MULTIPLICATIVE CONNECTIVITY
REVAN INDICES OF CERTAIN FAMILIES OF BENZENOID SYSTEMS

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ABSTRACT

In Chemical Science, the multiplicative connectivity indices are applied to measure the chemical and biological characteristics of chemical compounds. In this paper, we introduce the multiplicative product connectivity Revan index, multiplicative sum connectivity Revan index, first multiplicative atom bond connectivity Revan index and multiplicative geometric-arithmetic Revan index of a molecular graph and compute these multiplicative connectivity Revan indices of some important chemical structures like triangular benzenoid, benzenoid rhombus and benzonoid hourglass.

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1. INTRODUCTION

In this paper, we consider only a finite, simple connected graph. Let G be a graph with a vertex set V(G) and an edge set E(G). The degree \( d_G(v) \) of a vertex \( v \) is the number of vertices adjacent to \( v \). Let \( \Delta(G) \) (\( \delta(G) \)) denote the maximum (minimum) degree among the vertices of \( G \). The Revan vertex degree of a vertex \( v \) in \( G \) is defined as \( r_G(v) = \Delta(G) + \delta(G) - d_G(v) \). The Revan edge connecting the Revan vertices \( u \) and \( v \) will be denoted by \( uv \). For additional definitions and notations, the reader may refer to [1].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical chemistry which has an important effect on the development of the Chemical Sciences. A topological index is a numerical parameter mathematically derived from the graph structure. In organic chemistry, topological indices have been found to be useful in chemical documentation, isomer discrimination, structure property relationships, structure activity relationships and pharmaceutical drug design. There has been considerable interest in the general problem of determining topological indices.

Best known and used topological indices are the multiplicative connectivity indices, introduced by Kulli in [2]. Motivated by the definitions of the multiplicative connectivity indices and their wide applications, we introduce the multiplicative product connectivity Revan index, multiplicative sum connectivity Revan index, multiplicative atom bond connectivity Revan index and multiplicative geometric-arithmetic Revan index of a molecular graph as follows:

The multiplicative product connectivity Revan index of a graph \( G \) is defined as

\[
PRII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{r_G(u) r_G(v)}}
\]

The multiplicative sum connectivity Revan index of a graph \( G \) is defined as

\[
SRII(G) = \prod_{uv \in E(G)} \frac{1}{r_G(u) + r_G(v)}
\]

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The first multiplicative atom bond connectivity Revan index of a graph $G$ is defined as
\[
ABC,RII(G) = \prod_{u,v \in E(G)} \sqrt{r_G(u) + r_G(v) - 2 \over r_G(u) r_G(v)}
\]

The multiplicative geometric-arithmetic Revan index of a graph $G$ is defined as
\[
GARI(G) = \prod_{u,v \in E(G)} 2\sqrt{r_G(u)r_G(v)} \over r_G(u) + r_G(v)
\]

Recently many multiplicative topological indices were studied, for example, in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 14, 15, 16]. Also some connectivity indices were studied, for example, in [17, 18, 19, 20, 21, 22]. In this paper, we compute multiplicative connectivity Revan indices of triangular benzenoids, benzenoid rhombus and benzenoid hourglass. For more information about these benzenoids see [23, 24].

2. RESULTS FOR TRIANGULAR BENZENOID $T_p$

In this section, we consider the graph of triangular benzenoid $T_p$ which $p$ is the number of hexagons in the base graph. Clearly $T_p$ has $\frac{1}{2}p(p + 1)$ hexagons. The graph of triangular benzenoid $T_4$ is presented in Figure 1.

![Figure-1: The graph of triangular benzenoid $T_4$.](image)

Let $G$ be the graph of a triangular benzenoid $T_p$. By algebraic method, we obtain $|V(T_p)| = p^2 + 4p + 1$ and $|E(T_p)| = \frac{3}{2}p(p + 3)$. Also by algebraic method, we obtain that the edge set $E(G)$ can be divided into three partitions:

- $E_{22} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_{22}| = 6$.
- $E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 6p - 6$.
- $E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = \frac{3}{2}p(p - 1)$.

Clearly $\Delta(G) = 3$ and $\delta(G) = 2$. Therefore $r_G(u) = 5 - d_G(u)$. Thus we ensure that there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as follows:

- $RE_{33} = \{uv \in E(G) \mid r_G(u) = r_G(v) = 3\}$, $|RE_{33}| = 6$.
- $RE_{23} = \{uv \in E(G) \mid r_G(u) = 3, d_G(v) = 2\}$, $|RE_{23}| = 6(p - 1)$.
- $RE_{22} = \{uv \in E(G) \mid r_G(u) = r_G(v) = 2\}$, $|RE_{22}| = \frac{3}{2}p(p - 1)$.

In the following theorem, we compute the multiplicative product connectivity Revan index of $T_p$.

**Theorem 1:** The multiplicative product connectivity Revan index of a triangular benzenoid $T_p$ is given by
\[
PRII(T_p) = \left(\frac{1}{3}\right)^6 \times \left(\frac{1}{6}\right)^{3(p-1)} \times \left(\frac{1}{2}\right)^{2(p-1)}.
\]

**Proof:** By definition, we have $PRII(G) = \prod_{u,v \in E(G)} 1 \over \sqrt{r_G(u) r_G(v)}$. Thus
\[
PRII(T_p) = \left(\frac{1}{\sqrt{3 \times 3}}\right)^6 \times \left(\frac{1}{\sqrt{3 \times 2}}\right)^{6(p-1)} \times \left(\frac{1}{\sqrt{2 \times 2}}\right)^{3p(p-1)}.
\]

Therefore
\[
PRII(T_p) = \left(\frac{1}{3}\right)^6 \times \left(\frac{1}{6}\right)^{3(p-1)} \times \left(\frac{1}{2}\right)^{2(p-1)}.
\]
In the following theorem, we compute the multiplicative sum connectivity Revan index of $T_p$.

**Theorem 2:** The multiplicative sum connectivity Revan index of a triangular benzenoid $T_p$ is given by

$$SRII(T_p) = \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{5}\right)^{\frac{6}{3}(p-1)} \times \left(\frac{1}{2}\right)^{\frac{3}{2}(p-1)}.$$ 

**Proof:** By definition, we have $SRII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{r_G(u) + r_G(v)}}$.

Thus $SRII(T_p) = \left(\frac{1}{\sqrt{3+3}}\right)^6 \times \left(\frac{1}{\sqrt{3+2}}\right)^{\frac{6}{3}(p-1)} \times \left(\frac{1}{\sqrt{2+2}}\right)^{\frac{3}{2}(p-1)}$.

$$= \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{5}\right)^{\frac{6}{3}(p-1)} \times \left(\frac{1}{2}\right)^{\frac{3}{2}(p-1)}.$$ 

In the following theorem, we compute the multiplicative atom bond connectivity Revan index of $T_p$.

**Theorem 3:** The first multiplicative atom bond connectivity Revan index of a triangular benzenoid $T_p$ is given by

$$ABCRII(T_p) = \left(\frac{2}{3}\right)^6 \times \left(\frac{1}{2}\right)^{\frac{3}{2}(p-1)}.$$ 

**Proof:** By definition, we have $ABCRII(T_p) = \prod_{uv \in E(G)} \sqrt{r_G(u) + r_G(v) - 2} - r_G(u)r_G(v)$.

Thus $ABCRII(T_p) = \left(\frac{3+3-2}{3 \times 3}\right)^6 \times \left(\frac{3+2-2}{3 \times 2}\right)^{\frac{6}{3}(p-1)} \times \left(\frac{2+2-2}{2 \times 2}\right)^{\frac{3}{2}(p-1)}$.

$$= \left(\frac{2}{3}\right)^6 \times \left(\frac{1}{2}\right)^{\frac{3}{2}(p-1)}.$$ 

In the following theorem, we compute the multiplicative geometric-arithmetic Revan index of $T_p$.

**Theorem 4:** The multiplicative geometric-arithmetic Revan index of a triangular benzenoid $T_p$ is given by

$$GARII(T_p) = \left(\frac{2\sqrt{6}}{5}\right)^{6(p-1)}.$$ 

**Proof:** By definition, we have $GARII(T_p) = \prod_{uv \in E(G)} \frac{2r_G(u)r_G(v)}{r_G(u) + r_G(v)}$.

Thus $GARII(T_p) = \left(\frac{2\sqrt{3\times 3}}{3 + 3}\right)^6 \times \left(\frac{2\sqrt{3\times 2}}{3 + 3}\right)^{\frac{6}{3}(p-1)} \times \left(\frac{2\sqrt{2\times 2}}{2 + 2}\right)^{\frac{3}{2}(p-1)}$.

$$= \left(\frac{2\sqrt{6}}{5}\right)^{6(p-1)}.$$ 

3. RESULTS FOR BENZENOID RHOMBUS $R_p$

In this section, we consider the graph of benzenoid rhombus $R_p$ which is obtained from two copies of a triangular benzenoid $T_p$ by identifying hexagons in one of their base rows. The graph of benzenoid rhombus $R_4$ is presented in Figure 2.
Let \( G \) be the graph of a benzenoid rhombus \( R_p \). By algebraic method, we obtain \(|V(T_p)| = 2p^2 + 4p \) and \( |E(R_p)| = 3p^2 + 4p - 1 \). It is easy to see that the vertices of benzenoid rhombus \( R_p \) are either of degree 2 or 3, see Figure 2. Therefore \( \Delta(G)=3 \) and \( \delta(G)=2 \). Thus \( r_G(u) = 5 - d_G(u) \). By calculation, we obtain that the edge set \( E(R_p) \) can be divided into three partitions:

\[
E_{22} = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 2 \} \quad \text{and} \quad |E_{22}| = 6.
\]

\[
E_{23} = \{ uv \in E(G) \mid d_G(u) = 2, \text{ } d_G(v) = 3 \} \quad \text{and} \quad |E_{23}| = 8(p-1).
\]

\[
E_{33} = \{ uv \in E(G) \mid d_G(u) = d_G(v) = 3 \} \quad \text{and} \quad |E_{33}| = 3p^2 - 4p + 1.
\]

Thus there are three types of Revan edges as follows:

\[
RE_{33} = \{ uv \in E(G) \mid r_G(u) = r_G(v) = 3 \} \quad \text{and} \quad |RE_{33}| = 6.
\]

\[
RE_{32} = \{ uv \in E(G) \mid r_G(u) = 3, \text{ } r_G(v) = 2 \} \quad \text{and} \quad |RE_{32}| = 8(p-1).
\]

\[
RE_{22} = \{ uv \in E(G) \mid r_G(u) = r_G(v) = 2 \} \quad \text{and} \quad |RE_{22}| = 3p^2 - 4p + 1.
\]

Theorem 5: The multiplicative product connectivity Revan index of a benzenoid rhombus \( R_p \) is given by

\[
PRII(R_p) = \left( \frac{1}{3} \right)^6 \times \left( \frac{1}{6} \right)^{4(p-1)} \times \left( \frac{1}{2} \right)^{3p^2 - 4p + 1}.
\]

Proof: To compute \( PRII(R_p) \), we see that

\[
PRII(R_p) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) \cdot d_G(v)}} = \left( \frac{1}{\sqrt{3 \times 3}} \right)^6 \times \left( \frac{1}{\sqrt{3 \times 2}} \right)^{8(p-1)} \times \left( \frac{1}{\sqrt{2 \times 2}} \right)^{3p^2 - 4p + 1}.
\]

Theorem 6: The multiplicative sum connectivity Revan index of a benzenoid rhombus \( R_p \) is given by

\[
SRII(R_p) = \left( \frac{1}{6} \right)^3 \times \left( \frac{1}{5} \right)^{4(p-1)} \times \left( \frac{1}{2} \right)^{3p^2 - 4p + 1}.
\]

Proof: To compute \( SRII(R_p) \), we see that

\[
SRII(R_p) = \prod_{uv \in E(G)} \frac{1}{r_G(u) + r_G(v)} = \left( \frac{1}{\sqrt{3+3}} \right)^6 \times \left( \frac{1}{\sqrt{3+2}} \right)^{8(p-1)} \times \left( \frac{1}{\sqrt{2+2}} \right)^{3p^2 - 4p + 1}.
\]

Theorem 7: The first multiplicative atom bond connectivity Revan index of a benzenoid rhombus \( R_p \) is given by

\[
ABC_{1RII}(R_p) = \left( \frac{2}{3} \right)^6 \times \left( \frac{1}{2} \right)^{4(p-1)} \times \left( \frac{1}{\sqrt{2}} \right)^{3p^2 - 4p + 1}.
\]

Proof: To compute \( ABC_{1RII}(R_p) \), we see that
\[ ABC_{RII}(T_p) = \prod_{uv \in E(G)} \frac{r_G(u) + r_G(v) - 2}{r_G(u) r_G(v)}. \]

\[ = \left( \sqrt{\frac{3+3-2}{3\times3}} \right)^6 \times \left( \sqrt{\frac{3+2-2}{3\times2}} \right)^{8(p-1)} \times \left( \frac{3p^2-4p+1}{2+2} \right)^{3p^2-4p+1}. \]

\[ = \left( \frac{2}{3} \right)^6 \times \left( \frac{1}{2} \right)^{4(p-1)} \times \left( \frac{1}{\sqrt{2}} \right)^{3p^2-4p+1}. \]

In the following theorem, we compute the multiplicative atom bond connectivity Revan index of \( T_p \).

**Theorem 8:** The multiplicative geometric-arithmetic Revan index of a triangular benzenoid \( R_p \) is given by

\[ GARII(T_p) = \left( \frac{2\sqrt{6}}{5} \right)^{8(p-1)}. \]

**Proof:** To compute \( GARII(R_p) \), we see that

\[ GARII(R_p) = \prod_{uv \in E(G)} \frac{2r_G(u) r_G(v)}{r_G(u) + r_G(v)}. \]

\[ = \left( \frac{2\sqrt{3} \times 3}{3+3} \right)^6 \times \left( \frac{2\sqrt{3} \times 2}{3+2} \right)^{8(p-1)} \times \left( \frac{2\sqrt{2} \times 2}{2+2} \right)^{3p^2-4p+1}. \]

\[ = \left( \frac{2\sqrt{6}}{5} \right)^{8(p-1)}. \]

**4. RESULTS FOR BENZENOID HOURGLASS \( X_p \)**

In this section, we consider the graph of benzenoid hourglass \( X_p \) which is obtained from two copies of a triangular benzenoid \( T_p \) by overlapping hexagons. The graph of benzenoid hourglass is shown in Figure 3.

![Figure-3](image-url) The graph of benzenoid hourglass

Let \( G \) be the graph of a benzenoid hourglass \( X_p \). By algebraic method, we obtain \( |V(X_p)| = 2(p^2 + 4p - 2) \) and \( |E(X_p)| = 3p^2 + 9p - 4 \). It is easy to see that the vertices of benzenoid hourglass \( X_p \) are either of degree 2 or 3, see Figure 3. Therefore \( \Delta(G) = 3 \) and \( \delta(G) = 2 \). Thus \( d_G(u) = 5 - d_G(u) \). By algebraic method, we obtain that the edge set \( E(X_p) \) can be divided into three partitions:

- \( E_{22} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\} \quad |E_{22}| = 8 \)
- \( E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\} \quad |E_{23}| = 4(3p - 4) \)
- \( E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\} \quad |E_{33}| = 3p^2 - 3p + 4 \).

Thus there are three types of Revan edges as follows:

- \( RE_{23} = \{uv \in E(G) \mid r_G(u) = r_G(v) = 3\} \quad |RE_{23}| = 8 \)
- \( RE_{22} = \{uv \in E(G) \mid r_G(u) = 3, r_G(v) = 2\} \quad |RE_{22}| = 4(3p - 4) \)
- \( RE_{22} = \{uv \in E(G) \mid r_G(u) = r_G(v) = 2\} \quad |RE_{22}| = 3p^2 - 3p + 4 \).
We compute the multiplicative connectivity Revan indices of a benzenoid hourglass $X_p$.

**Theorem 9:** The multiplicative product connectivity Revan index of a benzenoid hourglass $X_p$ is given by

$$PRII(X_p) = \left(\frac{1}{3}\right)^8 \times \left(\frac{1}{6}\right)^{2(3p-4)} \times \left(\frac{1}{2}\right)^{3p^2-3p+4}.$$ 

**Proof:** To compute $PRII(X_p)$, we see that

$$PRII(R_p) = \prod_{uv \in E(G)} \frac{1}{\sqrt{r_G(u)r_G(v)}}$$

$$= \left(\frac{1}{\sqrt{3 \times 3}}\right)^8 \times \left(\frac{1}{\sqrt{3 \times 2}}\right)^{4(3p-4)} \times \left(\frac{1}{\sqrt{2 \times 2}}\right)^{3p^2-3p+4}.$$ 

$$= \left(\frac{1}{3}\right)^8 \times \left(\frac{1}{6}\right)^{2(3p-4)} \times \left(\frac{1}{2}\right)^{3p^2-3p+4}.$$ 

**Theorem 10:** The multiplicative sum connectivity Revan index of a benzenoid hourglass $X_p$ is given by

$$SRII(X_p) = \left(\frac{1}{6}\right)^4 \times \left(\frac{1}{5}\right)^{2(3p-4)} \times \left(\frac{1}{2}\right)^{3p^2-3p+4}.$$ 

**Proof:** To compute $SRII(X_p)$, we see that

$$SRII(X_p) = \prod_{uv \in E(G)} \frac{1}{\sqrt{r_G(u)+r_G(v)}}$$

$$= \left(\frac{1}{\sqrt{3+3}}\right)^8 \times \left(\frac{1}{\sqrt{3+2}}\right)^{4(3p-4)} \times \left(\frac{1}{\sqrt{2+2}}\right)^{3p^2-3p+4}.$$ 

$$= \left(\frac{1}{6}\right)^4 \times \left(\frac{1}{5}\right)^{2(3p-4)} \times \left(\frac{1}{2}\right)^{3p^2-3p+4}.$$ 

**Theorem 11:** The first multiplicative atom bond connectivity Revan index of a benzenoid hourglass $X_p$ is given by

$$ABC_1RII(X_p) = \left(\frac{2}{3}\right)^8 \times \left(\frac{1}{2}\right)^{2(3p-4)} \times \left(\frac{1}{\sqrt{2}}\right)^{3p^2-2p+4}.$$ 

**Proof:** To compute $ABC_1RII(X_p)$, we see that

$$ABC_1RII(X_p) = \prod_{uv \in E(G)} \sqrt{r_G(u)+r_G(v)-\frac{2}{r_G(u)r_G(v)}}$$

$$= \left(\frac{3+3-2}{3 \times 3}\right)^8 \times \left(\frac{3+2-2}{3 \times 2}\right)^{4(3p-4)} \times \left(\frac{2+2-2}{2 \times 2}\right)^{3p^2-3p+4}.$$ 

$$= \left(\frac{2}{3}\right)^8 \times \left(\frac{1}{2}\right)^{2(3p-4)} \times \left(\frac{1}{\sqrt{2}}\right)^{3p^2-4p+4}.$$ 

**Theorem 12:** The multiplicative geometric-arithmetic Revan index of a benzenoid hourglass $X_p$ is given by

$$GARII(T_p) = \left(\frac{2\sqrt{6}}{5}\right)^{4(3p-4)}.$$ 

**Proof:** To compute $GARII(T_p)$, we see that

$$GARII(X_p) = \prod_{uv \in E(G)} \frac{\sqrt{r_G(u)r_G(v)}}{r_G(u)+r_G(v)}$$

$$= \left(\frac{2\sqrt{6}}{5}\right)^{4(3p-4)}.$$


\[
\begin{align*}
&= \left( \frac{2\sqrt{3} \times 3}{3 + 3} \right)^8 \times \left( \frac{2\sqrt{2} \times 2}{3 + 2} \right)^{4(3p-4)} \times \left( \frac{2\sqrt{2} \times 2}{2 + 2} \right)^{3p^2 - 3p + 4} \\
&= \left( \frac{2\sqrt{6}}{5} \right)^{4(3p-4)}. 
\end{align*}
\]

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