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A STUDY ON SOME NEW OPERATIONS OF FUZZY SOFT SETS AND ESTABLISHED FORWARD A DECISION-MAKING PROBLEM

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ABSTRACT

Molodstov introduced the theory of soft sets, which can be seen as a new mathematical approach to vagueness. Maji et al. have also introduced several basic concepts of soft set theory. They have also introduced the concept of fuzzy soft set, a more generalized thought, which is a combination of fuzzy set and soft set. They introduced some properties concerning fuzzy soft union, intersection, the complement of a fuzzy soft set, De Morgan Laws etc. These results were also revised and improved by Ahmad and Kharal. They defined arbitrary fuzzy soft union and intersection and proved De Morgan Inclusions and De Morgan Laws in Fuzzy Soft Set Theory. The fuzzy soft set is one of the current topics incited for dealing with the doubtfulness present in most of our real-life circumstances. The parameterization tool of soft set theory improves the flexibility of its application. The motive of this paper is to study the concept of the disjunctive sum, difference and symmetric difference of fuzzy soft sets and their basic properties. Finally, we have established forward a decision-making problem using the concept of the cardinality of fuzzy soft sets.

Keywords: *Fuzzy Soft Set, Rough Set, Absolute Fuzzy Soft Set, Disjunctive Sum, Difference, Symmetric Difference, Cardinality.*

1. INTRODUCTION

In many complicated problems arising in the fields of engineering, social science, economics, environment, medical science etc. have various uncertainties. Fuzzy soft sets are very necessary structures growing in many areas of mathematics and computer science. Fuzzy sets have been proved to be an exactly suitable tool to handle doubtfulness. Zadeh [14] introduced fuzzy set theory in 1965 and it was specifically designed to represent uncertainty and vagueness with formalized logical tools in dealing with the imprecision inherent in many real-world problems. In 1999, Molodtsov [8] pointed out that the significance of existing theories namely Probability Theory, Fuzzy Soft Set Theory, Rough Set Theory etc. which are considered as mathematical tools in dealing with uncertainties, cannot be successfully used to solve complicated problems in the fields of engineering, social science, economics, medical science etc. He further pointed out that the reason for these difficulties is the inadequacy of the parameterization tool of the theory. Accordingly, he put forward a noble concept known as soft set as a new mathematical tool for behavior with doubtfulness. The soft set theory introduced by Molodstov is free of the difficulties present in these theories. The absence of any restriction on the approximate description in soft set theory creates this theory extremely suitable and easily applicable. Zadeh's fuzzy sets are considered as a special case of the soft sets. Since its introduction, the concept of the soft set has gained a considerable amount of attention including some successful applications in information processing, decision, demand analysis, clustering, and forecasting.

In 2003, P.K. Maji *et al.* [6] created a theoretical study on the soft set theory in more details. Particularly, they founded the concepts of subset, intersection, union, and complement of soft sets and discussed their properties. These operations construct it possible to make new soft sets from given soft sets.

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In recent times, researchers have contributed a lot to fuzzification of soft set theory. Maji *et al.* [7] combined fuzzy sets with soft sets and introduced the idea of fuzzy soft sets along with some properties regarding fuzzy soft sets union, intersection, the complement of a fuzzy soft set, De Morgan Law etc. These results were also revised and developed by Ahmad and Kharal [1]. In present times, works on the fuzzification of the soft set theory are going on and these works have founded different new mathematical models such as intuitionistic fuzzy soft set, generalized fuzzy soft set, possibility fuzzy soft set etc.

Neog and Sut have initiated some new operations of fuzzy soft sets in [4]. In case of fuzzy soft sets, this already has been pointed out by Ge and Yang in [13] and consequently, they have studied some basic results regarding fuzzy soft sets. In our work, we have put forward some more than concepts related to fuzzy soft sets. A decision problem solved with the support of cardinality of fuzzy soft sets has been proposed in our work.

2. BASIC CONCEPT OF FUZZY SOFT SETS

In this section, we first observe the basic definitions related to fuzzy soft sets which would be applied in the sequel.

Definition 2.1 [8]: A pair (F, E) is called a soft set (over U) iff F is a mapping of E into the set of all subsets of the set U.

In other hands, the soft set is a parameterized family of subsets of the set U. Every set $F(\epsilon) \in E$, from this family may be considered as the set of $\varepsilon - elements$ of the soft set (F, E), or as the set of $\varepsilon - approximate\ elements$ of the soft set.

Example 2.1: Let U= $\{c_1, c_2, c_3, c_4\}$ be the set of four cars under regard and
E= $\{e_1\ (Costly), e_2(Beautiful),\ e_3(Fuel\ Efficient),\ e_4(Modern\ Techonology),\ e_5(Luxurious)\}$ be the set of parameters and $A = \{e_1, e_2, e_3\} \subseteq E$.

Then, $(F, A) = \{F(e_1) = \{c_1, c_4\}, F(e_2) = \{c_1, c_2, c_4\}, F(e_3) = \{c_3\}\}$ is the soft set showing the 'attraction of the car' which Mr. A is going to buy. We can represent this soft set in a tabular form as shown below [7].

This style of representation will be essential for storing a soft set in a computer memory.

U	e_1	e_2	e_3
c_1	1	1	0
c_2	0	1	0
c_3	0	0	1
c_4	1	1	0

Definition 2.2 [10]: A pair (F, A) is called a fuzzy soft set over U where, $F: A \to \tilde{P}(U)$ is a mapping from A into $\tilde{P}\ (U)$. Here $\tilde{P}\ (U)$ denotes the fuzzy subsets of U.

Example 2.2: Let U= $\{c_1, c_2, c_3, c_4\}$ be the set of four cars under regard and
E= $\{e_1\ (Costly), e_2(Beautiful),\ e_3(Fuel\ Efficient),\ e_4(Modern\ Techonology),\ e_5(Luxurious)\}$ be the set of parameters and $A = \{e_1, e_2, e_3\} \subseteq E$. Then
$(F, A) = \{F(e_1) = \{{}^{c_1}/_{0.7}, {}^{c_2}/_{0.1}, {}^{c_3}/_{0.2}, {}^{c_4}/_{0.6}\}$, F$(e_2)$= $\{{}^{c_1}/_{0.8}, {}^{c_2}/_{0.6}, {}^{c_3}/_{0.1}, {}^{c_4}/_{0.5}\}$,
$F(e_3) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}\}$ is the fuzzy soft set showing the "attraction of the car" which Mr. A is going to buy.

Definition 2.3 [1]: Let U be a universe and E a set of attributes. Then the pair (U, E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

Definition 2.4 [8]: A soft set (F, A) over U is said to be null fuzzy soft set denoted by $\widetilde{\Phi}$ if $\forall\ \varepsilon \in A, F(\varepsilon)$ is a null fuzzy set $\tilde{0}$ of U, where $\tilde{0}(x) = 0\ \forall\ x \in U$.We would denoted ($\widetilde{\Phi}$, A) to represent the fuzzy soft null set with respect to the set of parameters A.

Definition 2.5 [8]: A soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall\ \varepsilon \in A$, $F(\varepsilon)\ is\ a$ absolute fuzzy soft set $\tilde{1}$ of U, where $\tilde{1}(x) = 1\ \forall\ x \in U$.We would denoted (U, A) to represent the absolute fuzzy soft set with respect to the set of parameters A.

Definition 2.6 [8]: For two fuzzy soft sets (F_1, A_1) and (F_2, A_2) in a fuzzy soft class (U, E), we say that (F_1, A_1) is a fuzzy soft subset of (F_2, A_2), if

(i) $A_1 \subseteq A_2$,

(ii) $for\ all\ \varepsilon \in A_1\ and\ written\ as\ (F_1, A_1) \subseteq (F_2, A_2). \varepsilon \in A_1, F_1(\varepsilon) \leq F_2$.

Example-2.6. Let U= $\{c_1, c_2, c_3, c_4\}$ be the set of four cars under regard and E= $\{e_1\ (Costly), e_2(Beautiful),\ e_3(Fuel\ Efficient),\ e_4(Modern\ Techonology),\ e_5(Luxurious)\}$ be the set of parameters and $A_1 = \{e_1, e_2, e_3\} \subseteq E\ and\ A_2 = \{e_1, e_2, e_3, e_5\} \subseteq E$. Then

$(F_1, A_1) = \{F_1(e_1) = \{{}^{c_1}/_{0.7}, {}^{c_2}/_{0.1}, {}^{c_3}/_{0.2}, {}^{c_4}/_{0.6}\}$,
$F_1(e_2) = \{{}^{c_1}/_{0.8}, {}^{c_2}/_{0.6}, {}^{c_3}/_{0.1}, {}^{c_4}/_{0.5}\}$,
$F_1(e_3) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}\}$ is the fuzzy soft set showing the "attraction of the car" which Mr. A is going to buy.

$(F_2, A_2) = \{F_1(e_1) = \{{}^{c_1}/_{0.7}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.2}, {}^{c_4}/_{0.7}\}$,
$F_1(e_2) = \{{}^{c_1}/_{0.9}, {}^{c_2}/_{0.6}, {}^{c_3}/_{0.5}, {}^{c_4}/_{1}\}$,
$F_1(e_3) = \{{}^{c_1}/_{0.3}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.8}, {}^{c_4}/_{0.3}\}$,
$F_1(e_5) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}\}$ is the fuzzy soft set showing the "attraction of the car" which Mr. B is going to buy.

Here, $A_1 \subseteq A_2$, and for all $\in A_1, F_1(\varepsilon) \leq F_2(\varepsilon). Thus\ (F_1, A_1) \tilde{\subseteq} (F_2, A_2)$.

Pei and Miao [10] modified this definition of soft subset in the following way –

Definition 2.7 Soft Subset Redefined [12]: For two fuzzy soft sets (F_1, A_1) and (F_2, A_2) in a fuzzy soft class (U, E), we say that $(F_1, A_1)$is a fuzzy soft subset of (F_2, A_2), if

(i) $A_1 \subseteq A_2$,

(ii) $\forall \varepsilon \in A_1, F_1(\varepsilon) \subseteq F_2(\varepsilon)$ and written as $(F_1, A_1) \tilde{\subseteq} (F_2, A_2)$.

(F_1, A_1) is said to be soft superset of (F_2, A_2) if (F_2, A_2) is a soft sub set of (F_1, A_1) and written as $(F_1, A_1) \tilde{\supseteq} (F_2, A_2)$.

Definition 2.8 [7]: For two fuzzy soft sets (F_1, A_1) and (F_2, A_2) over the common universe U is the fuzzy soft set (F_3, A_3), where $A_3 = A_1 \cup A_2\ and\ \forall \varepsilon \in A_3$,

$$F_3(\varepsilon) = \begin{cases} F_1(\varepsilon), & if\ \varepsilon \in A_1 - A_2 \\ F_2(\varepsilon), & if\ \varepsilon \in A_2 - A_1 \\ F_1(\varepsilon) \cup F_2(\varepsilon), & \text{if } \varepsilon \in A_1 \cap A_2 \end{cases}$$

And is written as $(F_1, A_1)\ \tilde{\cup}\ (F_2, A_2) = (F_3, A_3)$.

Example-2.8: Let U= $\{c_1, c_2, c_3, c_4\}$be the set of four cars under regard and E= $\{e_1\ (Costly), e_2(Beautiful),\ e_3(Fuel\ Efficient),\ e_4(Modern\ Techonology),\ e_5(Luxurious)\}$ be the set of parameters and $A_1 = \{e_1, e_2, e_3\} \subseteq E\ and\ A_2 = \{e_1, e_2, e_3, e_5\} \subseteq E$. We consider the fuzzy soft sets

$(F_1, A_1) = \{F_1(e_1) = \{{}^{c_1}/_{0.9}, {}^{c_2}/_{0.1}, {}^{c_3}/_{0.4}, {}^{c_4}/_{0.6}\}$,

$F_1(e_2) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0}, {}^{c_3}/_{0.9}, {}^{c_4}/_{0.5}\}$,
$F_1(e_3) = \{{}^{c_1}/_{0.8}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.6}\}\}$ and

$(F_2, A_2) = \{F_2(e_1) = \{{}^{c_1}/_{0.7}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.2}, {}^{c_4}/_{0.7}\}, F_2(e_2) = \{{}^{c_1}/_{0.9}, {}^{c_2}/_{0.6}, {}^{c_3}/_{0.5}, {}^{c_4}/_{1}\}$,
$F_2(e_3) = \{{}^{c_1}/_{0.3}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.8}, {}^{c_4}/_{0.3}\}, F_2(e_5) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}\}$.

Then, $(F_1, A_1)\ \tilde{\cup}\ (F_2, A_2) = (F_3, A_3)$, where $A_3 = A_1 \cup A_2 = \{e_1, e_2, e_3, e_5\}$ and
$(F_3, A_3) = \{F_3(e_1) = \{{}^{c_1}/_{0.9}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.4}, {}^{c_4}/_{0.7}\}, F_3(e_2) = \{{}^{c_1}/_{1}, {}^{c_2}/_{0.6}, {}^{c_3}/_{0.9}, {}^{c_4}/_{1}\}$,

$F_3(e_3) = \{{}^{c_1}/_{0.8}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.8}, {}^{c_4}/_{0.6}\}, F_3(e_5) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}\}$.

Definition 2.9 [1]: For two fuzzy soft sets (F_1, A_1) and (F_2, A_2) over the common universe U with $A_1 \cap A_2 \neq \emptyset$ is the fuzzy soft set (F_3, A_3), where $A_3 = A_1 \cap A_2\ and\ \forall \varepsilon \in A_3, F_3(\varepsilon) = F_1(\varepsilon) \cap F_2(\varepsilon)$ (as both are same set) and is written as $(F_1, A_1) \tilde{\cap} (F_2, A_2) = (F_3, A_3)$, where $\cap$ is the operations intersection of two fuzzy soft sets.

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Example-2.9: Let U= $\{c_1, c_2, c_3, c_4\}$be the set of four cars under regard and
E= $\{e_1\ (Costly), e_2(Beautiful),\ e_3(Fuel\ Efficient),\ e_4(Modern\ Techonology),\ e_5(Luxurious)\}$ be the set of parameters and $A_1 = \{e_1, e_2, e_3\} \subseteq E\ and\ A_2 = \{e_1, e_2, e_3, e_5\} \subseteq E$. We consider the fuzzy soft sets
(F_1, A_1) =$\{F_1(e_1) = \{{}^{c_1}/_{0.9}, {}^{c_2}/_{0.1}, {}^{c_3}/_{0.4}, {}^{c_4}/_{0.6}\}, F_1(e_2)= \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0}, {}^{c_3}/_{0.9}, {}^{c_4}/_{0.5}\},$
$F_1(e_3) = \{{}^{c_1}/_{0.8}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.6}\}\}$ and

(F_2, A_2) =$\{F_2(e_1) = \{{}^{c_1}/_{0.7}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.2}, {}^{c_4}/_{0.7}\}, F_2(e_2)= \{{}^{c_1}/_{0.9}, {}^{c_2}/_{0.6}, {}^{c_3}/_{0.5}, {}^{c_4}/_{1}\},$
$F_2(e_3) = \{{}^{c_1}/_{0.3}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.8}, {}^{c_4}/_{0.3}\}, F_2(e_5) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}\}$.

Then, $(F_1, A_1)\ \tilde{\cap}\ (F_2, A_2) = (F_3, A_3)$, where $A_3 = A_1 \cap A_2 = \{e_1, e_2, e_3\}$ and
(F_3, A_3) =$\{F_3(e_1) = \{{}^{c_1}/_{0.7}, {}^{c_2}/_{0.1}, {}^{c_3}/_{0.2}, {}^{c_4}/_{0.6}\}, F_3(e_2)= \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0}, {}^{c_3}/_{0.5}, {}^{c_4}/_{0.5}\},$
$F_3(e_3) = \{{}^{c_1}/_{0.3}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}$.

Definition 2.10 [9]: The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^C$and is defined by $(F, A)^C = (F^C, A)$, where $F^C : A \to \tilde{P}\ (U)$ is a mapping given by $F^C\ (\alpha)$=$[F(\alpha)]^C,\ \forall\ \alpha\ \in A$.

Example 2.10: Let U= $\{c_1, c_2, c_3, c_4\}$be the set of four cars under regard and
E= $\{e_1\ (Costly), e_2(Beautiful),\ e_3(Fuel\ Efficient),\ e_4(Modern\ Techonology),\ e_5(Luxurious)\}$ be the set of parameters and $A = \{e_1, e_2, e_3\} \subseteq E$. Then
(F, A) =$\{F(e_1) = \{{}^{c_1}/_{0.7}, {}^{c_2}/_{0.1}, {}^{c_3}/_{0.2}, {}^{c_4}/_{0.6}\},$
$F(e_2)= \{{}^{c_1}/_{0.8}, {}^{c_2}/_{0.6}, {}^{c_3}/_{0.1}, {}^{c_4}/_{0.5}\},$
$F(e_3) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}\}$ is the fuzzy soft set showing the "attraction of the car" which Mr. A is going to buy. Here,
$(F, A)^C = \{F^C(e_1) = \{{}^{c_1}/_{0.3}, {}^{c_2}/_{0.9}, {}^{c_3}/_{0.8}, {}^{c_4}/_{0.4}\},$
$F^C(e_2)= \{{}^{c_1}/_{0.2}, {}^{c_2}/_{0.4}, {}^{c_3}/_{0.9}, {}^{c_4}/_{0.5}\},$
$F^C(e_3) = \{{}^{c_1}/_{0.9}, {}^{c_2}/_{0.8}, {}^{c_3}/_{0.3}, {}^{c_4}/_{0.7}\}\}$

Definition 2.10[7]: If (F_1, A_1) and (F_2, A_2) be two fuzzy soft sets, then "$(F_1, A_1)\ AND\ (F_2, A_2)$" is a fuzzy soft set denoted by $(F_1, A_1) \wedge (F_2, A_2)$and is defined by $(F_1, A_1) \wedge (F_2, A_2) = (F_2, A_1 \times A_2)$, where $F_3(\alpha, \beta) = F_1(\alpha) \cap F_2(\beta), \forall\ \alpha \in A_1\ and\ \beta \in A_2$, where ∩ is the operation intersection of two fuzzy sets.

Definition 2.11[7]: If (F_1, A_1) and (F_2, A_2) be two fuzzy soft sets, then "$(F_1, A_1)\ OR\ (F_2, A_2)$" is a fuzzy soft set denoted by $(F_1, A_1) \vee (F_2, A_2)$and is defined by $(F_1, A_1) \vee (F_2, A_2) = (F_4, A_1 \times A_2)$, where $F_4(\alpha, \beta) = F_1(\alpha) \cup F_2(\beta), \forall\ \alpha \in A_1\ and\ \beta \in A_2$, where ∪ is the operation union of two fuzzy sets.

3. A STUDY ON SOME NEW OPERATIONS ON FUZZY SOFT SETS

In this section, we have put forward some more concepts related to fuzzy soft sets. A decision problem solved with the help of cardinality of fuzzy soft sets has been proposed in our work.

Definition 3.1 [11]: Let (F_1, A_1) and (F_2, A_2) be two fuzzy soft sets over (U, E). We define the disjunctive sum of (F_1, A_1) and (F_2, A_2)as the fuzzy soft set (F_3, A_3) over (U, E), written as
$(F_1, A_1)\ \tilde{\oplus}\ (F_2, A_2) = (F_3, A_3)$, Where $A_3 = A_1 \cap A_2 \neq \emptyset$ and $\forall \varepsilon \in A_3, x \in U$,
$\mu_{F_3(\varepsilon)}(x) = max\left(min\left(\mu_{F_1(\varepsilon)}(x), 1 - \mu_{F_2(\varepsilon)}(x)\right), min\left(1 - \mu_{F_1(\varepsilon)}(x), \mu_{F_2(\varepsilon)}(x)\right)\right)$.

Example 3.1: Let U= $\{c_1, c_2, c_3, c_4\}$be the set of four cars under regard and
E= $\{e_1\ (Costly), e_2(Beautiful),\ e_3(Fuel\ Efficient),\ e_4(Modern\ Techonology),\ e_5(Luxurious)\}$ be the set of parameters and $A_1 = \{e_1, e_2, e_3\} \subseteq E\ and\ A_2 = \{e_3, e_4\} \subseteq E$. Then
(F_1, A_1) =$\{F(e_1) = \{{}^{c_1}/_{0.7}, {}^{c_2}/_{0.1}, {}^{c_3}/_{0.2}, {}^{c_4}/_{0.6}\}$, F$(e_2)= \{{}^{c_1}/_{0.3}, {}^{c_2}/_{0.8}, {}^{c_3}/_{0.4}, {}^{c_4}/_{0.5}\},$
$F(e_3) = \{{}^{c_1}/_{0.1}, {}^{c_2}/_{0.2}, {}^{c_3}/_{0.7}, {}^{c_4}/_{0.3}\}\}$ is the fuzzy soft set showing the "attraction of the car" which Mr. A is going to buy. and
(F_2, A_2) =$\{F_2(e_1) = \{{}^{c_1}/_{0.2}, {}^{c_2}/_{0.5}, {}^{c_3}/_{0.1}, {}^{c_4}/_{0.7}\}, F_2(e_2)= \{{}^{c_1}/_{0.8}, {}^{c_2}/_{0.4}, {}^{c_3}/_{0.1}, {}^{c_4}/_{0.6}\}\}$, be the fuzzy soft set be showing a 'best car' according to the same person Mr. A.
Then, $(F_1, A_1)\ \tilde{\oplus}\ (F_2, A_2) = (F_3, A_3)$, Where $A_3 = A_1 \cap A_2 = \{e_3\}$ and

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$$(F_3,A_3)=\Big\{F_3(e_3)=\{(c_1\,/\max\Big(\min\Big(\mu_{F_1(e_3)}(c_1),1-\mu_{F_2(e_3)}(c_1)\Big),\min\Big(1-\mu_{F_1(e_3)}(c_1),\mu_{F_2(e_3)}(c_1)\Big)\Big)\Big),$$
$$(c_2\,/\max(\min\Big(\mu_{F_1(e_3)}(c_2),1-\mu_{F_2(e_3)}(c_2)\Big),\min\Big(1-\mu_{F_1(e_3)}(c_2),\mu_{F_2(e_3)}(c_2)\Big))),$$
$$(c_3\,/\max(\min\Big(\mu_{F_1(e_3)}(c_3),1-\mu_{F_2(e_3)}(c_3)\Big),\min\Big(1-\mu_{F_1(e_3)}(c_3),\mu_{F_2(e_3)}(c_3)\Big))),$$
$$(c_4\,/\max(\min\Big(\mu_{F_1(e_3)}(c_4),1-\mu_{F_2(e_3)}(c_4)\Big),\min\Big(1-\mu_{F_1(e_3)}(c_4),\mu_{F_2(e_3)}(c_4)\Big)))\}$$
$$=\{F_3(e_3)=\{(c_1\,/\max(\min(0.1,0.8),\min(0.9,0.2))),(c_2\,/\max(\min(0.2,0.5),\min(0.8,0.5))),(c_3\,/\max(\min(0.7,0.9),\min(0.3,0.1))),(c_4/\max(\min(0.3,0.3),\min(0.7,0.7)))\}.$$
$$=\{F_3(e_3)=\{(c_1/\max(0.1,0.2)),(c_2/\max(0.2,0.5)),(c_3/\max(0.7,0.1)),(c_4/\max(0.3,0.7))\}.$$
$$=\{F_3(e_3)=\{(c_1/0.2),(c_2/0.5),(c_3/0.7),(c_4/0.7)\}.$$

Proposition 3.1: Let (F_1,A_1) and (F_2,A_2) and (F_3,A_3) be three fuzzy soft sets over (U,E). Then the following results holds.

(i) (U,E) is closed with respect to the operation disjunctive sum of fuzzy soft sets.
(ii) $(F_1,A_1)\widetilde{\oplus}(F_2,A_2)=(F_2,A_2)\widetilde{\oplus}(F_1,A_1)$, (Commutative law).
(iii) $(F_1,A_1)\widetilde{\oplus}((F_2,A_2)\widetilde{\oplus}(F_3,A_3))=((F_1,A_1)\widetilde{\oplus}(F_2,A_2))\widetilde{\oplus}(F_3,A_3)$, (Associative Law).
(iv) $(F_1,A_1)\widetilde{\oplus}(\phi,A_1)=(F_1,A_1)$, (Law of Identity element, (ϕ,A_1) is identity of $\widetilde{\oplus}$).
(v) $(F_1,A_1)\widetilde{\oplus}(F_1,A_1)=(F_1,A_1)$, (Idempotent Law).
(vi) $(F_1,A_1)\widetilde{\oplus}(U,A_1)=(F_1,A_1)^C$, (Law of Complement)

Proof:

(i) It is obvious that the disjunctive sum of two fuzzy soft sets over the fuzzy soft class(U,E) is again a fuzzy soft set over (U,E).

(ii) Let, $(F_1,A_1)\widetilde{\oplus}(F_2,A_2)=(F_3,A_3)$, where $A_3=A_1\cap A_2\neq\emptyset$ and $\forall\varepsilon\in A_3,x\in U$, we have $\mu_{F_3(\varepsilon)}(x)=\max(\Big(\min\Big(\mu_{F_1(\varepsilon)}(x),1-\mu_{F_2(\varepsilon)}(x)\Big)\Big),\min\Big(1-\mu_{F_1(\varepsilon)}(x),\mu_{F_2(\varepsilon)}(x)\Big))$. Now consider $(F_2,A_2)\widetilde{\oplus}(F_1,A_1)=(F_4,A_3)$, where $A_3=A_1\cap A_2\neq\emptyset$ and $\forall\varepsilon\in A_3,x\in U$, Then $\mu_{F_4(\varepsilon)}(x)=\max(\Big(\min\Big(\mu_{F_2(\varepsilon)}(x),1-\mu_{F_1(\varepsilon)}(x)\Big)\Big),\min\Big(1-\mu_{F_2(\varepsilon)}(x),\mu_{F_1(\varepsilon)}(x)\Big))$.
It follows that $(F_1,A_1)\widetilde{\oplus}(F_2,A_2)=(F_2,A_2)\widetilde{\oplus}(F_1,A_1)$.

(iii) Associativity of the disjunctive sum of fuzzy soft sets is straightforward.

(iv) Let $(F_1,A_1)\widetilde{\oplus}(\phi,A_1)=(F_3,A_3)$, Where $A_3=A_1\cap A_1=A_1$ and $\forall\varepsilon\in A_3,x\in U$, we have $\mu_{F_3(\varepsilon)}(x)=\max(\big(\min\big(\mu_{F_1(\varepsilon)}(x),1-0\big)\big),\min\big(1-\mu_{F_1(\varepsilon)}(x),0\big))$.
$=\max(\big(\min\big(\mu_{F_1(\varepsilon)}(x),1\big)\big),\min\big(1-\mu_{F_1(\varepsilon)}(x),0\big))$.
$=\max\big(\mu_{F_1(\varepsilon)}(x),0\big)$.
$=\mu_{F_1(\varepsilon)}(x)$.
It follows that, $(F_1,A_1)\widetilde{\oplus}(\phi,A_1)=(F_1,A_1)$.

(v) Let $(F_1,A_1)\widetilde{\oplus}(F_1,A_1)=(F_3,A_3)$, Where $A_3=A_1\cap A_1=A_1$ and $\forall\varepsilon\in A_3,x\in U$, we have , $\mu_{F_3(\varepsilon)}(x)=\max(\Big(\min\Big(\mu_{F_1(\varepsilon)}(x),1-\mu_{F_1(\varepsilon)}(x)\Big)\Big),\min\Big(1-\mu_{F_1(\varepsilon)}(x),\mu_{F_1(\varepsilon)}(x)\Big))$.
$=\mu_{F_1(\varepsilon)}(x)$.
It follows that, $(F_1,A_1)\widetilde{\oplus}(F_1,A_1)=(F_1,A_1)$.

(vi) Let $(F_1,A_1)\widetilde{\oplus}(U,A_1)=(F_3,A_3)$, Where $A_3=A_1\cap A_1=A_1$ and $\forall\varepsilon\in A_3,x\in U$, we have
$\mu_{F_3(\varepsilon)}(x)=\max(\big(\min\big(\mu_{F_1(\varepsilon)}(x),1-1\big)\big),\min\big(1-\mu_{F_1(\varepsilon)}(x),1\big))$
$=\max(\big(\min\big(\mu_{F_1(\varepsilon)}(x),0\big)\big),\min\big(1-\mu_{F_1(\varepsilon)}(x),1\big))$
$=\max(0,\mu_{F_1(\varepsilon)}(x))$
$=1-\mu_{F_1(\varepsilon)}(x)$.
It follows that, $(F_1,A_1)\widetilde{\oplus}(U,A_1)=(F_1,A_1)^C$.

Definition 3.2 [4]: Let (F_1,A_1) and (F_2,A_2) be two fuzzy soft sets over (U,E). We define the difference of (F_1,A_1) and (F_2,A_2) as the Fuzzy soft set (F_3,A_3) over (U,E), written as $(F_1,A_1)\widetilde{\ominus}(F_2,A_2)=(F_3,A_3)$, where $A_3=A_1\cap A_2\neq\emptyset$ and $\forall\varepsilon\in A_3,x\in U$ and $\mu_{F_3(\varepsilon)}(x)=\min\Big(\mu_{F_1(\varepsilon)}(x),1-\mu_{F_2(\varepsilon)}(x)\Big)$.

Example 3.2: We consider the fuzzy soft sets (F_1,A_1) and (F_2,A_2) given in Example 3.1. Then the fuzzy soft set $(F_1,A_1)\widetilde{\ominus}(F_2,A_2)$ would represent the fuzzy soft set representing attractive but not good cars.
Let, $(F_1,A_1)\widetilde{\ominus}(F_2,A_2)=(F_3,A_3)$, where $A_3=A_1\cap A_2=\{e_3\}$ and

$$(F_3,A_3)=\Big\{F_3(e_3)=\{(c_1\,/\max\Big(\min\Big(\mu_{F_1(e_3)}(c_1),1-\mu_{F_2(e_3)}(c_1)\Big),\min\Big(1-\mu_{F_1(e_3)}(c_1),\mu_{F_2(e_3)}(c_1)\Big)\Big)\Big),$$
$$(c_2\,/\max(\min\Big(\mu_{F_1(e_3)}(c_2),1-\mu_{F_2(e_3)}(c_2)\Big),\min\Big(1-\mu_{F_1(e_3)}(c_2),\mu_{F_2(e_3)}(c_2)\Big))),$$

$(c_3$ / max (min $\left(\mu_{F_1(e_3)}(c_3), 1-\mu_{F_2(e_3)}(c_3)\right)$, min $\left(1-\mu_{F_1(e_3)}(c_3), \mu_{F_2(e_3)}(c_3)\right)$)),
$(c_4$ / max (min $\left(\mu_{F_1(e_3)}(c_4), 1-\mu_{F_2(e_3)}(c_4)\right)$, min $\left(1-\mu_{F_1(e_3)}(c_4), \mu_{F_2(e_3)}(c_4)\right)$))}
={ $F_3(e_3) = \{(c_1/\max(\min(0.1,0.8), \min(0.9,0.2))), (c_2 / \max(\min(0.2,0.5), \min(0.8,0.5))),$
$(c_3 / \max(\min(0.7,0.9), \min(0.3,0.1))), (c_4/\max(\min(0.3,0.3), \min(0.7,0.7)))\}$
={ $F_3(e_3) = \{(c_1/\max(0.1, 0.2)), (c_2/\max(0.2, 0.5)), (c_3/\max(0.7, 0.1)), (c_4/\max(0.3, 0.7))\}$
= { $F_3(e_3) = \{(c_1/0.2), (c_2/0.5), (c_3/0.7), (c_4/0.7)\}$

Proposition 3.2: Let (F_1, A_1) and (F_2, A_2) be two fuzzy soft sets over (U, E). Then the following results holds.

(i) (U, E) is closed with respect to the operation difference of fuzzy soft sets.
(ii) $(F_1, A_1) \widetilde{\ominus} (F_2, A_2) \neq (F_2, A_2) \widetilde{\ominus} (F_1, A_1)$.
(iii) $(F_1, A_1) \widetilde{\ominus} (\phi, A_1) = (F_1, A_1)$.
(iv) $(F_1, A_1) \widetilde{\ominus} (U, A_1) = (\emptyset, A)$.
(v) $(F_1, A_1) \widetilde{\ominus} (F_2, A_2) = (F_1, A_1) \widetilde{\cap} (F_2, A_2)^C$.

Proof: -

(i) It is obvious that the difference between two fuzzy soft sets over the fuzzy soft class (U, E) is again a fuzzy soft set over (U, E).
(ii) The proof follows from the definition.
(iii) Let $(F_1, A_1) \widetilde{\ominus} (\emptyset, A) = (F_3, A_3)$, where $A_3 = A_1 \cap A_1 = A_1$ *and* $\forall \varepsilon \in A_3, x \in U$, we have $\mu_{F_3(\varepsilon)}(x) = \min(\mu_{F_1(\varepsilon)}(x), 1-0) = \min(\mu_{F_1(\varepsilon)}(x), 1) = \mu_{F_1(\varepsilon)}(x)$.
It follows that $(F_1, A_1) \widetilde{\ominus} (\phi, A_1) = (F_1, A_1)$.
(iv) Let $(F_1, A_1) \widetilde{\ominus} (U, A) = (F_3, A_3)$, where $A_3 = A_1 \cap A_1 = A_1$ *and* $\forall \varepsilon \in A_3, x \in U$, we have
$\mu_{F_3(\varepsilon)}(x) = \min(\mu_{F_1(\varepsilon)}(x), 1-1) = \min(\mu_{F_1(\varepsilon)}(x), 0) = 0$
It follows that, $(F_1, A_1) \widetilde{\ominus} (U, A_1) = (\emptyset, A)$.
(v) Let $(F_1, A_1) \widetilde{\ominus} (F_2, A_2) = (F_3, A_3)$, where $A_3 = A_1 \cap A_2 \neq \emptyset$ *and* $\forall \varepsilon \in A_3 = A_1, x \in U$, we have, $\mu_{F_3(\varepsilon)}(x) = \min\left(\mu_{F_1(\varepsilon)}(x), 1-\mu_{F_2(\varepsilon)}(x)\right) = \min\left(\mu_{F_1(\varepsilon)}(x), \mu_{F_2^C(\varepsilon)}(x)\right)$.
Thus the proof follows the result.

Definition 3.3. Let (F_1, A_1) and (F_2, A_2) be two fuzzy soft sets over (U, E). We define the symmetric difference of (F_1, A_1) and (F_2, A_2) as the fuzzy soft set (F_3, A_3) over (U, E), written as $(F_1, A_1) \widetilde{\Delta} (F_2, A_2) = (F_3, A_3)$, where $A_3 = A_1 \cap A_2 \neq \emptyset$ *and* $\forall \varepsilon \in A_3, x \in U$
$\mu_{F_3(\varepsilon)}(x) = \max\left(\min\left(\mu_{F_1(\varepsilon)}(x), 1-\mu_{2(\varepsilon)}(x)\right)\right), \min\left(\mu_{F_2(\varepsilon)}(x), 1-\mu_{F_1(\varepsilon)}(x)\right)$).

We can write, $((F_1, A_1) \widetilde{\ominus} (F_2, A_2)) \widetilde{\cup} \left((F_2, A_2) \widetilde{\ominus} (F_1, A_1)\right) = (F_1, A_1) \widetilde{\Delta} (F_2, A_2)$.

Example 3.3: We consider the fuzzy soft sets (F_1, A_1) and (F_2, A_2) given in Example 3.1.
Let $(F_1, A_1) \widetilde{\ominus} (F_2, A_2) = (F_3, A_3)$, where $A_3 = A_1 \cap A_2 = \{e_3\}$ and
(F_3, A_3) ={ $F_3(e_3) = \{(c_1/\min(0.1,0.8)), (c_2/\min(0.2,0.5)), (c_3/\min(0.7,0.9)), (c_4/\min(0.3,0.3))\}$}
= { $F_3(e_3) = \{(c_1/0.1), (c_2/0.2), (c_3/0.7), (c_4/0.3)\}$}.

Let $(F_2, A_2) \widetilde{\ominus} (F_1, A_1) = (F_4, A_3)$, where $A_3 = A_1 \cap A_2 = \{e_3\}$ and
(F_4, A_3) ={ $F_4(e_3) = \{(c_1/\min(0.2,0.9)), (c_2/\min(0.5,0.8)), (c_3/\min(0.1,0.3)), (c_4/\min(0.7,0.7))\}$}
= { $F_4(e_3) = \{(c_1/0.2), (c_2/0.5), (c_3/0.1), (c_4/0.7)\}$}.

It follows that, $(F_1, A_1) \widetilde{\Delta} (F_2, A_2) = (F_3, A_3) \widetilde{\cup} (F_4, A_3) = (F_5, A_3)$.
(F_5, A_3) = { $F_5(e_3) = \{(c_1/0.2), (c_2/0.5), (c_3/0.7), (c_4/0.7)\}$.

3.4. APPLICATION OF FUZZY SOFT SETS IN A DECISION-MAKING PROBLEM

In this section, we take in a hypothetical case study to apply the concept of fuzzy soft sets in a decision-making problem. Our study is based on the cardinality of fuzzy soft sets introduce by Cagman et al. in [2].
Let (F, A) be a fuzzy soft set over (U, E) , where U is the universe and E is the set of attributes. Then the cardinal set of (F, A) is a fuzzy set over E, denoted by $\mathrm{CS}(F, A)$ and is defined as
$\mathrm{CS}(F, A) = \{x, \mu_{CS(F,A)}(x), x \in E\}$. The membership function $\mu_{CS(F,A)}$ of $\mathrm{CS}(F, A)$ is defined by $\mu_{CS(F,A)}: E \to [0,1]$, $\mu_{CS(F,A)}(x) = \frac{|\gamma_A(x)|}{|U|}$, where $|U|$ is the cardinality of universe U and $|\gamma_A(x)|$is the scalar cardinality of fuzzy set $\gamma_A(x)$.

Suppose, the authority of an institution wants to give the award to the performing group of students in a Mathematics project competition. Every group structure of three student members. Let
U= $\{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10}, m_{11}, m_{12}\}$ be our universal set containing 12 students. Herein the set of parameters E is the set of certain attributes determined by the authority.

Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8,\}$, where e_1 = Originality of idea and concept, e_2 = Relevance of the project to the theme, e_3 = Understanding of the issue, e_4=Data collection and analysis,e_5 = Experimentation and validation, e_6 = Interpretation and problem solving attempt, e_7 = Group work, e_8 = Oral Presentation. We construct the fuzzy soft sets $(F_1, E), (F_2, E)$, $(F_3, E)$and $(F_4, E)$for the 12 students under consideration as given below. The membership of the students except m_1, m_2 and m_3 in each $F(e)$ is zero indicating that this fuzzy soft set is corresponding to the group containing the students m_1, m_2 and m_3. In the same way, the fuzzy soft set (F_2, E) is corresponding to the group containing the students m_4, m_5 and m_6, the fuzzy soft set (F_3, E) is corresponding to the group containing the students m_7, m_8 and m_9 and the fuzzy soft set (F_4, E) is corresponding to the group containing the students m_{10}, m_{11} and m_{12}.

$(\boldsymbol{F_1}, \boldsymbol{E})$={ $F_1(e_1)$ = { (m_1 /0.8), (m_2 /0.4), (m_3 /0.3), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0), (m_{11} /0), (m_{12} /0)},
$F_1(e_2)$ = { (m_1 /0.5),(m_2 /0.3),(m_3 /0.4),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_1(e_3)$ = { (m_1 /0.3),(m_2 /0.1), (m_3 /0.6),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_1(e_4)$ = { (m_1 /0.5),(m_2 /0.4),(m_3 /0.6), (m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_1(e_5)$ = { (m_1 /0.4),(m_2 /0.7),(m_3 /0.8),(m_4 /0),(m_5 /0), (m_6 /0),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_1(e_6)$ = { (m_1 /0.7),(m_2 /0.8),(m_3 /0.7),(m_4 /0),(m_5 /0), (m_6 /0),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_1(e_7)$ = { (m_1 /0.3),(m_2 /0.3),(m_3 /0.3),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_1(e_8)$ = { (m_1 /0.2),(m_2 /0.3),(m_3 /0.8),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0),(m_8 /0), (m_9 /0) , (m_{10} /0),(m_{11} /0), (m_{12} /0)}.

$(\boldsymbol{F_2}, \boldsymbol{E}) =$ { $F_2(e_1)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0.3), (m_5 /0.7), (m_6 /0.6), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0), (m_{11} /0), (m_{12} /0)},
$F_2(e_2)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0.7),(m_5 /0.5),(m_6 /0.4),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_2(e_3)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0.6),(m_5 /0.5),(m_6 /0.2),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_2(e_4)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0.6),(m_5 /0.7),(m_6 /0.4),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_2(e_5)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0.8),(m_5 /0.9),(m_6 /0.7),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_2(e_6)$ = { (m_1 /0),(m_2 /0),(m_3 /0), (m_4 /0.3),(m_5 /0.4),(m_6 /0.4),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_2(e_7)$ = { (m_1 /0),(m_2 /0),(m_3 /0), (m_4 /0.3),(m_5 /0.2),(m_6 /0.7),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_2(e_8)$ = { (m_1 /0),(m_2 /0),(m_3 /0), (m_4 /0.1),(m_5 /0.3),(m_6 /0.9),(m_7 /0),(m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /0), (m_{12} /0)}.

$(\boldsymbol{F_3}, \boldsymbol{E}) =$ { $F_3(e_1)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0.6), (m_8 /0.3), (m_9 /0.8), (m_{10} /0), (m_{11} /0), (m_{12} /0)},
$F_3(e_2)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0.6),(m_8 /0.2), (m_9 /0.9), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_3(e_3)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0.7),(m_8 /0.6), (m_9 /0.9), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_3(e_4)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0.9),(m_8 /0.7), (m_9 /0.8), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_3(e_5)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0.5),(m_8 /0.9), (m_9 /0.7), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_3(e_6)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0.8),(m_8 /0.7), (m_9 /0.5), (m_{10} /0),(m_{11} /0), (m_{12} /0)},
$F_3(e_7)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0.8),(m_8 /0.8), (m_9 /0.2), (m_{10} /0),(m_{11} /0), (m_{12} /0)},

$F_3(e_8)$ = { (m_1 /0),(m_2 /0),(m_3 /0),(m_4 /0),(m_5 /0),(m_6 /0),(m_7 /0.7),(m_8 /0.3), (m_9 /0.5), (m_{10} /0),(m_{11} /0), (m_{12} /0)}.

and

$(\boldsymbol{F_4}, \boldsymbol{E})$ = { $F_4(e_1)$ = {(m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0.4), (m_{11} /0.9), (m_{12} /0.8)},

$F_4(e_2)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0.7), (m_{11} /0.8), (m_{12} /0.9)},

$F_4(e_3)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0.5),(m_{11} /0.8), (m_{12} /0.6)},

$F_4(e_4)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0),(m_{11} /1), (m_{12} /2)},

$F_4(e_5)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0.7),(m_{11} /0.6), (m_{12} /0.8)},

$F_4(e_6)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0.3),(m_{11} /0.7), (m_{12} /0.5)},

$F_4(e_7)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /0.1),(m_{11} /0.3), (m_{12} /0.8)},

$F_4(e_8)$ = { (m_1 /0), (m_2 /0), (m_3 /0), (m_4 /0), (m_5 /0), (m_6 /0), (m_7 /0), (m_8 /0), (m_9 /0), (m_{10} /1),(m_{11} /0.4), (m_{12} /0.8)}.

$$CS\ (F_1, E) = \{(e_1, 1.5/9), (e_2, 1.2/9), (e_3, 1.0/9), (e_4, 1.5/9), (e_5, 1.9/9), (e_6, 2.2/9), (e_7, 0.9/9), (e_8, 1.3/9).$$
$$= \{(e_1, 0.167), (e_2, 0.133), (e_3, 0.111), (e_4, 0.167), (e_5, 0.211), (e_6, 0.244), (e_7, 0.1), (e_8, 0.144).$$

It follows that, $|\ CS\ (F_1, E)|$ = 0.167 + 0.133 + 0.111 + 0.167 + 0.211 + 0.244 + 0.1 + 0.144 = **1.277**

$$CS\ (F_2, E) = \{(e_1, 1.6/9), (e_2, 1.6/9), (e_3, 1.3/9), (e_4, 1.7/9), (e_5, 2.4/9), (e_6, 1.1/9), (e_7, 1.2/9), (e_8, 1.3/9).$$
$$= \{(e_1, 0.178), (e_2, 0.178), (e_3, 0.144), (e_4, 0.189), (e_5, 0.267), (e_6, 0.122), (e_7, 0.133), (e_8, 0.144).$$

In the same way, $|\ CS\ (F_2, E)|$ = 0.178 + 0.178 + 0.144 + 0.189 + 0.267 + 0.122 + 0.133 + 0.144 = **1.355**

$$CS\ (F_3, E) = \{(e_1, 1.7/9), (e_2, 1.7/9), (e_3, 2.2/9), (e_4, 2.4/9), (e_5, 2.1/9), (e_6, 2.0/9), (e_7, 1.8/9), (e_8, 1.5/9).$$
$$= \{(e_1, 0.189), (e_2, 0.189), (e_3, 0.244), (e_4, 0.267), (e_5, 0.233), (e_6, 0.222), (e_7, 0.2), (e_8, 0.167).$$

In the same process, $|\ CS\ (F_3, E)|$ = 0.189 + 0.189 + 0.244 + 0.267 + 0.233 + 0.222 + 0.2 + 0.167 = **1.711**

$$CS\ (F_4, E) = \{(e_1, 2.1/9), (e_2, 2.4/9), (e_3, 2.1/9), (e_4, 1.2/9), (e_5, 2.1/9), (e_6, 1.5/9), (e_7, 1.2/9), (e_8, 2.2/9).$$
$$= \{(e_1, 0.233), (e_2, 0.267), (e_3, 0.233), (e_4, 0.133), (e_5, 0.233), (e_6, 0.167), (e_7, 0.133), (e_8, 0.244).$$

And $|\ CS\ (F_4, E)|$ = 0.233 + 0.267 + 0.233 + 0.133 + 0.233 + 0.167 + 0.133 + 0.244 = **1.643**

It is seen that the scalar cardinality corresponding to the fuzzy soft set (F_3, E) is the highest. It follows that the group consisting of the students m_7, m_8 and m_9 is in the first rank.

4. CONCLUSION

In our work, we have put forward few new concepts of fuzzy soft sets. Some related properties have been established with proof and examples. A decision-making problem has been considered to get the optimal solution with the help of cardinality of fuzzy soft sets. Future work in this regard would be required to study whether the concepts put forward in this paper yield a fruitful result.

REFERENCES

1. Ahmad, B., Kharal, A. (2009). "On Fuzzy Soft Sets". Advances in Fuzzy Systems.
2. Cagman, N., Enginoglu, S., & Citak, T. (2011). "Fuzzy soft set theory and its applications". Iranian Journal of Fuzzy Systems, 8(3), pp.137–147.
3. Klir, G. J., Yuan, B. (1995). "Fuzzy Sets and Fuzzy Logic: Theory and Applications". Englewood Clifis, NJ, Prentice-Hall.
4. Lin, T. Y. (1994). "Fuzzy Reasoning and Rough Sets, In Rough Sets, Fuzzy Sets and Knowledge Discovery". W. P. Ziarko, (Ed.), Springer-Verlag, London, pp.343-348.
5. Lin, T. Y., Liu, Q. (1994). "Rough Approximate Operators: Axiomatic Rough Set Theory, In Rough Sets, Fuzzy Sets and Knowledge Discovery". W. P. Ziarko, (Ed.), Springer-Verlag, London, pp.256-260.
6. Maji, P. K., Roy, A. R. (2003). "Soft Set Theory". Computers and Mathematics with Applications, 45, pp.555–562.
7. Maji, P. K., Biswas, R., & Roy, A.R. (2001). "Fuzzy Soft Sets". Journal of Fuzzy Mathematics, 9(3), pp.589 - 602.

8. Molodstov, D.A. (1999). "Soft Set Theory - First Result". Computers and Mathematics with Applications, 37, pp.19-31.
9. Neog, T.J., Sut, D.K. (2011). "Theory of Fuzzy Soft Sets from a New Perspective". International Journal of Latest Trends in Computing, 2(3), pp.439-450.
10. Neog, T.J., Sut, D.K. (2011). "On Union and Intersection of Fuzzy Soft Sets". International Journal of Computer Technology and Applications, 2(5), pp.1160-1176.
11. Neog, T.J., Sut, D.K. (2012). "Some New Operations of Fuzzy Soft Sets". Journal of Mathematical and Computational Science, 2 (5), pp.1186-1199.
12. Pei, D., Miao, D. (2005). "From soft sets to information systems". in Proceedings of the IEEE International Conference on Granular Computing. 2, pp. 617–621.
13. Ge, X., Yang, S. (2011). "Investigations on some operations of soft sets". World Academy of Science, Engineering, and Technology, 75, pp. 1113-1116.
14. Zadeh, L. A. (1965). "Fuzzy sets". Information and control, 8(3), pp.338-353.

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