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RELATIONS AMONG M-GONAL NUMBERS THROUGH SECOND ORDER RAMANUJAN EQUATION $X^2+Y^2=Z^2+W^2$

V. PANDICHELVI*[1] AND P. SIVAKAMASUNDARI[2]

[1]Assistant Professor, Department of Mathematics,
Urumu Dhanalakshmi college, Trichy, India.

[2]Guest Lecturer, Department of Mathematics, BDUCC, Lalgudi, Trichy, India.

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ABSTRACT

In this paper, we evaluate the ranks of the Triangular number, Hexagonal number, Octagonal number and Dodecagonal number such that the relation $32T_{3,m}+5T_{12,v}=12T_{8,u}+32T_{6,n}$ *is satisfied. Also, we determine the ranks of the Triangular number, Hexagonal number, Dodecagonal number and Icosagonal number such that the relation* $32T_{3,m}+9T_{20,v}=20T_{12,u}+32T_{6,n}$ *is satisfied by employing the standard solution of the second order Ramanujan equation* $X^2+Y^2=Z^2+W^2$.

Keywords: *Polygonal numbers, Ramanujan equation.*

NOTATIONS USED:

$T_{3,n}=\frac{1}{2}\left(n^2+n\right)$ be a triangular number of rank n.

$T_{6,n}=\left(2n^2-n\right)$ be a hexagonal number of rank n.

$T_{8,n}=3n^2-2n$ be a octagonal number of rank n.

$T_{12,n}=5n^2-4n$ be a dodecagonal number of rank n.

$T_{20,n}=9n^2-8n$ be a icosagonal number of rank n.

INTRODUCTION

We observe that $1^2+8^2=4^2+7^2$. Also, we observe that the following non-zero quadruples $\left(2rs-1,2rs-1,r^2+s^2,r^2-s^2\right)$, $\left(r^2-s^2-1,r^2-s^2-1,2rs,r^2+s^2\right)$, $\left(y+2,y,y+2,\pm1\right)$, $\left(-1,y,-1,\pm1\right)$, $\left(-1,y,\pm1,-1\right)$, $\left(-y,y,\pm1,-y\right)$ satisfy the equation $XY+X+Y+1=Z^2-W^2$. This equation motivates us to find the relation among M-gonal numbers through second order Ramanujan equation. In this communication, we find the relation between some polygonal numbers by using the standard solutions of second order Ramanujan equation $X^2+Y^2=Z^2+W^2$.

METHOD OF ANALYSIS

This paper consists of two sections. The detailed explanation of the connection between the Triangular Number, Hexagonal Number, Octagonal Number, Dodecagonal Number and Icosagonal Number is given in each of the following sections.

Corresponding Author: V. Pandichelvi[1]*

SECTION-1: RELATION BETWEEN TRIANGULAR NUMBER, HEXAGONAL NUMBER, OCTAGONAL NUMBER AND DODECAGONAL NUMBER.

The choices

$$X=4m+2\ ,\ Y=5v-2\ ,Z=6u-2\ ,W=8n-2 \tag{1}$$

leads the equation

$$32T_{3,m}+5T_{12,v}=12T_{8,u}+32T_{6,n} \tag{2}$$

to the second order Ramanujan equation

$$X^2+Y^2=Z^2+W^2 \tag{3}$$

which is satisfied by

$$\left.\begin{aligned} X&=rp+sq\\ Y&=rq-sp\\ Z&=rp-sq\\ W&=rq+sp \end{aligned}\right\} \tag{4}$$

Using (4) and (1), we note that

$$m=\frac{rp+sq-2}{4}$$
$$v=\frac{rq-sp+2}{5}$$
$$u=\frac{rp-sq+2}{6}$$
$$n=\frac{rq+sp+2}{8}$$

Since our aim is to find the integer values for the ranks of the polygonal numbers satisfying (1), we observe that the values of $m, n\ , u\ , and\ v$ are integers for the following choices of $r\ , s\ , p\ , and\ q$

$$r=240k+2$$
$$s=120k$$
$$p=30k+59$$
$$q=60k+59$$

Hence, the values of ranks of Triangular Number, Hexagonal Number, Octagonal Number and Dodecagonal Number satisfying our assumption are represented by

$$m=3600k^2+5325k+29$$
$$n=2250k^2+2670k+15$$
$$u=1190k+20$$
$$v=2160k^2+1440k+24$$

Using the values of $m, n\ , u\ , and\ v$, we obtain

$$T_{3,m}=\frac{1}{2}\left(12960000k^4+38340000k^3+28568025k^2+314175k+870\right)$$
$$T_{6,n}=10125000k^4+24030000k^3+14390550k^2+157530k+435$$
$$T_{8,u}=4248300k^2+140420k+1160$$
$$T_{12,v}=23328000k^4+31104000k^3+10877760k^2+339840k+2784$$

Thus, $32T_{3,m}+5T_{12,v}=324000000k^4+768960000k^3+511477200k^2+6726000k+27840$

$$12T_{8,u}+32T_{6,n}=324000000k^4+768960000k^3+511477200k^2+6726000k+27840$$

Hence,

$$32T_{3,m}+5T_{12,v}=12T_{8,u}+32T_{6,n}$$

SECTION-2: RELATION BETWEEN TRIANGULAR NUMBER, HEXAGONAL NUMBER, ODECAGONAL NUMBER AND ICOSAGONAL NUMBER.

The choices

$$X=4m+2\ ,\ Y=9v-4, Z=10u-4, W=8n-2 \qquad (5)$$

leads the equation

$$32T_{3,m}+9T_{20,v}=20T_{12,u}+32T_{6,n} \qquad (6)$$

to the second order Ramanujan equation

$$X^2+Y^2=Z^2+W^2 \qquad (7)$$

Employing the standard solutions to (7) as given in (4) and using (5), the values of $m, n\ , u, and\ v$ are exhibited by

$$m=\frac{rp+sq-2}{4}$$
$$v=\frac{rq-sp+4}{9}$$
$$u=\frac{rp-sq+4}{10}$$
$$n=\frac{rq+sp+4}{8}$$

Since our motivation is to find the integer values for the ranks of the polygonal numbers satisfying (6), we notice that the values of $m, n\ , u, and\ v$ are integers for the following choices of $r, s\ , p, and\ q$

$$r=720k+2$$
$$s=360k$$
$$p=270k+43$$
$$q=180k+43$$

Hence, the values of ranks of Triangular Number, Hexagonal Number, Dodecagonal Number and Icosagonal Number satisfying our assumption are signified by

$$m=64800k^2+11745k+21$$
$$n=28350k^2+5850k+11$$
$$u=12960k^2+1602k+9$$
$$v=3600k^2+1760k+10$$

Using the values of $m, n\ , u, and\ v$, we obtain

$$T_{3,m}=\frac{1}{2}\left(4199040000k^4+1522152000k^3+140731425k^2+505035k+462\right)$$
$$T_{6,n}=1607445000k^4+663390000k^3+69664050k^2+251550k+231$$
$$T_{12,u}=839808000k^4+207619200k^3+13946580k^2+137772k+369$$
$$T_{20,v}=116640000k^4+114048000k^3+28497600k^2+302720k+820$$

Thus,

$$32T_{3,m}+9T_{20,v}=68234400000k^4+25380864000k^3+2508181200k^2+10805040k+14772$$
$$20T_{12,u}+32T_{6,n}=68234400000k^4+25380864000k^3+2508181200k^2+10805040k+14772$$

Hence,

$$32T_{3,m}+9T_{20,v}=20T_{12,u}+32T_{6,n}$$

CONCLUSION

In this communication, we discover the ranks of the polygonal numbers through the second order Ramanujan equation. In this manner, one can find the relation between some recognizable numbers through third and fourth order Ramanujan equation**.**

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