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CERTAIN CLASSES OF ANALYTIC FUNCTIONS BY USING SĂLĂGEAN CARLSON-SHAFFER OPERATOR

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ABSTRACT

In this paper, we define the subclass of analytic function by using Sălăgean Carlson-Shaffer Operator. The objective of this article is to obtain the result concerning the coefficient estimates of the class $M_\lambda(a, c, m, \alpha)$.

Key words and phrases: *Analytic functions, Carlson-Shaffer operator, Sălăgean operator and Inclusion theorem.*

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1. INTRODUCTION

Let A denote the class of analytic functions f of the form

$$f(z) = z + \sum_{m=2}^{\infty} a_m z^m \tag{1.1}$$

which are analytic in the open unit disc $U = \{z; |z| < 1\}$. Let $M(\alpha)$ be the subclass of A consisting of functions f which satisfies the inequality,

$$\mathbb{R}\left\{\frac{zf'}{f}\right\} < \alpha, for\ some\ \alpha > 1.$$

Let $N(\alpha)$ be the subclass of A consisting of functions f which satisfies the inequality,

$$\mathbb{R}\left\{1 + \frac{zf''}{f'}\right\} < \alpha, for\ some\ \alpha > 1.$$

Then we observed that $f \in N(\alpha)$ if and only if $zf' \in M(\alpha)$. For $n \in \mathbb{N}_0\ and\ \lambda \geq 0, a, c, \in \mathbb{R}\setminus\mathbb{Z}$, let a linear operator [2] defined by

$$SL_\lambda f(z) = (1-\lambda)[(k * k * \cdots * k) * f] + \lambda[\phi(a,c) * f](z), \qquad z \in U, \tag{1.2}$$

where $k(z) = z(1-z)^{-2}$ is the Koebe function and

$$\phi(a, c; z) = \sum_{m=2}^{\infty} \frac{(a)_{m-1}}{(c)_{m-1}} z^m,\ \ |z| < 1,\ \ a, c \neq 0, -1, -2, \cdots,$$

is the incomplete beta function. For functions $f \in A$ of the form (1.1), we have

$$SL_\lambda f(z) = z + \sum_{m=2}^{\infty} B_\lambda(a, c, m, n)\, a_m z^m, \tag{1.3}$$

where

$$B_\lambda(a, c, m, n) = \left[(1-\lambda)m^n + \lambda \frac{(a)_{m-1}}{(c)_{m-1}}\right]. \tag{1.4}$$

Here $(a)_m$ is the Pochhammer symbol defined interms of the Gamma function by,

$$(a)_m = \frac{\Gamma(a+m)}{\Gamma(a)} = \begin{cases} 1, & for\ m = 0 \\ a(a+1)(a+2)\ldots\ldots(a+m-1) & for\ m \in \mathbb{N}. \end{cases}$$

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Now using the linear operator $\boldsymbol{SL_\lambda}$ we define the class $\boldsymbol{M_\lambda(a,c,m,\alpha)}$ consisting functions of the form (1.1) satisfying the condition

$$\mathbb{R}\left\{\frac{z[SL_\lambda f]'}{SL_\lambda f}\right\} < \alpha. \tag{1.5}$$

Note that $a = c = 1, and\ \lambda = 1$ the class reduces to $M(\alpha)$ and $a = 2, c = 1, and\ \lambda = 1$ the class reduces to $N(\alpha)$ defined by Owa and Nishwaki [3].

2. INCLUSION THEOREM INVOLVING COEFFIEICIENT INEQUALITIES

Theorem 2.1: If $f \in A$ satisfies

$$\sum_{m=2}^{\infty}\{(\boldsymbol{m}-\boldsymbol{k})+|\boldsymbol{m}+\boldsymbol{k}-2\alpha|\}\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})|\boldsymbol{a_m}| \le \boldsymbol{2}(\alpha-\boldsymbol{1}), \tag{2.1}$$

for some $0 \le k \le 1$, then $f \in M_\lambda(a,c,m,\alpha)$.

Proof: Let us suppose that

$$\sum_{m=2}^{\infty}\{(\boldsymbol{m}-\boldsymbol{k})+|\boldsymbol{m}+\boldsymbol{k}-2\alpha|\}\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})|\boldsymbol{a_m}| \le \boldsymbol{2}(\alpha-\boldsymbol{1}),\ \ \boldsymbol{f} \in \boldsymbol{A}.$$

It suffices to show that,

$$\left|\frac{\frac{z[SL_\lambda f]'}{SL_\lambda f}-k}{\frac{z[SL_\lambda f]'}{SL_\lambda f}-(2\alpha-k)}\right| < 1, (z \in U).$$

We note that,

$$\left|\frac{\frac{z[SL_\lambda f]'}{SL_\lambda f}-k}{\frac{z[SL_\lambda f]'}{SL_\lambda f}-(2\alpha-k)}\right| \le \left|\frac{(1-k)+\sum_{m=2}^{\infty}(m-k)\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})\boldsymbol{a_m}\boldsymbol{z^{m-1}}}{(1+k-2\alpha)+\sum_{m=2}^{\infty}(m+k-2\alpha)\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})\boldsymbol{a_m}\boldsymbol{z^{m-1}}}\right|$$

$$\le \frac{(1-k)+\sum_{m=2}^{\infty}(m-k)\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})|\boldsymbol{a_m}|\,|\boldsymbol{z^{m-1}}|}{(2\alpha-1-k)-\sum_{m=2}^{\infty}|(m+k-2\alpha)\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})||\boldsymbol{a_m}|\,|\boldsymbol{z^{m-1}}|}$$

$$< \frac{(1-k)+\sum_{m=2}^{\infty}(m-k)\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})|\boldsymbol{a_m}|}{(2\alpha-1-k)-\sum_{m=2}^{\infty}(m+k-2\alpha)\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})|\boldsymbol{a_m}|}$$

This expression is bounded above by 1 if,

$$(1-k)+\sum_{m=2}^{\infty}(m-k)\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})|\boldsymbol{a_m}| < (2\alpha-1-k)-\sum_{m=2}^{\infty}(m+k-2\alpha)\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})|\boldsymbol{a_m}|$$

which is equivalent to condition (2.1). Hence the proof.

Example: The function f given by

$$f(z) = z + \sum_{m=2}^{\infty}\frac{4(\alpha-1)}{m(m+1)\{(m-k)+|m+k-2\alpha|\}\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})}\,z^m$$ belong to the class $\boldsymbol{M_\lambda(a,c,m,\alpha)}$.

Now we discuss the coefficient estimates of functions $f \in M_\lambda(a,c,m,\alpha)$.

Theorem 2.2: $f \in M_\lambda(a,c,m,\alpha)$, then

$$|a_m| \le \frac{\prod_{j=1}^{m}(j+2\alpha-4)}{\boldsymbol{B}_\lambda(\boldsymbol{a},\boldsymbol{c},\boldsymbol{m},\boldsymbol{n})(\boldsymbol{m}-\boldsymbol{1})!} \tag{2.2}$$

Proof: Let us define the function $p(z)$ by,

$$p(z) = \frac{\alpha-\frac{z[SL_\lambda f]'}{SL_\lambda f}}{\alpha-1}$$

for $\boldsymbol{f} \in \boldsymbol{M_\lambda(a,c,m,\alpha)}$. Then $p(z)$ is analytic in U, $p(0) = 1\ and\ \mathbb{R}\{p(z)\} > 0$. If

$$p(z) = 1+\sum_{m=1}^{\infty}p_m z^m,\quad then\ |p_m| \le 2,\quad (m \ge 1).$$

Since,

$$\alpha SL_\lambda f - z[SL_\lambda f]' = (\alpha-1)p(z)SL_\lambda f$$

We obtain that,

$$(1-m)\boldsymbol{B_\lambda(a,c,m,n)a_m} = (\alpha-1)\{\boldsymbol{p_{m-1}} + \boldsymbol{p_{m-2}B_\lambda(a,c,2,n)a_2} + \cdots + \boldsymbol{p_1B_\lambda(a,c,m-1,n)a_{m-1}}\}.$$

If $m=2, then\ \boldsymbol{B_\lambda(a,c,2,n)a_2} \le (\alpha-1)p_1 implies\ that$

$$|a_2| \le \frac{(\alpha-1)|p_1|}{\boldsymbol{B_\lambda(a,c,2,n)}} \le \frac{2(\alpha-1)}{\boldsymbol{B_\lambda(a,c,2,n)}}.$$

Hence the coefficient estimate for (2.2) is true for $m=2$. Let us suppose that the coefficient estimate,

$$|a_k| \le \frac{\prod_{j=2}^{k}(j+2\alpha-4)}{\boldsymbol{B_\lambda(a,c,k,n)(k-1)}!}$$

is true for all $k = 2,3,4,\cdots,m$. Then we have,

$$-m\boldsymbol{B_\lambda(a,c,m+1,n)a_{m+1}} = (\alpha-1)\{\boldsymbol{p_m} + \boldsymbol{p_{m-2}B_\lambda(a,c,2,n)a_2} + \cdots + \boldsymbol{p_1B_\lambda(a,c,m,n)a_m}\}$$

So that,

$$\begin{aligned} m\boldsymbol{B_\lambda(a,c,m+1,n)}|\boldsymbol{a_{m+1}}| &\le \boldsymbol{(2\alpha-2)(1+B_\lambda(a,c,2,n)}|\boldsymbol{a_2}| + \cdots + \boldsymbol{B_\lambda(a,c,m,n)}|\boldsymbol{a_m}|) \\ &\le \boldsymbol{(2\alpha-2)}\left(\boldsymbol{1+(2\alpha-2)+\frac{(2\alpha-2)(2\alpha-1)}{2!}+\cdots+\frac{\prod_{j=2}^{m}(j+2\alpha-4)}{(m-1)!}}\right) \\ &= \boldsymbol{(2\alpha-2)}\left(\boldsymbol{\frac{(2\alpha-1)2\alpha(2\alpha+1)\cdots(2\alpha+m-4)}{(m-2)!}+\frac{(2\alpha-2)(2\alpha-1)2\alpha\cdots(2\alpha+m-4)}{(m-1)!}}\right) \\ &= \boldsymbol{\frac{\prod_{j=2}^{m+1}(j+2\alpha-4)}{(m-1)!}}. \end{aligned}$$

This implies

$$|\boldsymbol{a_{m+1}}| \le \boldsymbol{\frac{\prod_{j=2}^{m+1}(j+2\alpha-4)}{B_\lambda(a,c,m,n)m!}}.$$

Hence the coefficient estimate (2.2) holds true for the class $k = m+1$. Hence the theorem.

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