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Γ-SEMI NORMAL SUB NEAR-FIELD SPACES OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD PART IV

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ABSTRACT

In this paper, I Dr N V Nagendram as an author introduce the Gamma-semi normal sub near-field spaces in Γ-near-field space over a near-field PART IV, Dr. N V Nagendram together investigate the related properties of generalization of a Gamma-semi normal sub near-field spaces in Γ-near-field space over a near-field.

Keywords: *Γ-near-field space; Γ-Semi normal sub near-field space of Γ-near-field space; Semi near-field space of Γ-near-field space.*

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SECTION-1: INTRODUCTION

In this paper, Part IV consisting important sections I introduce the Γ-semi normal sub near-field spaces in Γ-near-field space over a near-field, and Dr. N V Nagendram being an author investigate the related properties of generalization of a Γ-semi normal sub near-field spaces in Γ-near-field space over a near-field.

As a generalization of a Γ-semi normal sub near-field spaces in Γ-near-field space over a near-field, introduced the notion of Γ-semi normal sub near-field spaces in Γ-near-field space over a near-field, extended many classical notions of Γ-semi normal sub near-field spaces in Γ-near-field space over a near-field. In this paper, I develop the algebraic theory of Γ-semi normal sub near-field spaces in Γ-near-field space over a near-field.

The notion of a Γ- semi normal sub near-field spaces in Γ-near-field space over a near-field is introduced and some examples are given. Further the terms; commutative Γ-semi normal sub near-field spaces in Γ-near-field space, quasi commutative Γ-semi normal sub near-field spaces in Γ-near-field space, normal Γ-semi normal sub near-field spaces in Γ-near-field space, left pseudo commutative Γ-semi normal sub near-field spaces in Γ-near-field space, right pseudo commutative Γ-semi normal sub near-field spaces in Γ-near-field space are introduced. It is proved that (1) if S is a commutative Γ-semi normal sub near-field spaces in Γ-near-field space then S is a quasi commutative Γ-semi normal sub near-field spaces in Γ-near-field space, (2) if S is a quasi commutative Γ-semi normal sub near-field spaces in Γ-near-field space then S is a normal Gamma-semi normal sub near-field spaces in Γ-near-field space, (3) if S is a commutative Γ-semi normal sub near-field spaces in Γ-near-field space, then S is both a left pseudo commutative and a right pseudo commutative Γ-semi normal sub near-field spaces in Γ-near-field space over a near-field. Further the terms; left identity, right identity, identity, left zero, right zero, zero of a Gamma-semi normal sub near-field spaces in Γ-near-field space over a near-field are introduced. It is proved that if a is a left identity and b is a right identity of a Γ-semi normal sub near-field spaces in Γ-near-field space, then a = b. It is also proved that any Γ-semi normal sub near-field spaces in Γ-near-field space has at most one identity. It is proved that if a is a left zero and b is a right zero of a Γ-semi normal sub near-field spaces in Γ-near-field space, then a = b and also it is proved that any Γ-semi normal sub near-field spaces in Γ-near-field space over a near-field has at most one zero element.

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SECTION-2: MAIN RESULTS ON SEMI NORMAL SUB NEAR-FIELD SPACES IN Γ-NEAR-FIELD SPACE OVER A NEAR-FIELD

Semi normal sub near-field spaces have greater importance in the theory of Γ-semi normal sub near-field spaces. In this section, the terms left Γ- semi normal sub near-field space, right Γ- semi normal sub near-field space, Γ- semi normal sub near-field space, proper Γ- semi normal sub near-field space, trivial Γ- semi normal sub near-field space, maximal left Γ- semi normal sub near-field space, maximal right Γ- semi normal sub near-field space, maximal Γ- semi normal sub near-field space, left Γ- semi normal sub near-field space generated by a sub near field space, right Γ- semi normal sub near-field space generated by a sub near-field space, Γ- semi normal sub near-field space generated by a sub near-field space, principal left Γ- semi normal sub near-field space, principal right Γ- semi normal sub near-field space, principal Γ- semi normal sub near-field space of a Γ-semi normal sub near-field space are introduced. Also left duo Γ- semi normal sub near-field space, right duo Γ- semi normal sub near-field space, duo Γ- semi normal sub near-field space, left simple Γ- semi normal sub near-field space, right simple Γ- semi normal sub near-field space, simple Γ- semi normal sub near-field space are introduced. It is proved that (1) the nonempty intersection of any two left Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a left Γ- semi normal sub near-field space of S, (2) the nonempty intersection of any family of left Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a left Γ- semi normal sub near-field space of S, (3) the union of any two left Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a left Γ- semi normal sub near-field space of S and (4) the union of any family of left Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a left Γ- semi normal sub near-field space of S.

It is also proved that (1) the nonempty intersection of any two right Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a right Γ- semi normal sub near-field space of S, (2) the nonempty intersection of any family of right Γ- semi normal sub near-field space of a Γ- semi normal sub near-field space S is a right Γ- semi normal sub near-field space of S, (3) the union of any two right Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a right Γ- semi normal sub near-field space of S and (4) the union of any family of right Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a right Γ- semi normal sub near-field space of S. Further it is proved that (1) the nonempty intersection of any two Γ-ideals of a Γ- semi normal sub near-field space S is a Γ- semi normal sub near-field space of S, (2) the nonempty intersection of any family of Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a Γ- semi normal sub near-field space of S, (3) the union of any two Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a semi normal sub near-field space l of S and (4) the union of any family of Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a Γ- semi normal sub near-field space of S. It is proved that if S is a Γ- semi normal sub near-field space and $a \in S$ then (i) $L(a) = a \cup S\Gamma a$, (ii) $R(a) = a \cup a\Gamma S$, (iii) $J(a) = a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma S$.

It is proved that a Γ- semi normal sub near-field space S is a duo Γ- semi normal sub near-field space if and only if $x\Gamma S_1 = S_1\Gamma x$ for all $x \in S$. Further it is also proved that every normal Γ- semi normal sub near-field space is a duo Γ- semi normal sub near-field space. It is proved that (1) a Γ- semi normal sub near-field space S is a left simple Γ- semi normal sub near-field space if and only if $S\Gamma a = S$ for all Γ- semi normal sub near-field spaces 47 $a \in S$, (2) a Γ- semi normal sub near-field space S is right a simple Γ- semi normal sub near-field space if and only if $a\Gamma S = S$ for all $a \in S$, (3) a Γ- semi normal sub near-field space S is a simple Γ- semi normal sub near-field space if and only if $S\Gamma a\Gamma S = S$ for all $a \in S$.

We now introduce the term of a right Γ-semi normal sub near-field space in a Γ-semi normal sub near-field space.

Definition 1.2.1: A nonempty sub near-field space A of a Γ- semi normal sub near-field space S is said to be a right Γ- semi normal sub near-field space of S if $s \in S,\ a \in A, \alpha \in \Gamma \Rightarrow a\alpha s \in A$.

Note 1.2.2: A nonempty sub near-field space A of a Γ- semi normal sub near-field space S is a left Γ-semi normal sub near-field space of S $\Leftrightarrow$ $A\Gamma S \subseteq A$.

Theorem 1.2.9: The nonempty intersection of any two right Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a right Γ- semi normal sub near-field space of S.

Proof: Let A, B be two right Γ-ideals of S. Let $a \in A \cap B$, $s \in S$ and $\gamma \in \Gamma$.
$a \in A \cap B \Rightarrow a \in A$ and $a \in B$.
$a \in A$, $s \in S$, $\gamma \in \Gamma$, A is a right Γ-ideal of S $\Rightarrow a\, \gamma\, s \in A$.
$a \in B$, $s \in S$, $\gamma \in \Gamma$, B is a right Γ-ideal of S $\Rightarrow a\, \gamma\, s \in B$.
$a\, \gamma\, s \in A$, $a\, \gamma\, s \in B \Rightarrow a\, \gamma\, s \in A \cap B$ and hence $A \cap B$ is a right Γ-ideal of S.

Theorem 1.2.10: The nonempty intersection of any family of right Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a right Γ- semi normal sub near-field space of S.

Proof: Let $\{A_\alpha\}$ $\alpha\in\Delta$ be a family of right Γ- semi normal sub near-field spaces of S and let $A = \bigcap_{\alpha\in\Delta} A_\alpha$

Let $a \in$ A, $s \in$ S, $\gamma \in \Gamma$.
$a \in A \Rightarrow a \in \bigcap_{\alpha\in\Delta} A_\alpha \Rightarrow a \in A_\alpha \ \forall\ \alpha \in \Delta$.
$a \in A_\alpha$, $s \in$ S, $\gamma \in \Gamma$, $\in A_\alpha$ is a right Γ-semi normal sub near-field space of S $\Rightarrow a\gamma s \in A_\alpha$.
$a\gamma s \in A_\alpha$ for all $\alpha \in \Delta \Rightarrow a\gamma s \in \bigcap_{\alpha\in\Delta} A_\alpha \Rightarrow a\,\gamma\, s \in$ A. Therefore A is a right Γ- semi normal sub near-field space of S.

Theorem 1.2.11: The union of any two right Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a right Γ- semi normal sub near-field space of S.

Proof: Let A_1, A_2 be two right Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S. Let A = $A_1 \cup A_2$.

Clearly A is a nonempty sub near-field space of S. Let $a \in$ A, $s \in$ S, $\gamma \in \Gamma$.
$a \in A \Rightarrow a \in A_1\cup A_2 \Rightarrow a \in A_1$ or $a \in A_2$.

If $a \in A_1$ then $a \in A_1$, $s \in$ S, $\gamma \in \Gamma$, A_1 is a right Γ-ideal of S
$\Rightarrow a\gamma s \in A_1 \subseteq A_1\cup A_2 = A \Rightarrow a\gamma s \in A$.

If $a \in A_2$ then $a \in A_2$, $s \in$ S, $\gamma \in \Gamma$, A_2 is a right Γ-ideal of S
$\Rightarrow a\gamma s \in A_2 \subseteq A_1\cup A_2 = A \Rightarrow a\gamma s \in A$.
$\therefore a \in$ A, $s \in$ S, $\gamma \in \Gamma$ then $a\gamma s \in$ A. Therefore A is a right Γ- semi normal sub near-field space of S. This completes the proof of the theorem.

Theorem 1.2.12: The union of any family of right Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a right Γ- semi normal sub near-field space of S.

Proof: Let $\{A_\alpha\}$ $\alpha\in\Delta$ be a family of right Γ- semi normal sub near-field spaces of S and let $A = \bigcup_{\alpha\in\Delta} A_\alpha$.

Clearly A is a nonempty subset of S. Let $a \in$ A, $s \in$ S, $\alpha \in \Gamma$.
$a \in A \Rightarrow a \in \bigcup_{\alpha\in\Delta} A_\alpha \Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta$.

$a \in A_\alpha$, $s \in$ S, $\gamma \in \Gamma$, A_α is a right Γ-semi normal sub near-field space of S $\Rightarrow a\gamma s \in A_\alpha \subseteq \bigcup_{\alpha\in\Delta} A_\alpha = A \Rightarrow a\gamma s \in A$.

Therefore A is a right Γ- semi normal sub near-field space of S.

This completes the proof of the theorem.

We now introduce the notion of a Γ- semi normal sub near-field space of a Γ- semi normal sub near-field space.

Definition 1.2.13: A nonempty subset A of a Γ- semi normal sub near-field space S is said to be a two sided Γ- semi normal sub near-field space or simply a Γ- semi normal sub near-field space of S if s ∈ S, a ∈ A, $\alpha \in \Gamma \Rightarrow s\alpha a \in A$, $a\alpha s \in A$.

Note 1.2.14: A nonempty subset A of a Γ- semi normal sub near-field space S is a two sided Γ- semi normal sub near-field space iff it is both a left Γ- semi normal sub near-field space and a right Γ- semi normal sub near-field space of S.

Example 1.2.15: Let N be the set of natural numbers and Γ = 2N. Then N is a Γ- semi normal sub near-field space and A = 3N is a Γ- ideal of the Γ- semi normal sub near-field space N.

Theorem 1.2.16: The nonempty intersection of any two Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S is a Γ- semi normal sub near-field space of S.

Proof: Let A, B be two Γ- semi normal sub near-field spaces of S. Let $a \in A\cap B$ and $s \in$ S, $\gamma \in \Gamma$.
$a \in A\cap B \Rightarrow a \in$ A and $a \in$ B.
$a \in$ A, $s \in$ S, $\gamma \in \Gamma$, A is a Γ-semi normal sub near-field space of S $\Rightarrow s\,\gamma\, a$, $a\,\gamma\, s \in$ A.
$a \in$ B, $s \in$ S, $\gamma \in \Gamma$, B is a Γ-semi normal sub near-field space of S $\Rightarrow s\,\gamma\, a$, $a\,\gamma\, s \in$ B.

$\therefore s\,\gamma\, a, a\,\gamma\, s \in$ A, $s\,\gamma\, a, a\,\gamma\, s \in$ B $\Rightarrow s\,\gamma\, a, a\,\gamma\, s \in$ A∩B.

Therefore A∩B is a Γ- semi normal sub near-field space of S.

Theorem 1.2.17: The nonempty intersection of any family of Γ-semi normal sub near-field spaces of a Γ -semi normal sub near-field space S is a Γ -semi normal sub near-field space of S.

Proof: Let $\{A_\alpha\}$ $\alpha \in \Delta$ be a family of Γ-ideals of S and let $A = \cap A_\alpha, \forall\ \alpha \in \Gamma$.

Let $a \in$ A, $s \in$ S, $\gamma \in \Gamma$.
$a \in$ A $\Rightarrow a \in \cap A_\alpha, \forall\ \alpha \in \Gamma \Rightarrow a \in A_\alpha, \forall\ \alpha \in \Gamma$, for each $\alpha \in \Delta$.
$a \in A_\alpha$, $s \in$ S, $\gamma \in \Gamma$, A_α is a Γ- semi normal sub near-field space of S $\Rightarrow s\,\gamma\, a, a\,\gamma\, s \in A_\alpha$.
$s\,\gamma\, a, a\,\gamma\, s \in A_\alpha$ for all $\alpha \in \Delta \Rightarrow s\, A_\alpha a, a\, A_\alpha s \in \cap A_\alpha, \forall\ \alpha \in \Gamma \Rightarrow s\,\gamma\, a, a\,\gamma\, s \in$ A.

Therefore A is a Γ- semi normal sub near-field space of S. This completes the proof of the theorem.

Theorem 1.2.18: The union of any two Γ- semi normal sub near-field spaces of a Γ-semi normal sub near-field space S is a Γ-semi normal sub near-field space of S.

Proof: Let A_1, A_2 be two Γ- semi normal sub near-field spaces of a Γ- semi normal sub near-field space S. Let $A = A_1 \cup A_2$.

Clearly A is a nonempty sub near-field space of S. Let $a \in$ A, $s \in$ S, $\gamma \in \Gamma$.
$a \in A \Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ or $a \in A_2$.

If $a \in A_1$ then $a \in A_1$, $s \in$ S, $\gamma \in \Gamma$, A_1 is a Γ-semi normal sub near-field space of S $\Rightarrow s\,\gamma\, a, a\,\gamma\, s \in A_1 \subseteq A_1 \cup A_2 = A$
$\Rightarrow s\,\gamma\, a, a\,\gamma\, s \in$ A.

If $a \in A_2$ then $a \in A_2$, $s \in$ S, $\gamma \in \Gamma$, A_2 is a Γ- semi normal sub near-field space of S $\Rightarrow s\,\gamma\, a, a\,\gamma\, s \in A_2 \subseteq A_1 \cup A_2 = A$
$\Rightarrow s\,\gamma\, a, a\,\gamma\, s \in$ A.

Thus, $a \in$ A, $s \in$ S, $\gamma \in \Gamma \Rightarrow s\,\gamma\, a, a\,\gamma\, s \in$ A. Therefore A is a Γ- semi normal sub near-field space of S. This completes the proof of the theorem.

Theorem 1.2.19: The union of any family of Γ - semi normal sub near-field spaces of a Γ - semi normal sub near-field space S is a Γ - semi normal sub near-field space of S.

Proof: Let $\{A_\alpha\}$ $\alpha \in \Delta$ be a family of Γ-ideals of S and let $A = \cup A_\alpha, \forall\ \alpha \in \Gamma$.Clearly A is a nonempty subset of S. Let $a \in$ A, $s \in$ S, $\in \Gamma$.

Let $a \in$ A, $s \in$ S, $\gamma \in \Gamma$.
$a \in$ A $\Rightarrow a \in \cup A_\alpha, \forall\ \alpha \in \Gamma \Rightarrow a \in A_\alpha, \forall\ \alpha \in \Gamma$, for each $\alpha \in \Delta$.
$a \in A_\alpha$, $s \in$ S, $\gamma \in \Gamma$, A_α is a Γ- semi normal sub near-field space of S $\Rightarrow s\,\gamma\, a, a\,\gamma\, s \in A_\alpha$.

$s\,\gamma\, a, a\,\gamma\, s \in A_\alpha$ for all $\alpha \in \Delta \Rightarrow s\, A_\alpha a, a\, A_\alpha s \in \cup A_\alpha, \forall\ \alpha \in \Gamma \Rightarrow s\,\gamma\, a, a\,\gamma\, s \in$ A.

Therefore A is a Γ- semi normal sub near-field space of S. This completes the proof of the theorem.

We now introduce a proper Γ- semi normal sub near-field space, trivial Γ- semi normal sub near-field space, maximal left Γ- semi normal sub near-field space, maximal right Γ- semi normal sub near-field space, maximal Γ- semi normal sub near-field space and globally idempotent Γ- semi normal sub near-field space of a Γ- semi normal sub near-field space.

Definition 1.2.20: A Γ- semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be a proper Γ- semi normal sub near-field space of S if A is different from S.

Definition 1.2.21: A Γ- semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be a trivial Γ- semi sub near-field space provided S\A is singleton.

Definition 1.2.22: A Γ- semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be a maximal left Γ-semi sub near-field space provided A is a proper left Γ-semi sub near-field space of S and is not properly contained in any proper left Γ-semi sub near-field space of S.

Definition 1.2.23: A Γ-semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be a maximal right Γ-semi sub near-field space provided A is a proper right Γ-semi sub near-field space of S and is not properly contained in any proper right Γ-semi sub near-field space of S.

Definition 1.2.24: A Γ-semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be a maximal Γ-semi sub near-field space provided A is a proper Γ-semi sub near-field space of S and is not properly contained in any proper Γ-semi sub near-field space of S.

Definition 1.2.25: A Γ-semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be globally idempotent if AΓA = A.

Theorem 1.2.26: If A is a Γ-semi sub near-field space of a Γ- semi normal sub near-field space S with unity 1 and 1 ∈ A then A = S.

Proof: Clearly A ⊆ S. Let *s* ∈ S.
1 ∈ A, *s* ∈ S, A is a Γ-semi sub near-field space of S ⇒ 1Γ*s* ⊆ A ⇒ *s* ∈ A. Thus S ⊆ A. A ⊆ S, S ⊆ A ⇒ S = A. This completes the proof of the theorem.

Theorem 1.2.27: If S is a Γ- semi normal sub near-field space with unity 1 then the union of all proper Γ-semi sub near-field spaces of S is the unique maximal Γ-semi sub near-field space of S.

Proof: Let M be the union of all proper Γ-semi sub near-field spaces of S. Since 1 is not an element of any proper Γ-semi sub near-field space of S, 1 ∉ M. Therefore M is a proper subset of S. By theorem 1.2.19, M is a Γ-semi sub near-field space of S. Thus M is a proper Γ-semi sub near-field space of S. Since M contains all proper Γ-semi sub near-field spaces of S,

M is a maximal Γ-semi sub near-field space of S. If M1 is any maximal Γ-semi sub near-field space of S, then $M_1 \subseteq M \subset S$ and hence $M_1 = M$. Therefore M is the unique maximal Γ-semi sub near-field space of S.

We now introducing left Γ-semi sub near-field space generated by a subset, right Γ-semi sub near-field space generated by a subset, Γ-semi sub near-field space generated by a subset of a Γ-semi normal sub near-field space.

Definition 1.2.28: Let S be a Γ- semi normal sub near-field space and A be a nonempty sub near-field space of S. The smallest left Γ-semi sub near-field space of S containing A is called left Γ-semi sub near-field space of S generated by A.

Theorem 1.2.29: The left Γ-semi sub near-field space of a Γ-semi normal sub near-field space S generated by a nonempty sub near-field space A is the intersection of all left Γ-semi sub near-field spaces of S containing A.

Proof: Let Δ be the set of all left Γ-semi sub near-field spaces of S containing A. Since S itself is a left Γ-semi sub near-field space of S containing A, S ∈ Δ.

So Δ ≠ Φ.

Let
$T^* = \bigcap_{\alpha \in \Delta} T_\alpha$.

Since A ⊆ T for all T ∈ Δ, $A \subseteq T^*$.

So, T* is a left Γ-semi sub near-field space of S.

Let K is a left Γ-semi sub near-field space of S containing A.

Clearly A ⊆ K and K is a left Γ-semi sub near-field space of S.

Therefore K ∈ Δ ⇒T* ⊆ K.

Therefore T* is the left Γ-semi sub near-field space of S generated by A. This completes the proof of the theorem.

Definition 1.2.30: Let S be a Γ- semi normal sub near-field space and A be a nonempty sub near-field space of S. The smallest right Γ-semi sub near-field space of S containing A is called right Γ-semi sub near-field space of S generated by A.

Theorem 1.2.31: The right Γ-semi sub near-field space of a Γ-semi normal sub near-field space S generated by a nonempty sub near-field space A is the intersection of all right Γ-semi sub near-field spaces of S containing A.

Proof: Let Δ be the set of all right Γ-semi sub near-field spaces of S containing A. Since S itself is a right Γ-semi sub near-field space of S containing A, S ∈ Δ. So Δ ≠ Φ.

Let $T^* = \bigcap_{\alpha \in \Delta} T_\alpha$. Since A ⊆ T for all T ∈ Δ, A⊆T^* .

By theorem 1.2.10, T* is a right Γ- semi sub near-field space of S.

Let K is a right Γ- semi sub near-field space of S containing A.

Clearly A ⊆ K and K is a right Γ- semi sub near-field space of S.

Therefore K ∈ Δ ⇒T* ⊆ K.

Therefore T* is the right Γ- semi sub near-field space of S generated by A. This completed the proof of the theorem.

Definition 1.2.32: Let S be a Γ- semi normal sub near-field space and A be a nonempty sub near-field space of S. The smallest Γ- semi sub near-field space of S containing A is called Γ- semi sub near-field space of S generated by A.

Theorem 1.2.33: The Γ- semi sub near-field space of a Γ- semi normal sub near-field space S generated by a nonempty sub near-field space A is the intersection of all Γ- semi sub near-field spaces of S containing A.

Proof: Let Δ be the set of all Γ-semi sub near-field spaces of S containing A.

Since S itself is a Γ-semi sub near-field space of S containing A, S ∈ Δ. So Δ ≠ Φ.

Let $T^* = \bigcap_{\alpha \in \Delta} T_\alpha$ Since $A \subseteq T$ for all T ∈ Δ, $A \subseteq T^*$.

By theorem 1.2.17, T* is a Γ- semi sub near-field space of S.

Let K is a Γ- semi sub near-field space of S containing A.

Clearly A ⊆ K and K is a Γ- semi sub near-field space of S.

Therefore K ∈ Δ ⇒T* ⊆ K.

Therefore T* is the Γ- semi sub near-field space of S generated by A. This completes the proof of the theorem.

We now introduce a principal left Γ- semi sub near-field space of a Γ- semi normal sub near-field space and characterize principal left Γ- semi sub near-field space.

Definition 1.2.34: A left Γ- semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be the principal left Γ- semi sub near-field space generated by a if A is a left Γ- semi sub near-field space generated by $\{a\}$ for some $a \in$ S. It is denoted by L(a).

Theorem 1.2.35: If S is a Γ- semi normal sub near-field space and a ∈ S then L(a) = a ∪ SΓa.

Proof: Let $s \in S$, $r \in a \cup S\Gamma a$ and $\gamma \in \Gamma$.
$r \in a \cup S\Gamma a \Rightarrow r = a$ or $r = t\alpha a$ for some $t \in S$, $\alpha \in \Gamma$.

If $r = a$ then $s\,\gamma\, r = s\,\gamma\, a \in S\Gamma a \subseteq a \cup S\Gamma a$.

If $r = t\,\alpha\, a$ then $s\,\gamma\, r = s\,\gamma\,(t\alpha a) = (s\,\gamma\, t)\alpha a \in S\Gamma a \subseteq a \cup S\Gamma a$.

Therefore $s\,\gamma\, a \in a \cup S\Gamma a$ and hence $a \cup S\Gamma a$ is a left Γ- semi sub near-field space of S. Let L be a left Γ- semi sub near-field space of S containing a.

Let $r \in a \cup S\Gamma a$. Then $r = a$ or $r = t\alpha a$ for some $t \in S$, $\alpha \in \Gamma$.

If $r = a$ then $r = a \in$ L. If $r = t\alpha a$ then $r = t\alpha a \in$ L.

Therefore $a \cup S\Gamma a \subseteq$ L and hence $a \cup S\Gamma a$ is the smallest left Γ- semi sub near-field space containing a. Therefore L(a) = $a \cup S\Gamma a$. This completes the proof of the theorem.

Note 1.2.36: If S is a Γ- semi normal sub near-field space and $a \in$ S then L (a) = S1Γa.

We now introduce principal right Γ- semi sub near-field space of a Γ- semi normal sub near-field space and characterize principal right Γ- semi sub near-field space.

Definition 1.2.37: A right Γ- semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be the principal right Γ- semi sub near-field space generated by a if A is a right Γ- semi sub near-field space generated by {a} for some a ∈S. It is denoted R(a).

Theorem 1.2.38: If S is a Γ - semi normal sub near-field space and a ∈ S then R(a) = a ∪ aΓS.

Proof: Let $s \in$ S, $r \in a \cup a\Gamma$S.

Now $r \in a \cup a\Gamma S \Rightarrow r = a$ or $r = a\alpha t$ for some $t \in$ S, $\alpha \in \Gamma$.

If $r = a$ then $r\,\gamma\, s = a\,\gamma\, s \in a\Gamma S \subseteq a \cup a\Gamma$S.

If $r = a\alpha t$ then $r\,\gamma\, s = (a\alpha t)\,\gamma\, s = a\alpha(t\,\gamma\, s) \in a\Gamma S \subseteq a \cup a\Gamma$S.

Therefore $r\,\gamma\, s \in a \cup a\Gamma$S and hence $a \cup a\Gamma$S is a right Γ- semi sub near-field space of S. Let R be a right Γ- semi sub near-field space of S containing a.

Let $r \in a \cup a\Gamma$S. Then $r = a$ or $r = a\alpha t$ for some $t \in$ S, $\alpha \in \Gamma$.

If $r = a$ then $r = a \in$ R. If $r = a\alpha t$ then $r = a\alpha t \in$ R.

Therefore $a \cup a\Gamma S \subseteq$ R and hence $a \cup S\Gamma a$ is the smallest right Γ- semi sub near-field space containing a. Therefore R(a) = $a \cup a\Gamma$S. This completes the proof of the theorem.

Note 1.2.39: If S is a Γ- semi normal sub near-field space and $a \in$ S then R (a) = aΓS1.

We now introduce a principal Γ- semi sub near-field space of a Γ-semi normal sub near-field space and characterize principal Γ- semi sub near-field space.

Definition 1.2.40: A Γ- semi sub near-field space A of a Γ- semi normal sub near-field space S is said to be a principal Γ- semi sub near-field space provided A is a Γ- semi sub near-field space generated by {a} for some $a \in$ S. It is denoted by J[a] or <a>.

Theorem 1.2.41: If S is a Γ- semi normal sub near-field space and a ∈ S then J(a) = a ∪ aΓS ∪ SΓa ∪ SΓaΓS.

Proof: Let $s \in$ S, $r \in a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma$S and $\gamma \in \Gamma$.
$r \in a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma S \Rightarrow r = a$ or $r = a\alpha t$ or $r = t\alpha a$ or $r = t\alpha a\beta u$ for some
$t, u \in$ S and $\alpha, \beta \in \Gamma$.

If $r = a$ then $r\gamma s = a\gamma s \in a\Gamma S$ and $s\gamma r = s\gamma a \in S\Gamma a$.

If $r = a\alpha t$ then $r\gamma s = (a\alpha t)\gamma s = a\alpha(t\gamma s) \in a\Gamma$S and $s\gamma r = s\gamma(a\alpha t) = s\gamma a\alpha t \in S\Gamma a\Gamma$S.

If $r = t\alpha a$ then $r\gamma s = (t\alpha a)\gamma s = t\alpha a\gamma s \in S\Gamma a\Gamma$S or $s\gamma r = s\gamma(t\alpha a) = (s\gamma t)\alpha a \in S\Gamma a$.

If $r = t\alpha a\beta u$ then $r\gamma s = (t\alpha a\beta u)\gamma s = t\alpha a\beta(u\gamma s) \in S\Gamma a\Gamma$S
and $s\gamma r = s\gamma(t\alpha a\beta u) = (s\gamma t)\alpha a\beta u \in S\Gamma a\Gamma$S.

But aΓS, SΓa, SΓaΓS are all sub near-field spaces of $a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma$S.

Therefore $r\gamma s$, $s\gamma r \in a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma$S and hence $a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma$S is a Γ- semi sub near-field space of S.

Let J be a Γ- semi sub near-field space of S containing a. Let $r \in a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma S$.

Then $r = a$ or $r = a\alpha t$ or $r = t\alpha a$ or $r = t\alpha a\beta u$ for some $t, u \in S$ and $\alpha, \beta \in \Gamma$.

If $r = a$ then $r = a \in J$. If $r = a\alpha t$ then $r = a\alpha t \in J$.

If $r = t\alpha a$ then $r = t\alpha a \in J$. If $r = t\alpha a\beta u$ then $r = t\alpha a\beta u \in J$.

Therefore, $a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma S \subseteq J$.

Hence $a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma S$ is the smallest Γ- semi sub near-field space of S containing a. Therefore $J(a) = a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma S$. This completes the proof of the theorem.

Note 1.2.42: If S is a Γ- semi normal sub near-field space and $a \in S$, then $<a> = a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma S = S1\Gamma a\,\Gamma S1$.

Theorem 1.2.43: In any Γ- semi normal sub near-field space S, the following are equivalent. (1) Principal Γ-semi sub near-field spaces of S form a chain. (2) Γ-semi sub near-field spaces of S form a chain.

Proof: To prove (1) ⇒ (2): Suppose that principal Γ- semi sub near-field spaces of S form a chain.

Let A, B be two Γ- semi sub near-field spaces of S. Suppose if possible $A \not\subset B$, $B \not\subset A$.
Then there exists $a \in A\backslash B$ and $b \in B\backslash A$.
$a \in A \Rightarrow < a > \subseteq A$ and $b \in B \Rightarrow < b > \subseteq B$.

Since principal Γ- semi sub near-field spaces form a chain, either $< a > \subseteq < b >$ or $< b > \subseteq < a >$.

If $< a > \subseteq < b >$, then $a \in < b > \subseteq B$. It is a contradiction.

If $< b > \subseteq < a >$, then $b \in < a > \subseteq A$. It is also a contradiction.

Therefore, either $A \subseteq B$ or $B \subseteq A$ and hence Γ- semi sub near-field spaces from a chain.

To prove (2) ⇒ (1): Suppose that Γ-semi sub near-field spaces of S form a chain.

Then clearly principal Γ-semi sub near-field space of S forms a chain.

Duo semi normal sub near-field spaces played an important role in the theory of semi normal sub near-field spaces. This completes the proof of the theorem.

We now introduce a left duo Γ-semi normal sub near-field space, a right duo Γ-semi normal sub near-field space and a duo Γ-semi normal sub near-field space.

Definition 1.2.44: A Γ- semi normal sub near-field space S is said to be a left duo Γ- semi normal sub near-field space provided every left Γ- semi sub near-field space of S is a two sided Γ- semi sub near-field space of S.

Definition 1.2.45: A Γ- semi normal sub near-field space S is said to be a right duo Γ- semi normal sub near-field space provided every right Γ-semi sub near-field space of S is a two sided Γ- semi sub near-field space of S.

Definition 1.2.46: A Γ- semi normal sub near-field space S is said to be a duo Γ- semi normal sub near-field space provided it is both a left duo Γ- semi normal sub near-field space and a right duo Γ- semi normal sub near-field space.

We now characterize duo Γ-semi normal sub near-field spaces.

Theorem 1.2.47: A Γ-semi normal sub near-field space S is a duo Γ- semi normal sub near-field space if and only if $x\Gamma S^1 = S^1\Gamma x$ for all $x \in S$.

Proof: Suppose that S is a duo Γ-semi normal sub near-field space and $x \in S$.

Let $t \in x\Gamma S^1$. Then $t=x\gamma s$ for some $s \in S^1$, $\gamma \in \Gamma$.

Since $S^1\Gamma x$ is a left Γ-semi sub near-field space of S, $S^1\Gamma x$ is a Γ-semi sub near-field space of S.

So $x \in S^1\Gamma x$, $\gamma \in \Gamma$, $s \in S$, $S^1\Gamma x$ is a Γ-semi sub near-field space
$\Rightarrow x\,\gamma\, s \in S^1\Gamma x \Rightarrow t \in S^1\Gamma x$.

Therefore, $x\Gamma S^1 \subseteq S^1\Gamma x$. Similarly we can prove that $S^1\Gamma x \subseteq x\Gamma S^1$ .Therefore $S^1\Gamma x = x\Gamma S^1$.

Conversely suppose that $S^1\Gamma x = x\Gamma S^1$ for all $x \in S$. Let A be a left Γ-semi sub near-field space of S.

Let $x \in A$, $s \in S$ and $\alpha \in \Gamma$. Then $x\alpha s \in x\Gamma S1 = S1\Gamma x \Rightarrow x\alpha s = t\beta x$ for some $t \in S^1$, $\beta \in \Gamma$.
$x \in A$, $t \in S$, $\beta \in \Gamma$, A is a left Γ-semi sub near-field space of $S \Rightarrow t\beta x \in A \Rightarrow x\alpha\, s \in A$.

Therefore A is a right Γ-semi sub near-field space of S and hence A is a Γ-semi sub near-field space of S.

Therefore S is left duo Γ-semi normal sub near-field space.

Similarly we can prove that S is a right duo Γ-semi normal sub near-field space. Hence S is duo Γ-semi normal sub near-field space. This completes the proof of the theorem.

Theorem 1.2.48: Every normal Γ- semi normal sub near-field space is a duo Γ- semi normal sub near-field space.

Proof: Suppose that S is normal Γ-semi normal sub near-field space.
Then $a\Gamma S = S\Gamma a$ for all $a \in S \Rightarrow a\Gamma S^1 = S^1\Gamma a$ for all $a \in S$. By theorem 1.2.47, S is a duo Γ-semi normal sub near-field space. This completes the proof of the theorem.

We now introduce a left simple Γ-semi normal sub near-field space and characterize left simple Γ-semi normal sub near-field spaces.

Definition 1.2.49: A Γ-semi normal sub near-field space S is said to be a left simple Γ-semi normal sub near-field space if S is its only left Γ-semi sub near-field space.

Theorem 1.2.50: A Γ-semi normal sub near-field space S is a left simple Γ -semi normal sub near-field space if and only if $S\,\Gamma\, a = S$ for all $a \in S$.

Proof: Suppose that S is a left simple Γ-semi normal sub near-field space and $a \in S$.

Let $t \in S\Gamma a$, $s \in S$, $\gamma \in \Gamma$.
$t \in S\Gamma a \Rightarrow t = s_1\alpha a$ where $s_1 \in S$ and $\alpha \in \Gamma$.

Now $s\gamma t = s\gamma(\, s_1\alpha a\,) = (s\Gamma s_1\,)\alpha a \in S\Gamma a \Rightarrow S\Gamma a$ is a left Γ-semi sub near-field space of S.

Since S is a left simple Γ-semi normal sub near-field space, $S\Gamma a = S$.

Therefore $S\Gamma a = S$ for all $a \in S$.

Conversely suppose that $S\Gamma a = S$ for all $a \in S$. Let L be a left Γ-semi sub near-field space of S.

Let $l \in L$. Then $l \in S$. By assumption $S\Gamma l = S$.

Let $s \in S$. Then $s \in S\Gamma l \Rightarrow s = t\alpha l$ for some $t \in S$, $\alpha \in \Gamma$.
$l \in L$, $t \in S$, $\alpha \in \Gamma$ and L is a left Γ-semi sub near-field space $\Rightarrow t\alpha l \in L \Rightarrow s \in L$.

Therefore, $S \subseteq L$. Clearly $L \subseteq S$ and hence $S = L$.

Therefore S is the only left Γ-semi sub near-field space of S. Hence S is left simple Γ-semi normal sub near-field space.

We now introduce a right simple Γ-semi normal sub near-field space and characterize right simple Γ-semi normal sub near-field spaces.

Definition 1.2.51: A Γ-semi normal sub near-field space S is said to be a right simple Γ-semi normal sub near-field space if S is its only right Γ-semi sub near-field space.

Theorem 1.2.52: A Γ-semi normal sub near-field space S is a right simple Γ-semi normal sub near-field space if and only if $a\Gamma S = S$ for all $a \in S$.

Proof: Suppose that S is a right simple Γ-semi normal sub near-field space and $a \in$ S. Let $t \in a\Gamma$S, $s \in$ S, $\gamma \in \Gamma$.
$t \in a\Gamma S \Rightarrow t = a\alpha s_1$ where $s_1 \in S$ and $\alpha \in \Gamma$.

Now $t\gamma s = (a\alpha\, s_1\,)\gamma\, s = a\alpha\,(s_1\;\gamma\, s) \in a\Gamma S \Rightarrow a\Gamma$S is a right Γ-semi sub near-field space of S.

Since S is a right simple Γ-semi normal sub near-field space, $a\Gamma$S = S.

Therefore $a\Gamma$S = S for all $a \in$ S.

Conversely suppose that $a\Gamma$S = S for all $a \in$ S.

Let R be a right Γ- semi sub near-field space of a Γ- semi normal sub near-field space S.

Let $r \in$ R. Then $r \in$ S. By assumption $r\Gamma$S = S.

Let $s \in$ S. Then $s \in r\Gamma S \Rightarrow s = r\alpha t$ for some $t \in$ S, $\alpha \in \Gamma$.
$r \in$ R, $t \in$ S, $\alpha \in \Gamma$ and R is a right Γ- semi sub near-field space $\Rightarrow r\alpha t \in R \Rightarrow s \in$ R.

Therefore, S ⊆ R. Clearly R ⊆ S and hence S = R.

Therefore S is the only right Γ- semi sub near-field space of S. Hence S is right simple Γ- semi normal sub near-field space S. This completes the proof of the theorem.

We now introduce a simple Γ-semi normal sub near-field space and characterize simple Γ-semi normal sub near-field spaces.

Definition 1.2.53: A Γ-semi normal sub near-field space S is said to be simple Γ-semi normal sub near-field space if S is its only two-sided Γ-semi sub near-field space.

Theorem 1.2.54: If S is a left simple Γ-semi normal sub near-field space or a right simple Γ-semi normal sub near-field space then S is a simple Γ-semi normal sub near-field space.

Proof: Suppose that S is a left simple Γ-semi normal sub near-field space. Then S is the only left Γ-semi sub near-field space of S.

If A is a Γ-semi sub near-field space of S, then A is a left Γ-semi sub near-field space of S and hence A = S.

Therefore S itself is the only Γ-semi sub near-field space of S and hence S is a simple Γ-semi normal sub near-field space.

Suppose that S is a right simple Γ-semi normal sub near-field space. Then S is the only right Γ-semi sub near-field space of S.

If A is a Γ-semi sub near-field space of S, then A is a right Γ-semi sub near-field space of S and hence A = S.

Therefore S itself is the only Γ-semi sub near-field space of S and hence S is a simple Γ-semi normal sub near-field space.

Theorem 1.2.55: A Γ-semi normal sub near-field space S is simple Γ-semi normal sub near-field space if and only if $S\Gamma a\Gamma S = S$ for all $a \in$ S.

Proof: Suppose that S is a simple Γ-semi normal sub near-field space and $a \in$ S.

Let $t \in S\Gamma a\Gamma S$, $s \in$ S and $\gamma \in \Gamma$.

$t \in S\Gamma a\Gamma S \Rightarrow t = s_1\alpha\, a\beta\, s_2$ where $s_1, s_2 \in S$ and $\alpha, \beta \in \Gamma$.

Now $t\gamma s = (s_1\alpha\, a\beta\, s_2)\gamma\, s = s_1\alpha\, a\beta\,(s_2\;\gamma\, s) \in S\Gamma a\Gamma S$ and
$s\gamma t = s\gamma(\, s_1\alpha\, a\beta\, s_2) = (\, s\gamma s_1\,)\alpha\, a\beta\, s_2 \in S\Gamma a\Gamma S$. Therefore $S\Gamma a\Gamma S$ is a Γ-semi sub near-field space of S.

Since S is a simple Γ-semi normal sub near-field space, S itself is the only Γ-semi sub near-field space of S and hence $S\Gamma a\Gamma S = S$.

Conversely suppose that $S\Gamma a\Gamma S = S$ for all $a \in S$. Let I be a Γ-semi sub near-field space of S.

Let $a \in I$. Then $a \in S$. So $S\Gamma a\Gamma S = S$.

Let $s \in S$. Then $s \in S\Gamma a\Gamma S \Rightarrow s = t_1 \alpha\, a\beta\, t_2$ for some $t_1, t_2 \in S$, $\alpha, \beta \in \Gamma$.
$a \in I$, $t_1, t_2 \in S$, $\alpha, \beta \in \Gamma$, I is a Γ- semi sub near-field space of S

$\Rightarrow t_1 \alpha\, a\beta\, t_2 \in I \Rightarrow s \in I$. Therefore $S \subseteq I$. Clearly $I \subseteq S$ and hence $S = I$.

Therefore S is the only Γ- semi sub near-field space of S. Hence S is a simple Γ- semi normal sub near-field space. This completes the proof of the theorem.

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