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EFFECT OF ROTATORY SYSTEM ON MHD FREE CONVECTION AND MASS TRANSFER FLOW OF OLDROYD FLUID THROUGH POROUS MEDIUM WITH CONSTANT HEAT AND MASS FLUX ACROSS MOVING PLATE

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ABSTRACT

The purpose of the problem is to study the effect of rotatory system on two-dimensional free convection and mass transfer flow of Oldroyd fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field. The expressions for velocity, temperature and concentration are obtained. The primary and secondary velocities are discussed with the help of graphs and tables.

Keywords: *Oldroyd Fluid, Porous medium, MHD Flow, Heat transfer, Mass transfer, Rotatory system.*

INTRODUCTION

The effect of variable permeability on combined free and forced convection in porous media was studied by Chandrasekhara and Namboodiri [3]. Later on mixed convection in porous media adjacent to a vertical uniform heat flux surface was studied by Joshi and Gebhart [5]. Heat and mass transfer in a porous medium was discussed by Bejan and Khair [2]. The above problem was studied in presence of buoyancy effect by Trevisan and Bejan [10]. Lai and Kulacki [6] studied the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. The free convection effect on the flow of an ordinary viscous fluid past an infinite vertical porous plate with constant suction and constant heat flux was investigated by Sharma [8]. The study two dimensional flow through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in presence of free convection current was studied by Sharma [9]. Convection in a porous medium with inclined temperature gradient was investigated by Nield [7]. The problem of mixed convection along an isothermal vertical plate in porous medium with injection and suction was studied by Hooper et al [4]. Acharya et al [1] have discussed magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Varshney and Kumar [12] have studied the unsteady effect on MHD free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Recently, Varshney et al [11] have studied unsteady effect on MHD free convection and mass transfer flow of oldroyd fluid through porous medium with constant suction and constant heat and mass flux.

In present study, we consider the problem of Varshney et al [11] in rotatory system.

FORMULATION OF THE PROBLEM

We consider two dimensional motion of Oldroyd fluid through a porous medium occupying semi-infinite region of space bounded by a vertical infinite surface at Z = 0 moving with velocity decreasing exponentially with time. Let the fluid and the surface be in a state of rigid rotation with constant angular velocity Ω about Z-axis, taken normal to the surface. A constant transverse magnetic field B_O is acting parallel to the axis of rotation. The effect of induced magnetic field is neglected. The Reynolds number is assumed to be small. Since the length of the surface is large, therefore, all the physical variables depend on Z only. In the present problem, magnetic field B_O is assumed to be constant throughout the motion. We further assume that the electric field is equal to zero. Hence, by the usual boundary layer approximation the basic equations for unsteady flow through porous medium in rotating system are :

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

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$$(1+\lambda_0 \frac{\partial}{\partial t})\frac{\partial U}{\partial t} + W\frac{\partial U}{\partial Z} - 2\Omega V = \nu\,(1+\alpha_1 \frac{\partial}{\partial t_0})\frac{\partial^2 U}{\partial Z^2} + g\beta(T-T_\infty) + g\beta'(C-C_\infty) - (\sigma\frac{B_0^2}{P} - \frac{\nu}{K_0})\,(1+\lambda_0 \frac{\partial}{\partial t})U \quad (2)$$

$$(1+\lambda_0 \frac{\partial}{\partial t})\frac{\partial V}{\partial t} + W\frac{\partial V}{\partial Z} - 2\Omega V = \nu\,(1+\alpha_1 \frac{\partial}{\partial t_0})\frac{\partial^2 V}{\partial Z^2}\,(\sigma\frac{B_0^2}{P} - \frac{\nu}{K_0})\,(1+\lambda_0 \frac{\partial}{\partial t})V \quad (3)$$

$$\frac{\partial T}{\partial t} + W\,\frac{\partial T}{\partial Z} = \frac{\lambda}{\rho c_p}\,\frac{\partial^2 T}{\partial Z^2} \quad (4)$$

$$\frac{\partial C}{\partial t} + W\,\frac{\partial C}{\partial Z} = D\,\frac{\partial^2 T}{\partial Z^2} \quad (5)$$

where ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β′ is the coefficient of concentration expansion, ν is the Kinematic viscosity, T_∞ is the temperature of the fluid in the free stream, C_∞ is the concentration at infinite, σ is the electric conductivity, B_o is the magnetic induction, K_o is the permeability of porous medium, λ is the thermal conductivity, D is the concentration diffusivity, C_p is the specific heat at constant pressure, Ω is the angular velocity, λ_o is the coefficient of viscoelastic and α_1 is the coefficient of Oldroyd viscoelastic.

METHOD OF SOLUTION

The equation of continuity (1) gives

$$W = \text{constant} = -W_0 \quad (6)$$

where $W_0 > 0$ corresponds to steady suction velocity at the surface. In view of equation (6), Equations (2) to (5) can be written as

$$(1+\lambda_0 \frac{\partial}{\partial t})\frac{\partial U}{\partial t} + W_0\,\frac{\partial U}{\partial Z} - 2\Omega V = \nu\,(1+\alpha_1 \frac{\partial}{\partial t_0})\frac{\partial^2 U}{\partial Z^2} + g\beta(T-T_\infty) + g\beta'(C-C_\infty) - (\sigma\frac{B_0^2}{P} - \frac{\nu}{K_0})\,(1+\lambda_0 \frac{\partial}{\partial t})U \quad (7)$$

$$(1+\lambda_0 \frac{\partial}{\partial t})\frac{\partial V}{\partial t} - W_0\,\frac{\partial V}{\partial Z} + 2\Omega V = \nu\,(1+\alpha_1 \frac{\partial}{\partial t_0})\frac{\partial^2 V}{\partial Z^2}\,(\sigma\frac{B_0^2}{P} - \frac{\nu}{K_0})\,(1+\lambda_0 \frac{\partial}{\partial t})V \quad (8)$$

$$\frac{\partial T}{\partial t} - W_0\,\frac{\partial T}{\partial Z} = \frac{\lambda}{\rho c_p}\,\frac{\partial^2 T}{\partial Z^2} \quad (9)$$

$$\frac{\partial C}{\partial t} + W_0\,\frac{\partial C}{\partial Z} = D\,\frac{\partial^2 C}{\partial Z^2} \quad (10)$$

The boundary conditions of the problem are

$$U = u_0 e^{-nt},\ V = 0,\ \partial T/\partial Z = -q/\lambda,\ \partial C/\partial Z = -m/D \text{ at } Z = 0,\ t = 0 \quad (11)$$

$$U \to 0\ ,\ V \to 0,\ T \to T_\infty,\ C \to C_\infty \quad \text{as} \quad Z \to \infty,\ t > 0$$

where q is the heat flux per unit area and m is the mass flux per unit area. On introducing the following non dimensional quantities into equations (7) to (10) :

$$U^* = \frac{U}{W_0},\quad V^* = \frac{V}{W_0},\quad Z^* = \frac{W_0 Z}{\nu},\ \theta = \frac{(T-T_\infty)W_0\lambda}{q\nu}$$

$$\phi = \frac{(C-C_\infty)W_0 D}{m\nu},\ \tau^* = \frac{W_0^2 t}{\nu}$$

$P_r = \frac{\pi C_p}{\lambda}$ (Prandtl Number)

$S_c = \frac{v}{\upsilon}$ (Schimdt Number),

$\propto = \frac{W_0^2\, K_0}{v^2}$ (Porosity Parameter)

$M = \frac{\sigma B_0^2\, v}{\rho W_0^2}$ (Magnetic Number),

$\Omega^* \frac{v\Omega}{W_0^2} =$ (Rotation velocity parameter)

$G_r = g\beta \frac{q v^2}{W_0^4\, \lambda}$ (Grashof number),

$G_m = g\beta' \frac{m\, v^2}{W_0^4\, D}$ (Modified Grashof number),

$\lambda_1 = \frac{\lambda_0\, V_0^2}{v}$ (Viscoelastic parameter)

$\alpha_2 = \frac{\alpha_1\, V_0^2}{v}$ (Oldroyd viscoelastic parameter)

We get the following equations after dropping star (*)

$$-(1 + \lambda_1 M_1)\frac{\partial Q}{\partial t} + \frac{\partial^2 Q}{\partial Z^2} + \frac{\delta Q}{\delta Z} - \lambda_1 \frac{\partial^2 Q}{\partial t^2} + \alpha_2 \frac{\partial}{\partial t}\left(\frac{\partial^2 Q}{\partial Z^2}\right)$$

$$-(M_1 + 2i\Omega) - -G_r O - G_m \Phi \tag{12}$$

$$-P_r \frac{\partial\theta}{\partial t} + \frac{\partial^2\theta}{\partial Z^2} + P_r \frac{\partial\theta}{\partial Z} = 0 \tag{13}$$

$$-S_c \frac{\partial\phi}{\partial t} + \frac{\partial^2\phi}{\partial Z^2} + S_c \frac{\partial\phi}{\partial Z} = 0 \tag{14}$$

Where $Q = U + iV, M_1 = \alpha^{-1} + M$

The corresponding boundary conditions become

$Z = 0, \quad Q = q_o e^{-nt}, \quad \theta' = -1, \quad \phi' = -1$ (15)

$Z \to \infty, \quad Q \to 0, \quad \theta \to 0, \quad \phi \to 0$

We assume the solution of

$$Q(Z,t) = Q_0(Z)e^{-nt}$$
$$\theta(Z,t) = \theta_0(Z)e^{-nt}$$
$$\phi(Z,t) = \phi_0(Z)e^{-nt} \tag{16}$$

Substituting equation (16) into equations (12), (13) and (14), we find

$$(1 - n\alpha_2)\, Q_0'' + Q_0' - Q_0\{(M_1 - n)(1 - n\lambda_1) + 2i\Omega\} = -G_r\theta_0 - G_m\Phi_0 \tag{17}$$

$$\theta_0'' + P_r\theta_0' + nP_r\theta_0 = 0 \tag{18}$$

$$\phi_0'' + S_c\phi_0' + nS_c\phi_0 = 0 \tag{19}$$

with corresponding boundary conditions

$$Q_o = q_o, \quad \theta_o' = -1, \quad \phi_o' = -1 \quad \text{at} \quad Z = 0$$
$$\theta_o \to 0, \quad \phi_o \to 0, \quad \text{as} \quad Z \to \infty \tag{20}$$

Solving equations (17)-(19) under boundary conditions (20), we get

$$Q_0 = (q_0 + A_4G_r + A_5G_m)e^{-A_1z} - A_4G_re^{-A_2z} - A_5G_me^{-A_3z} \tag{21}$$

$$\theta_0 \frac{1}{A_2} e^{-A_2z} \tag{22}$$

$$\phi_0 \frac{1}{A_3} e^{-A_3z} \tag{23}$$

where

$$A_1 = \frac{1 + [1 + 4(1 - n\alpha_2)\{(1 - n\lambda_1)(M_1 - n) + 2i\Omega\}]^{1/2}}{2(1 - n\alpha_2)}$$

$$A_2 = \frac{P_r + [P_r^2 - 4nP_r]^{1/2}}{2}$$

$$A_3 = \frac{S_c + [S_c^2 - 4nS_c]^{1/2}}{2}$$

$$A_4 = \frac{1}{A_2[(1 - n\alpha_2)A_2^2 - A_2 - \{(M_1 - n)(1 - n\lambda_1) + 2i\Omega\}]}$$

$$A_5 = \frac{1}{A_3[(1 - n\alpha_2)A_3^2 - A_3 - \{(M_1 - n)(1 - n\lambda_1) + 2i\Omega\}]}$$

Hence, The equations for Q, θ and ϕ will be as follows

$$Q - \left[(q_0 + A_4G_r + A_5G_m)e^{-A_1z} - A_4G_re^{-A_2z} - A_5G_me^{-A_3z}\right].e^{-nt} \tag{24}$$

$$\theta = \frac{1}{A_2} e^{-A_2z}.e^{-nt} \tag{25}$$

$$\phi = \frac{1}{A_3} e^{-A_3z}.e^{-nt} \tag{26}$$

The primary velocity U (real part of Q) and secondary velocity V (imaginary part of Q) from equation (24) are given as

$$U = [\{(q_0 + G_rK_1 + G_mK_3)\cos(b_1Z) + (G_rK_2 + G_mK_4)\sin(b_1Z)\}$$
$$e^{-a_1z} - G_rK_1e^{-a_2z} - G_mK_3e^{-a_3z}].e^{-nt} \tag{27}$$

$$V = [\{(G_rK_2 + G_mK_4)\cos(b_1Z) - (G_rK_1 + G_mK_3)\sin(b_1Z)\}e^{-a_1z}$$
$$-G_rK_2e^{-a_2z} - G_mK_4e^{-a_3z}].e^{-nt} \tag{28}$$

where

$$a = \left[\frac{\{(1+4M_3)^2 + 64\,\Omega_1^2\}^{1/2} + \{1+4M_3\}}{2}\right]^{1/2}$$

$$b = \left[\frac{\{(1+4M_3)^2 + 64\,\Omega_1^2\}^{1/2} - \{1+4M_3\}}{2}\right]^{1/2}$$

$$M_2 = (M_1 - n)(1 - n\lambda_1), \qquad M_3 = M_2(1 - n\alpha_2), \qquad \Omega_1 = (1 - n\alpha_2)\Omega$$

$$a_1 = \frac{1+a}{2(1-n\alpha_2)}, \qquad b_1 = \frac{b}{2(1-n\alpha_2)}$$

$$K_1 = \frac{\{(1-n\alpha_2)A_2^2 - A_2 - M_2\}}{A_2[\{(1-n\alpha_2)A_2^2 - A_2 - M_2\}^2 + 4\Omega^2]},$$

$$K_2 = \frac{2\Omega}{A_2[\{(1-n\alpha_2)A_2^2 - A_2 - M_2\}^2 + 4\Omega^2]}$$

$$K_3 = \frac{\{(1-n\alpha_2)A_3^2 - A_3 - M_2\}}{A_3[\{(1-n\alpha_2)A_3^2 - A_3 - M_2\}^2 + 4\Omega^2]},$$

$$K_4 = \frac{2\Omega}{A_3[\{(1-n\alpha_2)A_3^2 - A_3 - M_2\}^2 + 4\Omega^2]}$$

RESULTS AND DISCUSSION

The primary and secondary velocity profiles are plotted in Fig.-1 and 2 having Graphs from 1 to 4 for $P_r = 0.71$, $\alpha = 10$, $S_c = 0.6$, $n = 0.1$, $G_r = 5$, $G_m = 4$, $\lambda_1 = 2$, $\alpha_2 = 2$, $n = 0.1$, $q_o = 1$, $t = 2$ and different values of M (Hartman No.), Ω (Rotation velocity parameter) .

	M	Ω
For Graph-1	2	0
For Graph-2	2	0.5
For Graph-3	2	1.0
For Graph-4	6	0.5

From Graphs-1 to 4 of Fig.-1, it is found that the primary velocity U increases sharply till Z = 1.2 (near the wall) after it U decreases sharply till Z = 3.5 then after it primary velocity decreases continuously with the increase in Z. It is also observed that primary velocity U decreases with the increase in M and Ω.

From Graph-1 of Fig.-2 it is noticed that secondary velocity is zero in the absence of rotation velocity. From Graphs - 2 to 4 of Fig.-2, it is found that the secondary velocity V decreases sharply till Z = 1.2 (near the wall) after it secondary velocity increases sharply till Z = 3.5 then after it secondary velocity increases continuously with the increase in Z and merges to Z-axis. It is also observed that secondary velocity V increases with the increase in M, but decreases with the increase in Ω.

PARTICULAR CASE

When Ω is equal to zero, this problem reduces to the problem of Varshney et al [11].

CONCLUSION

The primary and secondary velocities of Oldroyd fluid decrease with the increase in Ω.

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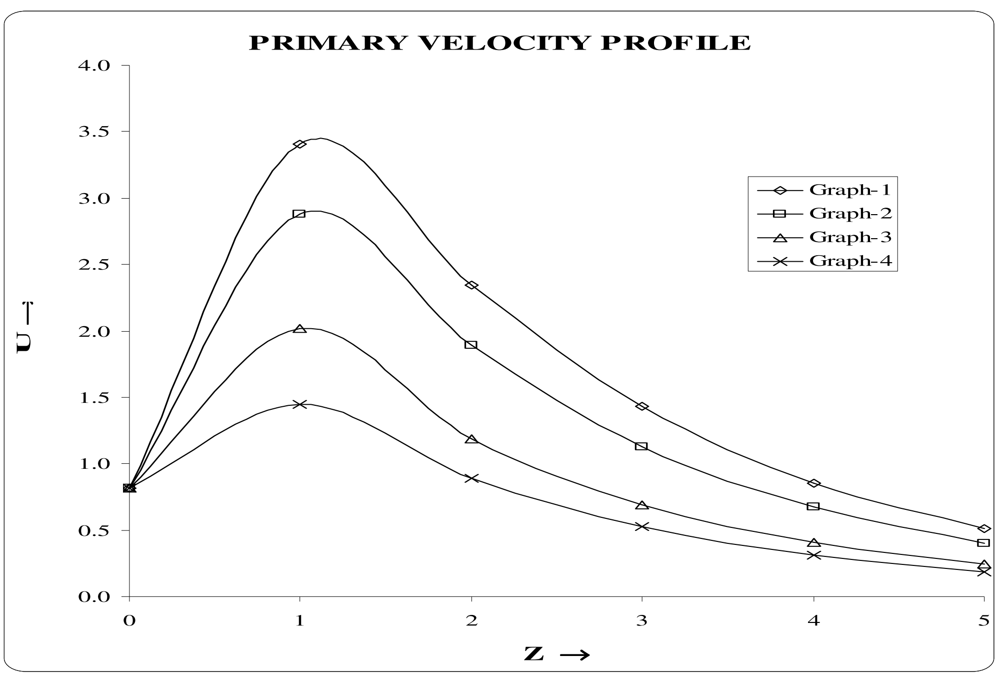

Fig.-1

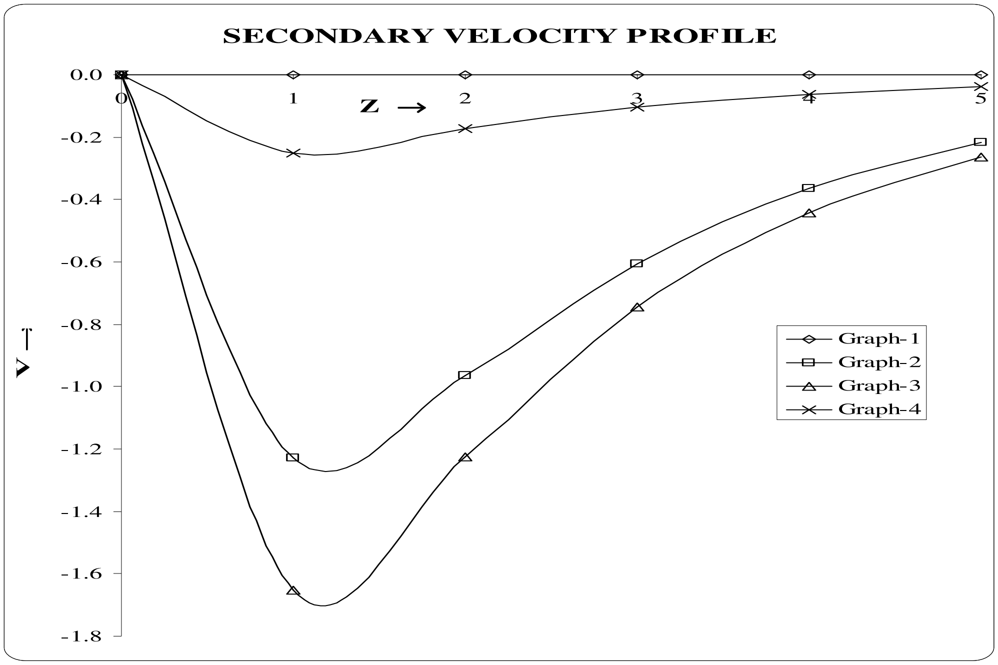

Fig.-2

Table-1 : Values of Primary velocity U at $P_r = 0.71$, $\alpha = 10$, $S_c = 0.6$, $n = 0.1$, $G_r = 5$, $G_m = 4$, $\lambda_1 = 2$, $\alpha_2 = 2$, $n = 0.1$, $q_o = 1$, $t = 2$ and different values of *M and Ω*

Z	*Graph-1*	*Graph-2*	*Graph-3*	*Graph-4*
0	***0.81873***	***0.81873***	***0.81873***	***0.81873***
1	***3.40582***	***2.88118***	***2.01513***	***1.44795***
2	***2.34690***	***1.89145***	***1.19064***	***0.89238***
3	***1.42931***	***1.13070***	***0.69109***	***0.53019***
4	***0.85430***	***0.67205***	***0.40882***	***0.31531***
5	***0.51013***	***0.40075***	***0.24372***	***0.18810***

Table-2 **: Values of Secondary velocity V at $P_r = 0.71$, $\alpha = 10$, $S_c = 0.6$, $n = 0.1$, $G_r = 5$, $G_m = 4$, $\lambda_1 = 2$, $\alpha_2 = 2$, $n = 0.1$, $q_o = 1$, $t = 2$ and different values of *M and Ω***

Z	*Graph-1*	*Graph-2*	*Graph-3*	*Graph-4*
0	***0.00000***	***0.00000***	***0.00000***	***0.00000***
1	***0.00000***	***-1.22710***	***-1.65287***	***-0.25002***
2	***0.00000***	***-0.96475***	***-1.22403***	***-0.17288***
3	***0.00000***	***-0.60560***	***-0.74573***	***-0.10408***
4	***0.00000***	***-0.36409***	***-0.44404***	***-0.06198***
5	***0.00000***	***-0.21762***	***-0.26480***	***-0.03697***

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