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ON SOME NANO APPROXIMATION OPERATORS

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ABSTRACT

The object of this study is to introduce Nano approximation operators called Nano Lower and Nano Upper approximations in Nano topologized approximation space which is a generalization of Pawlak approximation space. Nano open sets and Nano-closed sets are used to construct Nano approximation operators. Some of the properties of Nano approximation operators are discussed. Further Nwg- Lower and Nwg-upper approximations are defined using Nwg-open sets and Nwg-closed sets. Also Nano and Nwg-accuracy measures are defined and observed that the degree of exactness of a set using Nwg lower and upper approximation is better than the other accuracy measures.

Keywords: *Nano topological space, Nwg-closed set, rough set, Approximation space, topologized approximation space, Nwg approximation operator.*

I. INTRODUCTION

In 1982, Pawlak[16] introduced the concept of rough sets in which a subset of universe is described by a pair of ordinary sets called the lower and upper approximations . In rough set theory equivalence relation plays a important role as they are the building blocks for constructing lower and upper approximations. Rough set theory is extended by replacing equivalence relation by binary relation [3], neighborhood systems [13, 14] and coverings [9, 10]. Yao [15] defined a pair of generalized approximation operators by replacing the equivalence relation with the family of open sets for lower approximation operator “Interior operator” and family of closed sets for upper approximation operator “Closure operators”. Many authors have generalized Pawlak approximation space using β -open sets [2], χ open sets [1], $\delta\beta$-open set [4]. In 2010 Jamal [5] topologized the approximation space by associating a topology with it.

Lellis Thivagar [6] observed that universal set, lower and upper approximations of a subset of universal set forms a basis for topology and named it as Nano topology because of its small size. The author [7] has also defined different neighbourhoods based on Nano topology and using indiscernibility matrix found CORE in an information system. He [8] also topologized the approximation space with Nano topology. Nagaveni [12] introduced a class of sets called weakly generalized closed sets in general topology in 1999. Nagaveni and Bhuvaneswari [11] applied the concept of weakly generalization in Nano topology and introduced Nwg-closed sets.

In this paper we introduce new approximation operators called Nano Lower and Nano Upper approximations. Some of the properties are investigated. Also Nwg-Lower and Nwg-Upper approximations are defined. Using these approximation operators Pawlak approximation space is generalized.

II. PRELIMINARIES

This section is to recall some definitions and properties which are useful in this study.

Definition 2.1[16]: $K=(X,R)$ is called the approximation space, where X is a set called the universe and R is an equivalence relation. The equivalence classes of R are also known as the granules, elementary sets, or blocks.

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Definition 2.2[16]: Lower and upper approximations of a subset of universe are defined by $\overline{R}(A)=\{x\in X : R_x \cap A \neq \phi\}$, $\underline{R}(A)=\{x\in X : R_x \subseteq A\}$.

$b(A)=\overline{R}(A)-\underline{R}(A)$ is called the τ-boundary of a subset $A\subseteq X$. A subset A is exact if $b(A)=\phi$, otherwise A is rough. A is exact if and only if $\overline{R}(A)=\underline{R}(A)$

Definition 2.3[16]: Let X be a finite nonempty universe, $A\subseteq X$, the degree of completeness can also be characterized by the accuracy measures as $\alpha_R(A)=\dfrac{|\underline{R}(A)|}{|\overline{R}(A)|}$, $A\neq\phi$, where |.| represents the cardinality of set.

Definition 2.4[16]: we can measure inexactness and express topological characterization of imprecision with the following.

(i) If $\underline{R}(A)\neq\phi$ and $\overline{R}(A)\neq X$, then A is roughly R-definable.

(ii) If $\underline{R}(A)=\phi$ and $\overline{R}(A)\neq X$, then A is internally R-undefinable.

(iii) If $\underline{R}(A)\neq\phi$ and $\overline{R}(A)=X$, then A is externally R-undefinable.

(iv) If $\underline{R}(A)=\phi$ and $\overline{R}(A)=X$, then A is totally R-un definable.

The set of all roughly R-definable (resp., internally R-undefinable, externally R-undefinable and totally R-undefinable) sets by RD(X), (resp., IUD(X), EUD(X) and TUD(X)).

Definition 2.5[5]: Let $K=(X,R)$ be an approximation space and $\tau_R(X)$ is the topology associated with K then the triple $(X,R,\tau_R(X))$ is called a topologized approximation space.

Definition 2.6[15]: Let X be a finite universe, $A\subseteq X$, the lower approximation and upper approximation of A is defined by $i(X)=\{Y/Y\in O(U), Y\subseteq X\}$, $c(X)=\{Y/Y\in C(U), X\subseteq Y\}$.

Definition 2.7[6]: Let U be the universe, R be an equivalence relation on U , $X\subseteq U$. then $\{U, \overline{R}(X)\ and\ \underline{R}(X)\}$ forms a basis for a topology called Nano topology, $\tau_R(X)=\{U,\phi,U, \overline{R}(X), \underline{R}(X), B(X)\}$ where $X\subseteq U$. We call $(U,\tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. Elements of $[\tau_R(X)]^c$ are called Nano closed sets.

Definition 2.8[8]: Let U be the universe of finite elements, $X\subseteq U$ and R denote the relation which is used to get a base for Nano topology $\tau_R(X)$ and the triple $(U,R,\tau_R(X))$ is called a Nano topologized approximation space.

III. NANO TOPOLOGIZED APPROXIMATION SPACES

In this section we introduce Nano and Nwg-approximation operators and study some of the properties.

Definition 3.1: Let U be the universe of finite elements, $X\subseteq U$ and R denote the relation which is used to get a base for Nano topology $\tau_R(X)$. Then Nano-Lower and Nano upper approximations of any nonempty subset X of U is defined as $L_N(X)=\bigcup\{G : G\ is\ Nano\ open\ set, G\subseteq X\}$, $U_N(X)=\bigcap\{F : F\ is\ Nano\ closed\ set, X\subseteq F\}$.

Definition 3.2: Let U be the universe of finite elements, $X\subseteq U$ and R denote the relation which is used to get a base for Nano topology $\tau_R(X)$ .Then Nano weakly generalized-lower (Nwg –Lower) and Nano weakly generalized-Upper (Nwg-Upper) approximations are defined as $L_{Nwg}(X)=\bigcup\{G : G\ is\ Nwg\ open\ set, G\subseteq X\}$, $U_{Nwg}(X)=\bigcap\{F : F\ is\ Nwg\ closed\ set, X\subseteq F\}$.

Example 3.3: Let $U=\{a,b,c,d\}$ be a universe and a relation R defined by $R=\{(a,a),(a,c),(a,d),(b,b),(b,d),(c,b),(c,d),(d,a)\}$, *thus* $aR=\{a,c,d\}$, $bR=\{b,d\}$, $cR=\{b,d\}$ and $dR=\{a\}$. Let $X=\{a,c\}$, then $\underline{R}(X)=\{a\}$, $\overline{R}(X)=\{a,c,d\}$, $B(X)=\overline{R}(X)-\underline{R}(X)$ $\tau_R(X)=\{U,\phi,\{a\},\{a,c,d\},\{c,d\}\}$. $(U,R,\tau_R(X))$ is a Nano topologized approximation space.

$NWGOS(U) = \{\{a\},\{c\},\{d\},\{a,c\},\{a,d\},\{c,d\},\{a,c,d\},\{a,b,d\},\{a,b,c\}\}$. Nano approximation operators are $L_N(X) = \{a\}, U_N(X) = \{a,b,c,d\}$, Nwg approximation operators are $L_{Nwg}(X) = \{a\}, U_{Nwg}(X) = \{a,b,c\}$.

Proposition 3.4: Let $(U, R, \tau_R(X))$ be a Nano topologized approximation space and $A, B \subseteq U$.

1. $L_N(A) \subseteq A \subseteq U_N(A)$
2. $L_N(\phi) = \phi = U_N(\phi), L_N(U) = U = U_N(U)$
3. If $A \subseteq B$ then $L_N(A) \subseteq L_N(B)$ and $U_N(A) \subseteq U_N(B)$
4. $L_N(U - A) \subseteq X - U_N(B)$
5. $U_N(U - A) \subseteq X - L_N(B)$
6. $L_N(L_N(A)) = L_N(A)$
7. $U_N(U_N(A)) = U_N(A)$
8. $L_N(L_N(A)) \subseteq U_N(A)$
9. $L_N(U_N(A)) \subseteq U_N(U_N(A))$
10. $L_N(A \cup B) \supseteq L_N(A) \cup L_N(B)$
11. $L_N(A \cap B) = L_N(A) \cap L_N(B)$
12. $U_N(A \cup B) \supseteq U_N(A) \cup U_N(B)$
13. $U_N(A \cap B) \supseteq U_N(A) \cap U_N(B)$

Proposition 3.5: Let $(U, R, \tau_R(X))$ be a Nano topologized approximation space and $A, B \subseteq U$.

1. $L_{Nwg}(A) \subseteq A \subseteq U_{Nwg}(A)$
2. $L_{Nwg}(\phi) = \phi = U_{Nwg}(\phi), L_{Nwg}(U) = U = U_{Nwg}(U)$
3. If $A \subseteq B$ then $L_{Nwg}(A) \subseteq L_{Nwg}(B)$ and $U_{Nwg}(A) \subseteq U_{Nwg}(B)$
4. $L_{Nwg}(U - A) \subseteq X - U_{Nwg}(B)$
5. $U_{Nwg}(U - A) \subseteq X - L_{Nwg}(B)$
6. $L_{Nwg}(L_{Nwg}(A)) = L_{Nwg}(A)$
7. $U_{Nwg}(U_{Nwg}(A)) = U_{Nwg}(A)$
8. $L_{Nwg}(L_{Nwg}(A)) \subseteq U_{Nwg}(A)$
9. $L_{Nwg}(U_{Nwg}(A)) \subseteq U_{Nwg}(U_{Nwg}(A))$
10. $L_{Nwg}(A \cup B) \supseteq L_{Nwg}(A) \cup L_{Nwg}(B)$
11. $L_{Nwg}(A \cap B) = L_{Nwg}(A) \cap L_{Nwg}(B)$
12. $U_{Nwg}(A \cup B) \supseteq U_{Nwg}(A) \cup U_{Nwg}(B)$
13. $U_{Nwg}(A \cap B) \supseteq U_{Nwg}(A) \cap U_{Nwg}(B)$

Theorem 3.6: *For any Nano topologized approximation space* $(U, R, \tau_R(X))$, we have, $L_N(A) \subseteq L_{Nwg}(A) \subseteq A \subseteq U_{Nwg}(A) \subseteq U_N(A)$.

Proof: $L_N(A) = \bigcup\{G : G \text{ is Nano open}, G \subseteq A\} \subseteq \bigcup\{G : G \text{ is } Nwg - \text{open}, G \subseteq A\} = L_{Nwg}(A)$.
$U_N(A) = \bigcup\{G : G \text{ is Nano closed}, A \subseteq G\} \supseteq \bigcup\{G : G \text{ is } Nwg - \text{closed}, A \subseteq G\} = U_{Nwg}(A)$. By Proposition 3.5(1) $L_{Nwg}(A) \subseteq A \subseteq U_{Nwg}(A)$. Hence $L_N(A) \subseteq L_{Nwg}(A) \subseteq A \subseteq U_{Nwg}(A) \subseteq U_N(A)$.

Definition 3.7: Let $(U, R, \tau_R(X))$ be a Nano topologized approximation space. The universe U can be divided into 15 region with respect to any $A \subseteq U$ as follows.(figure 3.1)

(1) The interior of A, $L_N(A))$

(2) The Positive region of A, $U_N(A)$

(3) The negative region of A, The exterior of A, $U - U_N(A)$

(4) The Nwg-positive region of A, $U_{Nwg}(A)$

(5) The Nwg-interior of A, $L_{Nwg}(A)$

(6) The internal edge of A, $\underline{Edge(A)} = A - L_N(A)$

(7) The Nwg-internal edge of A, $\underline{NwgEdge(A)} = A - L_{Nwg}(A)$

(8) The external edge of A, $\overline{Edge(A)} = U_N(A) - A$

(9) The Nwg-external edge of A, $\overline{NwgEdge(A)} = U_{Nwg}(A) - A$

(10) The boundary of A, $b(A) = U_N(A) - L_N(A)$

(11) The Nwg-boundary of A, $Nwgb(A) = U_{Nwg}(A) - L_{Nwg}(A)$

(12) The Nwg-exterior of A, $U - U_{Nwg}(A)$

(13) $U_{Nwg}(A) - L_N(A)$

(14) $L_{Nwg}(A) - L_N(A)$

(15) $U_N(A) - U_{Nwg}(A)$

The elements of the region $L_{Nwg}(A) - L_N(A)$ are well defined in A and the elements of the region $U_N(A) - U_{Nwg}(A)$ do not belong to A while these elements were not defined in Pawlak's approximation space. Boundary region is decreased using nano and Nwg approximations.

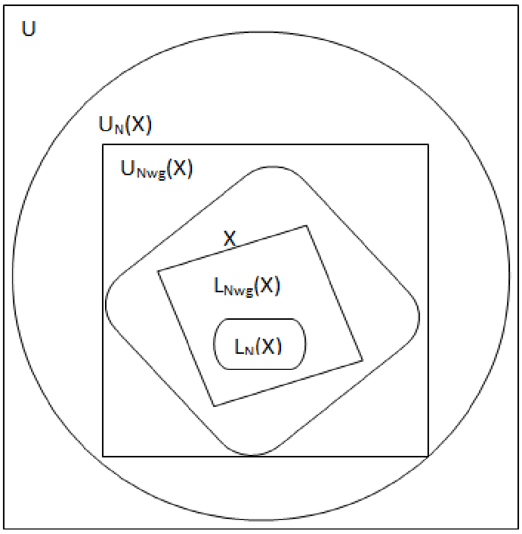

Figure-3.1

Definition 3.8: Let $(U, R, \tau_R(X))$ be a Nano topologized approximation space and $A \subseteq U$, we define Nano strong, Nwg-strong , Nano weak Nwg-weak memberships as (i) $x \underline{\in}_N A$ iff $x \in L_N(A)$ (ii) $x \underline{\in}_{Nwg} A$ iff $x \in L_{Nwg}(A)$ (iii) $x \overline{\in}_N A$ iff $x \in U_N(A)$ (iv) $x \overline{\in}_{Nwg} A$ iff $x \in U_{Nwg}(A)$

Remark 3.9: According to the above definition Nano and Nwg-approximation operators of a set $A \subseteq U$ can be written as

(i) $L_N(X) = \{x \in X : x \underline{\in}_N A\}$ (ii) $L_{Nwg}(X) = \{x \in X : x \underline{\in}_{Nwg} A\}$ (iii) $U_N(X) = \{x \in X : x \overline{\in}_N A\}$.

(iv) $U_{Nwg}(X) = \{x \in X : x \overline{\in}_{Nwg} A\}$.

Remark 3.10: Let $(U, R, \tau_R(X))$ be a Nano topologized approximation space and $A \subseteq U$. Then

$(i)\quad x \underline{\in}_N A \Rightarrow x \underline{\in}_{Nwg} A \qquad (ii)\quad x \overline{\in}_{Nwg} A \Rightarrow x \overline{\in}_N A$

Definition 3.11: Let U be a finite nonempty universe, $X \subseteq U$, we define Nano accuracy measure and Nwg accuracy measure which shows the degree of completeness of X as follows

$$(i)\qquad \alpha_N(X) = \left|\frac{L_N(X)}{U_N(X)}\right|,\ X \neq \phi.\quad 0 \leq \alpha_N(X) \leq 1$$

$$(ii)\qquad \alpha_{Nwg}(X) = \left|\frac{L_{Nwg}(X)}{U_{Nwg}(X)}\right|,\ X \neq \phi.\quad 0 \leq \alpha_{Nwg}(X) \leq 1$$

Example 3.12: For the example 3.3, the various accuracy measures of any subset of universe U is given in the following table. From the following table (table3.1) we see that Nwg accuracy measure $\alpha_{Nwg}(A)$ is better than $\alpha_R(A)$) and $\alpha_N(A)$.

S.No	$A \subseteq U$	$\alpha_R(A)$	$\alpha_N(A)$	$\alpha_{Nwg}(A)$
1	$\{a\}$	½	½	½
2	$\{b\}$	*0*	*0*	*0*
3	$\{c\}$	*0*	*0*	*1*
4	$\{d\}$	*1/3*	*0*	*1*
5	$\{a,b\}$	*1/3*	½	½
6	$\{b,c\}$	*0*	*0*	½
7	$\{c,d\}$	*1/3*	*2/3*	*2/3*
8	$\{a,d\}$	½	¼	*2/3*
9	$\{a,c\}$	½	¼	*2/3*
10	$\{b,d\}$	*2/3*	*0*	½
11	$\{a,b,c\}$	*1/3*	¼	*1*
12	$\{b,c,d\}$	*2/3*	*2/3*	*2/3*
13	$\{a,c,d\}$	¾	¾	¾
14	$\{a,b,d\}$	¾	¼	*1*

Table-3.1

Definition 3.13: Let $(U, R, \tau_R(X))$ be a Nano topologized approximation space and $A, B \subseteq U$. Then we say that A and B are

(i) Nano-roughly bottom equal $(\underline{A} =_N \underline{B})$ if $L_N(A) = L_N(B)$

(ii) Nano-roughly top equal $(\overline{A} =_N \overline{B})$ if $U_N(A) = U_N(B)$

(iii) Nano-roughly equal $(A =_N B)$ if $L_N(A) = L_N(B)$ and $U_N(A) = U_N(B)$

(iv) Nwg-roughly bottom equal $(\underline{A} =_{Nwg} \underline{B})$ if $L_{Nwg}(A) = L_{Nwg}(B)$

(v) Nwg-roughly top equal $(\overline{A} =_{Nwg} \overline{B})$ if $U_{Nwg}(A) = U_{Nwg}(B)$

(vi) Nwg-roughly equal $(A =_{Nwg} B)$ if $L_{Nwg}(A) = L_{Nwg}(B)$ and $U_{Nwg}(A) = U_{Nwg}(B)$

Definition 3.14: Let $(U, R, \tau_R(X))$ be a Nano-topologized approximation space and $A, B \subseteq U$. Then we say that

(i) A is Nano-roughly bottom included in B $(\underline{A} \subset_N \underline{B})$ if $L_N(A) \subseteq L_N(B)$

(ii) A is Nano-roughly top included in B $(\overline{A} \subset_N \overline{B})$ if $U_N(A) \subseteq U_N(B)$

(iii) A is Nano-roughly included in B $(A \subseteq_N B)$ if $L_N(A) \subseteq L_N(B)$ and $U_N(A) \subseteq U_N(B)$

(iv) A is Nwg-roughly bottom included in B $(\underline{A} \subset_{Nwg} \underline{B})$ if $L_{Nwg}(A) \subseteq L_{Nwg}(B)$

(v) A is Nwg-roughly top included in B $(\overline{A} \subset_{Nwg} \overline{B})$ if $U_{Nwg}(A) \subseteq U_{Nwg}(B)$

(vi) A is Nwg-roughly included in B $(A \subseteq_{Nwg} B)$ if $L_{Nwg}(A) \subseteq L_{Nwg}(B)$ and $U_{Nwg}(A) \subseteq U_{Nwg}(B)$

Definition 3.15: for any Nano topologized approximation space $(U, R, \tau_R(X))$ a subset $X \subseteq U$ is called

(i) Nano-definable (Nano-exact) if $L_N(X) = U_N(X)$.

(ii) Nano-Rough if $L_N(X) \neq U_N(X)$.

Definition 3.16: for any Nano topologized approximation space $(U, R, \tau_R(X))$ a subset $X \subseteq U$ is called

(i) Nwg-definable (Nwg-exact) if $L_{Nwg}(X) = U_{Nwg}(X)$.

(ii) Nwg-Rough if $L_{Nwg}(X) \neq U_{Nwg}(X)$.

Remark 3.17: A set is Nano exact iff it is both Nano open and Nano closed.

Remark 3.18: A set is Nwg exact iff it is both Nwg open and Nwg closed.

Proposition 3.19: Let $(U, R, \tau_R(X))$ be any Nano topologized approximation space, then

(i) Every Nano exact set in U is Nwg-exact.

(ii) Every Nwg-rough is Rough

Remark 3.20: Every Nwg-exact need not be Nano exact as given in the following example.

Example 3.21: In example 3.3 $X = \{d\}$ is Nwg-exact but not Nano exact.

Definition 3.22: For any Nano topologized -approximation space $(U, R, \tau_R(X))$ a subset $X \subseteq U$ is called .

(i) Roughly Nano-definable if $L_N(X) \neq \phi$ *and* $U_N(X) \neq U$

(ii) Internally Nano-un definable if $L_N(X) = \phi$ *and* $U_N(X) \neq U$

(iii) Externally Nano-un definable if $L_N(X) \neq \phi$ *and* $U_N(X) = U$

(iv) Totally Nano-un definable if $L_N(X) = \phi$ *and* $U_N(X) = U$

The set of all roughly Nano-definable (resp., internally Nano-undefinable, externally Nano-undefinable and totally Nano-undefinable) sets by RND(X), (resp., INUD(X), ENUD(X) and TNUD(X)).

Definition 3.23: For any Nano topologized -approximation space $(U, R, \tau_R(X))$ a subset $X \subseteq U$ is called .

(v) Roughly Nwg-definable if $L_{Nwg}(X) \neq \phi$ *and* $U_{Nwg}(X) \neq U$

(vi) Internally Nwg-un definable if $L_{Nwg}(X) = \phi$ *and* $U_{Nwg}(X) \neq U$

(vii) Externally Nwg-un definable if $L_{Nwg}(X) \neq \phi$ *and* $U_{Nwg}(X) = U$

(viii) Totally Nwg-un definable if $L_{Nwg}(X) = \phi$ *and* $U_{Nwg}(X) = U$

The set of all roughly Nwg-definable (resp., internally Nwg-undefinable, externally Nwg-undefinable and totally Nwg-undefinable) sets by RNWGD(X), (resp., INWGUD(X), ENWGUD(X) and TNWGUD(X)).

Example 3.24: In example 3.3

(i) $X = \{a\}$ is Nano and Nwg definable.

(ii) $X = \{d\}$ is Internally Nano un definable and Nwg-exact.

(iii) $X = \{a, b, d\}$ externally un definable.

(iv) $X = \{b, c\}$ is Nano and Nwg-rough set.

Example 3.25: In example 3.3

(i) $A = \{a, b\}$ and $B = \{a, b, d\}$ are Nano bottom equal but not Nwg-bottom equal.

(ii) $A = \{a, c\}$ and $B = \{a, d\}$ are Nano top equal but not Nwg-top equal.

Remark 3.26: For any Nano topologized -approximation space $(U, R, \tau_R(X))$ a subset $X \subseteq U$ if

(i) $\alpha_N(X) = 0$, then A is internally Nano undefinable or totally Nano undefinable.

(ii) $\alpha_{Nwg}(X) = 0$, then A is internally Nwg undefinable or totally Nwg undefinable.

(iii) $\alpha_N(X) = 1$, then A is Nano exact (Nano definable).

(iv) $\alpha_{Nwg}(X) = 1$, then A is Nwg exact (Nwg definable).

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