= d_{G_1 \times G_2}((w_1, u_2), (x_1, u_2)) \\
 d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2) &= d_{G_1}(w_1, x_1) + 2d_{G_2}(u_2) \\
 d_{G_1}(u_1, v_1) + 2(c_1, c_2) &= d_{G_1}(w_1, x_1) + 2(c_1, c_2). \\
 d_{G_1}(u_1, v_1) &= d_{G_1}(w_1, x_1)
 \end{aligned}$$

So,  $G_1$  is an edge regular intuitionistic fuzzy graph.

Similarly  $G_2$  is an edge regular intuitionistic fuzzy graph.

Now, suppose that  $G_1$  is  $(s_1, s_2)$ -edge regular intuitionistic fuzzy graph and  $G_2$  is  $(s_3, s_4)$ -edge regular intuitionistic fuzzy graph with  $(s_1, s_2) \neq (s_3, s_4)$

Then  $d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2, v_2) = 2(c_1, c_2) + (s_3, s_4)$ .

Also  $d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) = 2d_{G_2}(u_2) + d_{G_1}(u_1, v_1) = 2(c_1, c_2) + (s_1, s_2)$

So,  $d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) \neq d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2))$

which is a contradiction to  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph. Hence  $G_1$  and  $G_2$  are edge regular intuitionistic fuzzy graph of same degree.

**Theorem 2.6:** Let  $G_1 : (A_1, B_1)$  and  $G_2 : (A_2, B_2)$  be two intuitionistic fuzzy graphs such that  $G_1^* : (V, E)$  and  $G_2^* : (V, E)$  are regular graphs with  $\mu_1 \geq \mu_2'$ ,  $\gamma_1 \leq \gamma_2'$  and  $B_1$  is constant function. Then  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph.

**Proof:** Let  $G_1$  and  $G_2$  be two regular intuitionistic fuzzy graph on regular graph and  $B_1(u, v) = (c_1, c_2)$ . Then  $G_1$  is both regular and edge regular intuitionistic fuzzy graph.

Let  $d_{G_1}(u_1, v_1) = (k_1, k_2)$  and  $d_{G_1}(u_1) = (m_1, m_2)$ ,  $d_{G_2}^*(u_2) = n$ , for all  $u_1, v_1 \in V_1, u_2 \in V_2$ .

**Case-(i):** When  $u_1 = v_1, (u_2, v_2) \in E_2$ , by theorem 2.2,

$$\begin{aligned} d_{G_1 \times G_2}((u_1, u_2), (u_1, v_2)) &= 2d_{G_1}(u_1) + (c_1, c_2)(d_{G_2}^*(u_2) + d_{G_2}^*(v_2) - 2) \\ &= 2(m_1, m_2) + (c_1, c_2)(n + n - 2) \\ &= 2((m_1, m_2) + (c_1, c_2)(n - 1)) \end{aligned}$$

**Case-(ii):** When  $u_2 = v_2, (u_1, v_1) \in E_1$

$$\begin{aligned} d_{G_1 \times G_2}((u_1, u_2), (v_1, u_2)) &= d_{G_1}(u_1, v_1) + 2(c_1, c_2) d_{G_2}^*(u_2) \\ &= (k_1, k_2) + 2(c_1, c_2)n \\ &= 2(m_1, m_2) - 2(c_1, c_2) + 2(c_1, c_2)n \\ &= 2((m_1, m_2) + (c_1, c_2)(n - 1)). \end{aligned}$$

Hence  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph.

**Corollary 2.7:** Let  $G_1$  and  $G_2$  be two intuitionistic fuzzy graph with  $\mu_1 \leq \mu_2'$ ,  $\gamma_1 \geq \gamma_2'$  and  $A_1$  is a constant function with  $A_1(u) = (c_1, c_2)$  for all  $u \in V_1$ . Let  $G_1^*$  and  $G_2^*$  be two regular graphs. If  $G_1$  is strong, then  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph.

**Proof:** Let  $A_1(u) = (c_1, c_2)$  for all  $u \in V_1$ . Since  $G_1$  is strong, we have  $B_1(uv) = (c_1, c_2)$  for all  $(u, v) \in E_1$ . Also given  $\mu_1 \leq \mu_2'$ ,  $\gamma_1 \geq \gamma_2'$  Hence by theorem 2.6, the result follows.

**Theorem 2.8:** Let  $G_1 : (A_1, B_1)$  and  $G_2 : (A_2, B_2)$  be two intuitionistic fuzzy graphs such that  $G_1^* : (V, E)$  and  $G_2^* : (V, E)$  are regular graphs with  $\mu_1 \geq \mu_2'$ ,  $\gamma_1 \leq \gamma_2'$  and  $B_2$  is constant function. Then  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph.

**Proof:** Proof is similar to Theorem 2.6

**Corollary 2.9:** Let  $G_1$  and  $G_2$  be two intuitionistic fuzzy graph with  $\mu_1 \geq \mu_2'$ ,  $\gamma_1 \leq \gamma_2'$  and  $A_2$  is a constant function with  $A_2(u) = (c_1, c_2)$  for all  $u \in V_2$ . Let  $G_1^*$  and  $G_2^*$  be two regular graphs. If  $G_2$  is strong, then  $G_1 \times G_2$  is an edge regular intuitionistic fuzzy graph.

**Proof:** Let  $A_2(u) = (c_1, c_2)$  for all  $u \in V_2$ . Since  $G_2$  is strong, we have  $B_2(uv) = (c_1, c_2)$ , for all  $(u, v) \in E$ . Also given  $\mu_1 \geq \mu_2'$ ,  $\gamma_1 \leq \gamma_2'$  Hence by theorem 2.8, the result follows.

### 3. EDGE REGULAR PROPERTY OF COMPOSITION OF INTUITIONISTIC FUZZY GRAPHS

In this section, degree of an edge in composition product of two intuitionistic fuzzy graphs is defined and we see about the edge regular property of composition product of two intuitionistic fuzzy graph.

#### Degree of an edge

By definition for any  $((u_1, u_2)(v_1, v_2)) \in E$ , we have

$$\begin{aligned} d_{G_1 \circ G_2}((u_1, u_2), (u_1, v_2)) &= \sum_{(w_1, w_2) \neq (v_1, v_2)} (\mu_2 \circ \mu_2')(\gamma_2 \circ \gamma_2')((u_1, u_2), (w_1, w_2)) \\ &\quad + \sum_{(w_1, w_2) \neq (u_1, u_2)} (\mu_2 \circ \mu_2')(\gamma_2 \circ \gamma_2')((w_1, w_2), (v_1, v_2)) \end{aligned}$$

If  $u_1=v_1, u_2 \neq v_2$  then

$$\begin{aligned} d_{G_1 \circ G_2}((u_1, u_2), (u_1, v_2)) &= \sum_{(w_1, w_2) \neq (u_1, v_2)} (\mu_2 \circ \mu'_2)(\gamma_2 \circ \gamma'_2)((u_1, u_2), (w_1, w_2)) \\ &\quad + \sum_{(w_1, w_2) \neq (u_1, u_2)} (\mu_2 \circ \mu'_2)(\gamma_2 \circ \gamma'_2)((w_1, w_2), (v_1, v_2)) \\ &= \sum_{w_2 \neq v_2, w_2 u_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{w_2=u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\ &\quad + \sum_{w_2 \neq u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\ &\quad + \sum_{w_2 \neq u_2, w_2 v_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(w_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(w_2, v_2))\} \\ &\quad + \sum_{w_2=v_2, w_1 u_1 \in E_1} \{\min(\mu_2(w_1, u_1), \mu'_1(v_2)), \max(\gamma_2(w_1, u_1), \gamma'_1(v_2))\} \\ &\quad + \sum_{w_2 \neq v_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(w_2), \mu'_1(v_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(w_2), \gamma'_1(v_2))\} \end{aligned}$$

If  $u_1 \neq v_1, u_2 = v_2$  then

$$\begin{aligned} d_{G_1 \circ G_2}((u_1, u_2), (v_1, u_2)) &= \sum_{(w_1, w_2) \neq (v_1, u_2)} (\mu_2 \circ \mu'_2)(\gamma_2 \circ \gamma'_2)((u_1, u_2), (w_1, w_2)) \\ &\quad + \sum_{(w_1, w_2) \neq (u_1, u_2)} (\mu_2 \circ \mu'_2)(\gamma_2 \circ \gamma'_2)((w_1, w_2), (v_1, v_2)) \\ &= \sum_{u_1=w_1, w_2 u_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{w_2=u_2, w_1 u_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\ &\quad + \sum_{w_2 \neq u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\ &\quad + \sum_{u_1=w_1, w_2 u_2 \in E_2} \{\min(\mu_1(v_1), \mu'_2(w_2, u_2)), \max(\gamma_1(v_1), \gamma'_2(w_2, u_2))\} \\ &\quad + \sum_{w_2=u_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(u_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(u_2))\} \\ &\quad + \sum_{w_2 \neq u_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(w_2), \mu'_1(u_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(w_2), \gamma'_1(u_2))\} \end{aligned}$$

If  $u_1 \neq v_1, u_2 \neq v_2$  then

$$\begin{aligned} d_{G_1 \circ G_2}((u_1, u_2), (v_1, v_2)) &= \sum_{(w_1, w_2) \neq (v_1, u_2)} (\mu_2 \circ \mu'_2)(\gamma_2 \circ \gamma'_2)((u_1, u_2), (w_1, w_2)) \\ &\quad + \sum_{(w_1, w_2) \neq (u_1, u_2)} (\mu_2 \circ \mu'_2)(\gamma_2 \circ \gamma'_2)((w_1, w_2), (v_1, v_2)) \\ &= \sum_{u_1=w_1, w_2 u_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{w_2=u_2, w_1 u_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\ &\quad + \sum_{w_2 \neq u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\ &\quad + \sum_{u_1=w_1, w_2 v_2 \in E_2} \{\min(\mu_1(v_1), \mu'_2(w_2, v_2)), \max(\gamma_1(v_1), \gamma'_2(w_2, v_2))\} \\ &\quad + \sum_{w_2=v_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(v_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(v_2))\} \\ &\quad + \sum_{w_2 \neq u_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(w_2), \mu'_1(v_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(w_2), \gamma'_1(v_2))\} \end{aligned}$$

**Theorem 3.1:** Let  $G_1$  and  $G_2$  be two intuitionistic fuzzy graph such that  $\mu_1 \geq \mu'_2, \gamma_1 \leq \gamma'_2, \mu'_1 \geq \mu_2, \gamma'_1 \leq \gamma_2$ , then

- a)  $d_{G_1 \circ G_2}((u_1, u_2), (u_1, v_2)) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2, v_2)$
- b)  $d_{G_1 \circ G_2}((u_1, u_2), (v_1, u_2)) = d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1))$
- c)  $d_{G_1 \circ G_2}((u_1, u_2), (v_1, v_2)) = d_{G_1}(u_1, v_1) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + d_{G_2}(u_2) + d_{G_2}(v_2)$

**Proof:**

$$\begin{aligned} a) d_{G_1 \circ G_2}((u_1, u_2), (u_1, v_2)) &= \sum_{w_2 \neq v_2, u_2 w_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{w_2=u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\ &\quad + \sum_{w_2 \neq u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\ &\quad + \sum_{w_2 \neq u_2, w_2 v_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(w_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(w_2, v_2))\} \\ &\quad + \sum_{w_2=v_2, w_1 u_1 \in E_1} \{\min(\mu_2(w_1, u_1), \mu'_1(v_2)), \max(\gamma_2(w_1, u_1), \gamma'_1(v_2))\} \\ &\quad + \sum_{w_2 \neq v_2, u_1 w_1 \in E_1} \{\min(\mu_2(w_1, u_1), \mu'_1(w_2), \mu'_1(v_2)), \max(\gamma_2(w_1, u_1), \gamma'_1(w_2), \gamma'_1(v_2))\} \\ &= \sum_{w_2 \neq v_2, u_2 w_2 \in E_2} \{\mu'_2(u_2, w_2), \gamma'_2(u_2, w_2)\} + \sum_{w_2=u_2, u_1 w_1 \in E_1} \{\mu_2(u_1, w_1), \gamma_2(u_1, w_1)\} \\ &\quad + \sum_{w_2 \neq u_2, u_1 w_1 \in E_1} |\mathcal{V} - \{u_2\}| \{\mu_2(u_1, w_1), \gamma_2(u_1, w_1)\} \\ &\quad + \sum_{w_2 \neq u_2, w_2 v_2 \in E_2} \{\mu'_2(w_2, v_2), \gamma'_2(w_2, v_2)\} + \sum_{w_2=v_2, w_1 u_1 \in E_1} \{\mu_2(w_1, u_1), \gamma_2(w_1, u_1)\} \\ &\quad + \sum_{w_2 \neq v_2, u_1 w_1 \in E_1} |\mathcal{V} - \{v_2\}| \{\mu_2(u_1, w_1), \gamma_2(u_1, w_1)\} \\ &= \sum_{w_2 \neq v_2, u_2 w_2 \in E_2} \{\mu'_2(u_2, w_2), \gamma'_2(u_2, w_2)\} + \sum_{w_2 \neq u_2, w_2 v_2 \in E_2} \{\mu'_2(w_2, v_2), \gamma'_2(w_2, v_2)\} \\ &\quad + d_{G_1}(u_1) + (p_2 - 1)d_{G_1}(u_1) + d_{G_1}(u_1) + (p_2 - 1)d_{G_1}(u_1) \end{aligned}$$

$$d_{G_1 \circ G_2}((u_1, u_2), (u_1, v_2)) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2, v_2)$$

$$\begin{aligned} b) d_{G_1 \circ G_2}((u_1, u_2), (v_1, u_2)) &= \sum_{u_1=w_1, w_2 u_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{w_2=u_2, w_1 u_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\ &\quad + \sum_{w_2 \neq u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\ &\quad + \sum_{u_1=w_1, w_2 u_2 \in E_2} \{\min(\mu_1(v_1), \mu'_2(w_2, u_2)), \max(\gamma_1(v_1), \gamma'_2(w_2, u_2))\} \end{aligned}$$

$$\begin{aligned}
 & + \sum_{w_2=u_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(u_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(u_2))\} \\
 & + \sum_{w_2 \neq u_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(w_2), \mu'_1(u_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(w_2), \gamma'_1(u_2))\} \\
 & = \sum_{w_2 \in V_2} \{\mu'_2(u_2, w_2), \gamma'_2(u_2, w_2)\} + \sum_{w_1 \in V_1} \{\mu_2(u_1, w_1), \gamma_2(u_1, w_1)\} \\
 & + |V - \{u_2\}| \sum_{w_1 \in V_1} \{\mu_2(u_1, w_1), \gamma_2(u_1, w_1)\} \\
 & + \sum_{w_2 \in V_2} \{\mu'_2(w_2, u_2), \gamma'_2(w_2, u_2)\} + \sum_{w_2=v_2, w_1 u_1 \in E_1} \{\mu_2(w_1, v_1), \gamma_2(w_1, v_1)\} \\
 & + |V - \{u_2\}| \sum_{w_1 \in V_1} \{\mu_2(w_1, v_1), \gamma_2(w_1, v_1)\}
 \end{aligned}$$

$$d_{G_1 \circ G_2}((u_1, u_2), (v_1, u_2)) = d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1))$$

$$\begin{aligned}
 c) d_{G_1 \circ G_2}((u_1, u_2), (v_1, v_2)) &= \sum_{u_1=w_1, w_2 u_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\
 &+ \sum_{w_2=u_2, w_1 u_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2))\} \\
 &+ \sum_{w_2 \neq u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\
 &+ \sum_{u_1=w_1, w_2 v_2 \in E_2} \{\min(\mu_1(v_1), \mu'_2(w_2, v_2)), \max(\gamma_1(v_1), \gamma'_2(w_2, v_2))\} \\
 &+ \sum_{w_2=v_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(v_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(v_2))\} \\
 &+ \sum_{w_2 \neq v_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(w_2), \mu'_1(v_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(w_2), \gamma'_1(v_2))\} \\
 &= \sum_{w_2 \in V_2} \{\mu'_2(u_2, w_2), \gamma'_2(u_2, w_2)\} + \sum_{w_1 \in V_1} \{\mu_2(u_1, w_1), \gamma_2(u_1, w_1)\} \\
 &+ \sum_{w_2 \neq u_2, u_1 w_1 \in E_1} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\
 &- (\mu_2(u_1, v_1), \gamma_2(u_1, v_1)) \\
 &+ \sum_{w_2 \in V_2} \{\mu'_2(w_2, v_2), \gamma'_2(w_2, v_2)\} + \sum_{w_2=v_2, w_1 u_1 \in E_1} \{\mu_2(w_1, v_1), \gamma_2(w_1, v_1)\} \\
 &+ \sum_{w_2 \neq v_2, w_1 v_1 \in E_1} \{\min(\mu_2(w_1, v_1), \mu'_1(w_2), \mu'_1(v_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(w_2), \gamma'_1(v_2))\} \\
 &- (\mu_2(u_1, v_1), \gamma_2(u_1, v_1))
 \end{aligned}$$

$$d_{G_1 \circ G_2}((u_1, u_2), (v_1, v_2)) = d_{G_1}(u_1, v_1) + (p_2 - 1) (d_{G_1}(u_1) + d_{G_1}(v_1)) + d_{G_2}(u_2) + d_{G_2}(v_2)$$

**Remark 3.2:** If  $G_1$  and  $G_2$  are two edge regular intuitionistic fuzzy graph, then  $G_1 \circ G_2$  need not be edge regular intuitionistic fuzzy graph.

**Remark 3.3:** If  $G_1 \circ G_2$  is an edge regular intuitionistic fuzzy graph, then  $G_1$  and  $G_2$  need not be edge regular intuitionistic fuzzy graph

**Theorem 3.4:** Let  $G_1 : (A_1, B_1)$  and  $G_2 : (A_2, B_2)$  be two regular intuitionistic fuzzy graph of same degree on  $G_1^* : (V, E)$  and  $G_2^* : (V, E)$  such that  $\mu_1 \geq \mu'_2, \gamma_1 \leq \gamma'_2, \mu'_1 \geq \mu_2, \gamma'_1 \leq \gamma_2$ , Then  $G_1$  and  $G_2$  are edge regular intuitionistic fuzzy graph of same degree if and only if  $G_1 \circ G_2$  is an edge regular intuitionistic fuzzy graph.

**Proof:** Let  $d_{G_1}(u_1) = d_{G_2}(u_2) = (k_1, k_2)$ , for all  $u_1 \in V_1, u_2 \in V_2$  and  $(k_1, k_2)$  is a constant.

Assume that  $G_1$  and  $G_2$  are  $(c_1, c_2)$ - edge regular intuitionistic fuzzy graph. Then,

$$d_{G_1}(u_1, v_1) = d_{G_2}(u_2, v_2) = (c_1, c_2), \text{ for all } (u_1, v_1) \in E_1, (u_2, v_2) \in E_2.$$

When  $u_1 = v_1, (u_2, v_2) \in E_2$  we have

$$d_{G_1 \circ G_2}((u_1, u_2), (u_1, v_2)) = 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2, v_2) = 2p_2(k_1, k_2) + (c_1, c_2).$$

When  $u_2 = v_2, (u_1, v_1) \in E_1$  we have,

$$\begin{aligned}
 d_{G_1 \circ G_2}((u_1, u_2), (v_1, u_2)) &= 2d_{G_2}(u_2) + d_{G_1}(u_1, v_1) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) \\
 &= (c_1, c_2) + 2(k_1, k_2) + (p_2 - 1)((k_1, k_2) + (k_1, k_2)) \\
 &= (c_1, c_2) + 2p_2(k_1, k_2).
 \end{aligned}$$

When  $(u_1, v_1) \in E_1, (u_2, v_2) \notin E_2$  we have

$$\begin{aligned}
 d_{G_1 \circ G_2}((u_1, u_2), (v_1, v_2)) &= d_{G_1}(u_1, v_1) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) + d_{G_2}(u_2) + d_{G_2}(v_2) \\
 &= (c_1, c_2) + 2p_2(k_1, k_2).
 \end{aligned}$$

Hence  $G_1 \circ G_2$  is an edge regular intuitionistic fuzzy graph.

Conversely assume that  $G_1 \circ G_2$  is an edge regular intuitionistic fuzzy graph.

Let  $(u_1, v_1), (w_1, x_1) \in E_1$  be two edges of  $G_1$  and let  $u \in V_2$ . Then  $((u_1, u), (v_1, u)), ((w_1, u), (x_1, u)) \in E$ .

$$\begin{aligned}
 \text{So, } d_{G_1 \circ G_2}((u_1, u), (v_1, u)) &= d_{G_1 \circ G_2}((w_1, u), (x_1, u)) \\
 \Rightarrow d_{G_1}(u_1, v_1) + 2d_{G_2}(u) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) &= d_{G_1}(w_1, x_1) + 2d_{G_2}(u) + (p_2 - 1)(d_{G_1}(w_1) + d_{G_1}(x_1)) \\
 \Rightarrow d_{G_1}(u_1, v_1) + (p_2 - 1)(2(k_1, k_2)) &= d_{G_1}(w_1, x_1) + (p_2 - 1)(2(k_1, k_2)) \\
 \Rightarrow d_{G_1}(u_1, v_1) &= d_{G_1}(w_1, x_1)
 \end{aligned}$$

Hence  $G_1$  is an edge regular intuitionistic fuzzy graph.

Similarly  $G_2$  is an edge regular intuitionistic fuzzy graph.

Now, suppose that  $G_1$  is  $(s_1, s_2)$ -edge regular intuitionistic fuzzy graph and  $G_2$  is  $(s_3, s_4)$ -edge regular intuitionistic fuzzy graph with  $(s_1, s_2) \neq (s_3, s_4)$

$$\begin{aligned}
 \text{Then } d_{G_1 \circ G_2}((u_1, u_2), (u_1, v_2)) &= 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2, v_2) \\
 &= 2p_2(k_1, k_2) + (s_3, s_4).
 \end{aligned}$$

$$\begin{aligned}
 \text{Also, } d_{G_1 \circ G_2}((u_1, u_2), (v_1, u_2)) &= d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2) + (p_2 - 1)(d_{G_1}(u_1) + d_{G_1}(v_1)) \\
 &= (s_1, s_2) + 2(k_1, k_2) + (p_2 - 1)((k_1, k_2) + (k_1, k_2)) \\
 &= (s_1, s_2) + 2p_2(k_1, k_2)
 \end{aligned}$$

$$\text{So, } d_{G_1 \circ G_2}((u_1, u_2), (u_1, v_2)) \neq d_{G_1 \circ G_2}((u_1, u_2), (v_1, u_2))$$

which is a contradiction to  $G_1 \circ G_2$  is an edge regular intuitionistic fuzzy graph. Hence  $G_1$  and  $G_2$  are edge regular intuitionistic fuzzy graph of same degree.

#### 4. EDGE REGULAR PROPERTY OF ALPHA PRODUCT OF INTUITIONISTIC FUZZY GRAPHS

In this section, we see about the edge regular property of alpha product of two intuitionistic fuzzy graph.

**Definition 4.1:** Let  $G_1 : (A_1, B_1)$  and  $G_2 : (A_2, B_2)$  where  $A_1 = (\mu_1, \gamma_1)$ ,  $B_1 = (\mu_2, \gamma_2)$ ,  $A_2 = (\mu'_1, \gamma'_1)$ ,  $B_2 = (\mu'_2, \gamma'_2)$  be two intuitionistic fuzzy graph on  $G^*(V, E)$  Where  $V = V_1 \times V_2$  and  $E = \{(u_1, u_2)(v_1, v_2) : u_1 = v_1, u_2 v_2 \in E_2 \text{ or } u_2 = v_2, u_1 v_1 \in E_1 \text{ or } u_1 v_1 \in E_1, u_2 v_2 \notin E_2 \text{ or } u_1 v_1 \notin E_1, u_2 v_2 \in E_2\}$ . Then the alpha product of  $G_1$  and  $G_2$  is defined as  
 $G_1 \times_\alpha G_2 = \{((\mu_1 \times_\alpha \mu'_1)(\gamma_1 \times_\alpha \gamma'_1), (\mu_2 \times_\alpha \mu'_2)(\gamma_2 \times_\alpha \gamma'_2))\}$

where

$$\begin{aligned}
 ((\mu_1 \times_\alpha \mu'_1)(\gamma_1 \times_\alpha \gamma'_1))(u_1, u_2) &= \{\min(\mu_1(u_1), \mu'_1(u_2)), \max(\gamma_1(u_1), \gamma'_1(u_2))\} \\
 ((\mu_2 \times_\alpha \mu'_2)(\gamma_2 \times_\alpha \gamma'_2))(u_1, u_2)(v_1, v_2) &= \begin{cases} \{\min(\mu_1(u_1), \mu'_2(u_2, v_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, v_2))\} & u_1 = v_1, u_2 v_2 \in E_2 \\ \{(\min(\mu'_1(u_2), \mu_2(u_1, v_1)), \max(\gamma'_1(u_2), \gamma_2(u_1, v_1)))\} & u_2 = v_2, u_1 v_1 \in E_1 \\ \{\min(\mu_2(u_1, v_1), \mu'_1(u_2), \mu'_1(v_2)), \max(\gamma_2(u_1, v_1), \gamma'_1(u_2), \gamma'_1(v_2))\} & u_1 v_1 \in E_1, u_2 v_2 \notin E_2 \\ \{\min(\mu'_2(u_2, v_2), \mu_1(u_1), \mu_1(v_1)) \max(\gamma'_2(u_2, v_2), \gamma_1(u_1), \gamma_1(v_1))\} & u_1 v_1 \notin E_1, u_2 v_2 \in E_2 \end{cases}
 \end{aligned}$$

#### Degree of an edge

$$\begin{aligned}
 d_{G_1 \times_\alpha G_2}((u_1, u_2), (v_1, v_2)) &= \sum_{((u_1, u_2), (w_1, w_2)) \in E} (\mu_2 \times_\alpha \mu'_2)(\gamma_2 \times_\alpha \gamma'_2)((u_1, u_2), (w_1, w_2)) \\
 &\quad + \sum_{(w_1, w_2), (v_1, v_2) \in E} (\mu_2 \times_\alpha \mu'_2)(\gamma_2 \times_\alpha \gamma'_2)(w_1, w_2), (v_1, v_2)) \\
 &\quad - 2(\mu_2 \times_\alpha \mu'_2)(\gamma_2 \times_\alpha \gamma'_2)((u_1, u_2), (v_1, v_2)) \\
 &= \sum_{u_1=w_1, u_2=w_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\
 &\quad + \sum_{u_2=w_2, u_1=w_1 \in E_1} \{\min(\mu'_1(u_2), \mu_2(u_1, w_1)), \max(\gamma'_1(u_2), \gamma_2(u_1, w_1))\} \\
 &\quad + \sum_{u_1 w_1 \in E_1, u_2 w_2 \notin E_2} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\
 &\quad + \sum_{u_1 w_1 \notin E_1, u_2 w_2 \in E_2} \{\min(\mu'_2(u_2, w_2), \mu_1(u_1), \mu_1(w_1)) \max(\gamma'_2(u_2, w_2), \gamma_1(u_1), \gamma_1(w_1))\} \\
 &\quad + \sum_{u_1=w_1, v_2 w_2 \in E_2} \{\min(\mu_1(v_1), \mu'_2(v_2, w_2)), \max(\gamma_1(v_1), \gamma'_2(v_2, w_2))\} \\
 &\quad + \sum_{v_2=w_2, v_1 w_1 \in E_1} \{\min(\mu'_1(v_2), \mu_2(v_1, w_1)), \max(\gamma'_1(v_2), \gamma_2(v_1, w_1))\} \\
 &\quad + \sum_{w_1 v_1 \in E_1, w_2 v_2 \notin E_2} \{\min(\mu_2(w_1, v_1), \mu'_1(w_2), \mu'_1(v_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(w_2), \gamma'_1(v_2))\} \\
 &\quad + \sum_{w_1 v_1 \notin E_1, w_2 v_2 \in E_2} \{\min(\mu'_2(w_2, v_2), \mu_1(w_1), \mu_1(v_1)) \max(\gamma'_2(w_2, v_2), \gamma_1(w_1), \gamma_1(v_1))\} \\
 &\quad - 2(\mu_2 \times_\alpha \mu'_2)(\gamma_2 \times_\alpha \gamma'_2)((u_1, u_2), (v_1, v_2))
 \end{aligned}$$

**Remark 4.2:** If  $G_1$  and  $G_2$  are two edge regular intuitionistic fuzzy graphs, then  $G_1 \times_\alpha G_2$  need not be edge regular intuitionistic fuzzy graph.

**Remark 4.3:** If  $G_1 \times_\alpha G_2$  is an edge regular intuitionistic fuzzy graph, then  $G_1$  and  $G_2$  need not be edge regular intuitionistic fuzzy graph

**Theorem 4.4:** Let  $G_1:(A_1,B_1)$  and  $G_2 :(A_2,B_2)$  be two intuitionistic fuzzy graphs such that  $G_1^*: (V,E)$  and  $G_2^*: (V,E)$  are complete graphs. Suppose  $\mu_1 \geq \mu'_2, \gamma_1 \leq \gamma'_2, \mu'_1 \geq \mu_2, \gamma'_1 \leq \gamma_2$ , Then for any  $(u_1, u_2), (v_1, v_2) \in E$

1. When  $u_1 = v_1, u_2 v_2 \in E_2, d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, v_2)) = d_{G_2}(u_2, v_2) + 2 d_{G_1}(u_1)$
2. When  $u_2 = v_2, u_1 v_1 \in E_1, d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, v_2)) = d_{G_1}(u_1, v_1) + 2 d_{G_2}(u_2)$

**Proof:** 1. By definition for any  $((u_1, u_2), (v_1, v_2)) \in E$

$$\begin{aligned} d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, v_2)) &= \sum_{((u_1, u_2), (w_1, w_2)) \in E} (\mu_2 \times_{\alpha} \mu'_2)(\gamma_2 \times_{\alpha} \gamma'_2)((u_1, u_2), (w_1, w_2)) \\ &\quad + \sum_{(w_1, w_2), (v_1, v_2) \in E} (\mu_2 \times_{\alpha} \mu'_2)(\gamma_2 \times_{\alpha} \gamma'_2)(w_1, w_2), (v_1, v_2)) \\ &\quad - 2(\mu_2 \times_{\alpha} \mu'_2)(\gamma_2 \times_{\alpha} \gamma'_2)((u_1, u_2), (v_1, v_2)) \\ &= \sum_{u_1=w_1, u_2=w_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{u_2=w_2, u_1=w_1 \in E_1} \{\min(\mu'_1(u_2), \mu_2(u_1, w_1)), \max(\gamma'_1(u_2), \gamma_2(u_1, w_1))\} \\ &\quad + \sum_{u_1 w_1 \in E_1, u_2 w_2 \notin E_2} \{\min(\mu_2(u_1, w_1), \mu'_1(u_2), \mu'_1(w_2)), \max(\gamma_2(u_1, w_1), \gamma'_1(u_2), \gamma'_1(w_2))\} \\ &\quad + \sum_{u_1 w_1 \notin E_1, u_2 w_2 \in E_2} \{\min(\mu'_2(u_2, w_2), \mu_1(u_1), \mu_1(w_1)) \max(\gamma'_2(u_2, w_2), \gamma_1(u_1), \gamma_1(w_1))\} \\ &\quad + \sum_{u_1=w_1, v_2 w_2 \in E_2} \{\min(\mu_1(v_1), \mu'_2(v_2, w_2)), \max(\gamma_1(v_1), \gamma'_2(v_2, w_2))\} \\ &\quad + \sum_{v_2=w_2, v_1 w_1 \in E_1} \{\min(\mu'_1(v_2), \mu_2(v_1, w_1)), \max(\gamma'_1(v_2), \gamma_2(v_1, w_1))\} \\ &\quad + \sum_{w_1 v_1 \in E_1, w_2 v_2 \notin E_2} \{\min(\mu_2(w_1, v_1), \mu'_1(w_2), \mu'_1(v_2)), \max(\gamma_2(w_1, v_1), \gamma'_1(w_2), \gamma'_1(v_2))\} \\ &\quad + \sum_{w_1 v_1 \notin E_1, w_2 v_2 \in E_2} \{\min(\mu'_2(w_2, v_2), \mu_1(w_1), \mu_1(v_1)) \max(\gamma'_2(w_2, v_2), \gamma_1(w_1), \gamma_1(v_1))\} \\ &\quad - 2(\mu_2 \times_{\alpha} \mu'_2)(\gamma_2 \times_{\alpha} \gamma'_2)((u_1, u_2), (v_1, v_2)) \end{aligned}$$

Given  $G_1^*$  and  $G_2^*$  are complete graphs. So the edge set is given by

$$E = \{(u_1, u_2), (v_1, v_2) : u_1 = w_1, u_2 v_2 \in E_2 \text{ or } u_2 = w_2, u_1 v_1 \in E_1\}$$

$$\begin{aligned} d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, v_2)) &= \sum_{u_1=w_1, u_2=w_2 \in E_2} \{\min(\mu_1(u_1), \mu'_2(u_2, w_2)), \max(\gamma_1(u_1), \gamma'_2(u_2, w_2))\} \\ &\quad + \sum_{u_2=w_2, u_1=w_1 \in E_1} \{\min(\mu'_1(u_2), \mu_2(u_1, w_1)), \max(\gamma'_1(u_2), \gamma_2(u_1, w_1))\} \\ &\quad + \sum_{u_1=w_1, v_2 w_2 \in E_2} \{\min(\mu_1(v_1), \mu'_2(w_2, v_2)), \max(\gamma_1(v_1), \gamma'_2(w_2, v_2))\} \\ &\quad + \sum_{v_2=w_2, v_1 w_1 \in E_1} \{\min(\mu'_1(v_2), \mu_2(v_1, w_1)), \max(\gamma'_1(v_2), \gamma_2(v_1, w_1))\} \\ &\quad - 2(\mu_2 \times_{\alpha} \mu'_2)(\gamma_2 \times_{\alpha} \gamma'_2)((u_1, u_2), (v_1, v_2)) \\ &= \sum_{u_2 w_2 \in E_2} \{\mu'_2(u_2, w_2), \gamma'_2(u_2, w_2)\} + \sum_{u_1 w_1 \in E_1} \{\mu_2(u_1, w_1), \gamma_2(u_1, w_1)\} \\ &\quad + \sum_{w_2 v_2 \in E_2} \{\mu'_2(w_2, v_2), \gamma'_2(w_2, v_2)\} + \sum_{w_1 v_1 \in E_1} \{\mu_2(w_1, v_1), \gamma_2(w_1, v_1)\} \\ &\quad - 2(\mu_2(u_2, v_2), \gamma_2(u_2, v_2)) \\ &= \sum_{u_2 w_2 \in E_2} \{\mu'_2(u_2, w_2), \gamma'_2(u_2, w_2)\} + \sum_{w_2 v_2 \in E_2} \{\mu'_2(w_2, v_2), \gamma'_2(w_2, v_2)\} \\ &\quad - 2(\mu_2(u_2, v_2), \gamma_2(u_2, v_2)) + d_{G_1}(u_1) + d_{G_2}(u_2) \end{aligned}$$

$$d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, v_2)) = d_{G_2}(u_2, v_2) + 2 d_{G_1}(u_1).$$

Proof of (2) is similar to (1).

**Theorem 4.5:** Let  $G_1:(A_1,B_1)$  and  $G_2 :(A_2, B_2)$  be two regular intuitionistic fuzzy graph of same degree on  $G_1^*: (V, E)$  and  $G_2^*: (V, E)$  such that  $\mu_1 \geq \mu'_2, \gamma_1 \leq \gamma'_2, \mu'_1 \geq \mu_2, \gamma'_1 \leq \gamma_2$ , Then  $G_1$  and  $G_2$  are edge regular intuitionistic fuzzy graph of same degree if and only if  $G_1 \times_{\alpha} G_2$  is an edge regular intuitionistic fuzzy graph.

**Proof:** Let  $d_{G_1}(u_1) = d_{G_2}(u_2) = (m_1, m_2)$ , for all  $u_1 \in V_1, u_2 \in V_2$  and  $(k_1, k_2)$  is a constant.

Assume that  $G_1$  and  $G_2$  are  $(k_1, k_2)$ - edge regular intuitionistic fuzzy graph. Then,

$$d_{G_1}(u_1, v_1) = d_{G_2}(u_2, v_2) = (k_1, k_2), \text{ for all } (u_1, v_1) \in E_1, (u_2, v_2) \in E_2.$$

When  $u_1 = v_1, (u_2, v_2) \in E_2$  we have

$$d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, v_2)) = d_{G_2}(u_2, v_2) + 2d_{G_1}(u_1) = (k_1, k_2) + 2(m_1, m_2).$$

When  $u_2 = v_2, (u_1, v_1) \in E_1$  we have,

$$d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, u_2)) = 2d_{G_2}(u_2) + d_{G_1}(u_1, v_1) = (k_1, k_2) + 2(m_1, m_2).$$

Hence  $G_1 \times_{\alpha} G_2$  is an edge regular intuitionistic fuzzy graph.

Conversely assume that  $G_1 \times_{\alpha} G_2$  is an edge regular intuitionistic fuzzy graph.

Let  $(u_1, v_1), (w_1, x_1) \in E_1$  be two edges of  $G_1$  and let  $u \in V_2$ . Then  $((u_1, u), (v_1, u)), ((w_1, u), (x_1, u)) \in E$ .

$$\begin{aligned} \text{So, } d_{G_1 \times_{\alpha} G_2}((u_1, u), (v_1, u)) &= d_{G_1 \times_{\alpha} G_2}((w_1, u), (x_1, u)) \\ \Rightarrow d_{G_1}(u_1, v_1) + 2d_{G_2}(u) &= d_{G_1}(w_1, x_1) + 2d_{G_2}(u) \\ \Rightarrow d_{G_1}(u_1, v_1) + 2(m_1, m_2) &= d_{G_1}(w_1, x_1) + 2(m_1, m_2) \\ \Rightarrow d_{G_1}(u_1, v_1) &= d_{G_1}(w_1, x_1) \end{aligned}$$

Hence  $G_1$  is an edge regular intuitionistic fuzzy graph.

Similarly  $G_2$  is an edge regular intuitionistic fuzzy graph.

Now, suppose that  $G_1$  is  $(k_1, k_2)$ -edge regular intuitionistic fuzzy graph and  $G_2$  is  $(k_3, k_4)$ -edge regular intuitionistic fuzzy graph with  $(k_1, k_2) \neq (k_3, k_4)$

$$\begin{aligned} \text{Then } d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (u_1, v_2)) &= 2p_2 d_{G_1}(u_1) + d_{G_2}(u_2, v_2) \\ &= 2p_2(k_1, k_2) + (s_3, s_4). \end{aligned}$$

$$\begin{aligned} \text{Also, } d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, u_2)) &= d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2) = (k_1, k_2) + 2(m_1, m_2) \\ d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (u_1, v_2)) &= d_{G_2}(u_2, v_2) + 2d_{G_1}(u_1) = (k_3, k_4) + 2(m_1, m_2) \end{aligned}$$

$$\text{So, } d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (u_1, v_2)) \neq d_{G_1 \times_{\alpha} G_2}((u_1, u_2), (v_1, u_2))$$

which is a contradiction to  $G_1 \times_{\alpha} G_2$  is an edge regular intuitionistic fuzzy graph. Hence  $G_1$  and  $G_2$  are edge regular intuitionistic fuzzy graph of same degree.

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