In this paper, I introduce a new class of sets in generalized topological spaces called \( \mu \)-\( \alpha \)-semi generalized closed sets. Also, I investigate some of their basic properties and obtained some interesting theorems.

**Keywords:** Generalized topological spaces, \( \mu \)-\( \alpha \)-semi closed sets, \( \mu \)-\( \alpha \)-semi generalized closed sets.

1. **INTRODUCTION**

The concept of generalized topological spaces was introduced and investigated by A. Csaszar [1]. Many \( \mu \)-closed sets like \( \mu \)-semi closed sets, \( \mu \)-pre closed sets etc., in generalized topological spaces are introduced by him. In this paper, I introduce a new class of sets in generalized topological spaces called \( \mu \)-\( \alpha \)-semi generalized closed sets. Also, I investigate some of their basic properties and produced many interesting theorems.

2. **PRELIMINARIES**

**Definition 2.1:** [1] Let \( X \) be a nonempty set. A collection \( \mu \) of subsets of \( X \) is a generalized topology (or briefly GT) on \( X \) if it satisfies the following:

1. \( \emptyset, X \in \mu \) and
2. If \( \{M_i : i \in I\} \subseteq \mu \), then \( \bigcup_{i \in I} M_i \in \mu \).

If \( \mu \) is a GT on \( X \), then \( (X, \mu) \) is called a generalized topological space (or briefly GTS) and the elements of \( \mu \) are called \( \mu \)-open sets and their complement are called \( \mu \)-closed sets.

**Definition 2.2:** [1] Let \( (X, \mu) \) be a GTS and let \( A \subseteq X \). Then the \( \mu \)-closure of \( A \), denoted by \( c_\mu(A) \), is the intersection of all \( \mu \)-closed sets containing \( A \).

**Definition 2.3:** [1] Let \( (X, \mu) \) be a GTS and let \( A \subseteq X \). Then the \( \mu \)-interior of \( A \), denoted by \( i_\mu(A) \), is the union of all \( \mu \)-open sets contained in \( A \).

**Definition 2.4:** [1] Let \( (X, \mu) \) be a GTS. A subset \( A \) of \( X \) is said to be

1. \( \mu \)-semi-closed set if \( i_\mu(c_\mu(A)) \subseteq A \)
2. \( \mu \)-pre-closed set if \( c_\mu(i_\mu(A)) \subseteq A \)
3. \( \mu \)-\( \alpha \)-closed set if \( c_\mu(i_\mu(c_\mu(A))) \subseteq A \)
4. \( \mu \)-\( \beta \)-closed set if \( i_\mu(c_\mu(i_\mu(A))) \subseteq A \)
5. \( \mu \)-regular-closed set if \( A = c_\mu(i_\mu(A)) \)

**Definition 2.5:** [3] Let \( (X, \mu) \) be a GTS. A subset \( A \) of \( X \) is said to be

1. \( \mu \)-regular generalized closed set if \( c_\mu(A) \subseteq U \) whenever \( A \subseteq U \), where \( U \) is \( \mu \)-regular open in \( X \)
2. \( \mu \)-generalized closed set if \( c_\mu(A) \subseteq U \) whenever \( A \subseteq U \), where \( U \) is \( \mu \)-open in \( X \)
3. \( \mu \)-generalized-\( \alpha \)-closed set if \( a_\mu(c_\mu(A)) \subseteq U \) whenever \( A \subseteq U \), where \( U \) is \( \mu \)-open in \( X \)

The complement of \( \mu \)-semi closed (resp., \( \mu \)-pre closed, \( \mu \)-\( \alpha \)-closed, \( \mu \)-\( \beta \)-closed, \( \mu \)-regular closed, \( \mu \)-generalized closed) is said to be \( \mu \)-semi open (resp., \( \mu \)-pre open, \( \mu \)-\( \alpha \)-open, \( \mu \)-\( \beta \)-open, \( \mu \)-regular open, \( \mu \)-generalized open) in \( X \).
3. \(\mu\alpha\)-SEMI GENERALIZED CLOSED SETS

In this section I introduce \(\mu\alpha\)-semi generalized closed sets in generalized topological spaces and studied some of their basic properties. Some interesting and important theorems are also obtained.

**Definition 3.1:** Let \((X, \mu)\) be a GTS. Then a non-empty subset \(A\) is said to be a \(\mu\alpha\)-semi generalized closed set (briefly \(\mu\alpha\)-SGCS) if \(sc_{\mu}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\mu\alpha\)-open in \(X\).

**Example 3.2:** Let \(X = \{a, b, c\}\) and let \(\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\). Then \((X, \mu)\) is a GTS. Now, \(\mu\alpha\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\).

\(\mu\alpha\)-SGCS in \((X, \mu)\), then \(A = \{a\}\) is a \(\mu\alpha\)-SGCS in \((X, \mu)\).

**Theorem 3.3:** Every \(\mu\alpha\)-closed set in \((X, \mu)\) is a \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\) but not conversely in general.

**Proof:** Let \(A\) be a \(\mu\alpha\)-closed set in \((X, \mu)\), then \(c_{\mu}(A) = A\). Now let \(A \subseteq U\) and \(U\) be \(\mu\alpha\)-open in \(X\). Then \(sc_{\mu}(A) = A \cup i_{\mu}(c_{\mu}(A)) = A \cup i_{\mu}(A) \subseteq A \subseteq U\), by hypothesis. Therefore \(sc_{\mu}(A) \subseteq U\). This implies, \(A\) is a \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\).

**Example 3.4:** Let \(X = \{a, b, c, d\}\) and let \(\mu = \{\emptyset, \{b\}, \{d\}, \{b, d\}, X\}\). Then \((X, \mu)\) is a GTS. Now, \(\mu\alpha\alpha O(X) = \{\emptyset, \{b\}, \{d\}, \{b, d\}, X\}\).

\(\mu\alpha\)-SGCS in \((X, \mu)\), then \(A = \{b\}\) is a \(\mu\alpha\)-SGCS in \((X, \mu)\).

**Theorem 3.5:** Every \(\mu\alpha\)-closed set in \((X, \mu)\) is a \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\).

**Proof:** Let \(A\) be a \(\mu\alpha\)-closed set in \((X, \mu)\). Then \(i_{\mu}(c_{\mu}(A)) = c_{\mu}(i_{\mu}(A)) = \emptyset\). Now let \(A \subseteq U\) and \(U\) be \(\mu\alpha\)-open in \(X\). Then \(sc_{\mu}(A) = A \cup i_{\mu}(c_{\mu}(A)) = A \cup i_{\mu}(A) \subseteq A \subseteq U\), by hypothesis. Therefore \(A\) is a \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\).

**Remark 3.6:** Every \(\mu\alpha\)-generalized closed sets and \(\mu\alpha\)-pre closed sets are independent in general in \((X, \mu)\).

**Example 3.7:** Let \(X = \{a, b, c\}\) and let \(\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\). Then \((X, \mu)\) is a GTS. Now, \(\mu\alpha\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\).

\(\mu\alpha\)-SGCS in \((X, \mu)\), then \(A = \{a\}\) is a \(\mu\alpha\)-SGCS but not a \(\mu\alpha\)-pre closed set as \(c_{\mu}(i_{\mu}(A)) = c_{\mu}(i_{\mu}(\{a\})) = \{a\} \not\subseteq A\).

**Theorem 3.8:** Every \(\mu\alpha\)-closed set in \((X, \mu)\) is a \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\) but not conversely in general.

**Proof:** Let \(A\) be a \(\mu\alpha\)-closed set in \((X, \mu)\). Then \(c_{\mu}(i_{\mu}(A)) = c_{\mu}(i_{\mu}(\{a\})) = \emptyset\). Now let \(A \subseteq U\) and \(U\) be \(\mu\alpha\)-open in \(X\). Then \(sc_{\mu}(A) = A \cup i_{\mu}(c_{\mu}(A)) = A \cup i_{\mu}(A) \subseteq A \subseteq U\), by hypothesis. Therefore \(A\) is a \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\).

**Remark 3.9:** A \(\mu\beta\)-closed set is not \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\) in general.

**Example 3.10:** Let \(X = \{a, b, c\}\) and let \(\mu = \{\emptyset, \{a\}, \{b\}, \{a, c\}, X\}\). Then \((X, \mu)\) is a GTS. Now, \(\mu\alpha\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, c\}, X\}\).

\(\mu\alpha\)-SGCS in \((X, \mu)\), then \(A = \{c\}\) is a \(\mu\alpha\)-SGCS but not a \(\mu\beta\)-closed set as \(A \subseteq U = \{a, b\}\) where \(U\) is a \(\mu\alpha\)-open set and now, \(\mu\alpha\alpha O(X) = \{\emptyset, \{a, b\}, X\}\).

**Theorem 3.11:** Every \(\mu\alpha\)-closed set in \((X, \mu)\) is a \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\) but not conversely in general.

**Proof:** Let \(A\) be a \(\mu\alpha\)-closed set in \((X, \mu)\). Then \(c_{\mu}(i_{\mu}(A)) = c_{\mu}(i_{\mu}(\{a\})) = \emptyset\). Now let \(A \subseteq U\) and \(U\) be \(\mu\alpha\)-open in \(X\). Then \(sc_{\mu}(A) = A \cup i_{\mu}(c_{\mu}(A)) = A \cup i_{\mu}(A) \subseteq A \subseteq U\), by hypothesis. Therefore \(A\) is a \(\mu\alpha\)-semi generalized closed set in \((X, \mu)\).
Example 3.14: Let \(X = \{a, b, c, d\}\) and let \(\mu = \{\emptyset, \{a\}, \{c\}, \{b, d\}, X\}\). Then \((X, \mu)\) is a GTS.

Now, \(\mu-\alpha O(X) = \{\emptyset, \{a\}, \{c\}, \{b, d\}, X\}\). Then \(A = \{c\}\) is a \(\mu-\alpha\)-semi generalized closed set but not a \(\mu\)-regular closed set as \(c_\mu(i_\mu(A)) = c_\mu(i_\mu(\{c\})) = \{a, c\} \neq \{c\} = A\).

Remark 3.15: Every \(\mu-\alpha\)-semi generalized closed set and \(\mu\)-generalized closed set in \((X, \mu)\) are independent to each other in general.

Example 3.16: Let \(X = \{a, b, c, d\}\) and let \(\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}\). Then \((X, \mu)\) is a GTS.

Now, 
\[\mu-\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}\].
Then \(A = \{a\}\) is a \(\mu-\alpha\)-semi generalized closed set in \((X, \mu)\), but not a \(\mu\)-generalized closed set as \(A \subseteq U\) and \(\mu-\alpha\)-regular closed set as \(\mu-\alpha\)-open set and now,

\[\mu-\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}\] and \(sc_\mu(A) = X \not\subseteq \{a\} = U\).

Example 3.17: Let \(X = \{a, b, c\}\) and let \(\mu = \{\emptyset, \{a, b\}, X\}\). Then \((X, \mu)\) is a GTS. Let \(A = \{a, b\}\) and \(U = X\). Then \(A \subseteq U\) and \(c_\mu(A) = c_\mu(\{a, b\}) = X \subseteq U\). Therefore \(A\) is a \(\mu\)-generalized closed set, but not a \(\mu-\alpha\)-semi generalized closed set as \(A \subseteq U\) and \(\mu-\alpha\)-open set and now,

\[\mu-\alpha O(X) = \{\emptyset, \{a, b\}, X\}\] and \(sc_\mu(A) = X \not\subseteq \{a, b\} = U\).

Remark 3.18: Every \(\mu-\alpha\)-semi generalized closed set and \(\mu\)-generalized-\(\alpha\)-closed set in \((X, \mu)\) are independent to each other in general.

Example 3.19: Let \(X = \{a, b, c, d\}\) and let \(\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}\). Then \((X, \mu)\) is a GTS.

Now, \(\mu-\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}\). Then \(A = \{b\}\) is a \(\mu-\alpha\)-semi generalized closed set in \((X, \mu)\), but not a \(\mu\)-generalized-\(\alpha\)-closed set as \(A \subseteq U\) and \(\mu-\alpha\)-open set and now,

\[\mu-\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}\] and \(sc_\mu(A) = X \not\subseteq \{a, b\} = U\).

Example 3.20: Let \(X = \{a, b, c\}\) and let \(\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\). Then \((X, \mu)\) is a GTS. Let \(A = \{a\}\) and \(U = X\). Then \(A \subseteq U\) and \(c_\mu(A) = c_\mu(\{a\}) = \{a\}\). Therefore \(A\) is a \(\mu\)-generalized-\(\alpha\)-closed set, but not a \(\mu-\alpha\)-semi generalized closed set as \(A \subseteq U\) and \(\mu-\alpha\)-open set and now,

\[\mu-\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\] and \(sc_\mu(A) = X \not\subseteq \{a\} = U\).

Remark 3.21: Union of any two \(\mu-\alpha\)-semi generalized closed sets in \((X, \mu)\) need not be a \(\mu-\alpha\)-semi generalized closed set.

Example 3.22: Let \(X = \{a, b, c\}\) and let \(\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\). Then \((X, \mu)\) is a GTS. Now,

\[\mu-\alpha O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\].
Then \(A = \{a\}\) and \(B = \{b\}\) are \(\mu-\alpha\)-semi generalized closed sets in \((X, \mu)\). But, \(A \cup B = \{a, b\}\) is not a \(\mu-\alpha\)-semi generalized closed set.

Theorem 3.23: If a subset \(A\) in \(X\) is a \(\mu-\alpha\)-semi generalized closed set in \((X, \mu)\), then \(sc_\mu(A) - A\) contains no non-empty \(\mu-\alpha\)-closed set in \(X\).
Proof: Let A be a $\mu$-$\alpha$-semi generalized closed set in $(X, \mu)$ and let $sc_{\mu}(A) - A$ contains a non-empty $\mu$-$\alpha$-closed set say F. That is, $F \subseteq sc_{\mu}(A) - A$. This implies, $F \subseteq sc_{\mu}(A) \cap A^c$ and hence $F \subseteq sc_{\mu}(A) \& F \subseteq A^c$ (1)

Now $F \subseteq A^c$ implies $A \subseteq F^c$. Since $F^c$ is $\mu$-$\alpha$-open and A is a $\mu$-$\alpha$- semi generalized closed set, we have, $sc_{\mu}(A) \subseteq F^c$. This implies $F \subseteq (sc_{\mu}(A))^c$ (2)

From (1) & (2), $F \subseteq sc_{\mu}(A)$ and $F \subseteq (sc_{\mu}(A))^c$. This implies $F \subseteq sc_{\mu}(A) \cap (sc_{\mu}(A))^c = \emptyset$. Therefore $F = \emptyset$. Thus $sc_{\mu}(A) - A$ contains no non-empty $\mu$-$\alpha$-closed set in X.

Theorem 3.24: Let $(X, \mu)$ be a GTS. Then for every $A \in sc_{\mu}(X)$ and for every set B $\subseteq X$, $A \subseteq B \subseteq sc_{\mu}(A)$ implies $B \in sc_{\mu}(X)$.

Proof: Let B $\subseteq U$ and U be a $\mu$-$\alpha$-open set in $(X, \mu)$. Then since A $\subseteq B$ and A $\subseteq U$. By hypothesis $B \subseteq sc_{\mu}(A)$. Therefore $sc_{\mu}(B) \subseteq sc_{\mu}(sc_{\mu}(A)) = sc_{\mu}(A) \subseteq U$ as A is a $\mu$-$\alpha$-semi generalized closed set of X. Hence B $\in sc_{\mu}(X)$.

Theorem 3.25: In a GTS X, for each $x \in X$, $\{x\}$ is a $\mu$-$\alpha$-closed set or its complement X–$\{x\}$ is a $\mu$-$\alpha$- semi generalized closed set in $(X, \mu)$.

Proof: Suppose that $\{x\}$ is not a $\mu$-$\alpha$ closed set in $(X, \mu)$. Then $X - \{x\}$ is not a $\mu$-$\alpha$-open set in $(X, \mu)$. The only $\mu$-$\alpha$-open set containing $X - \{x\}$ is X. Thus $X - \{x\} \subseteq X$ and so $sc_{\mu}(X-\{x\}) \subseteq sc_{\mu}(X) = X$. Therefore $sc_{\mu}(X-\{x\}) \subseteq X$ and so $X - \{x\}$ is a $\mu$-$\alpha$-semi generalized closed set in $(X, \mu)$.

Theorem 3.26: If A is both a $\mu$-$\alpha$-open set and a $\mu$-$\alpha$-semi generalized closed set in $(X, \mu)$, then A is a $\mu$-semi closed set in $(X, \mu)$.

Proof: Let A be $\mu$-$\alpha$-open and $\mu$-$\alpha$-semi generalized closed set in $(X, \mu)$. Then, $sc_{\mu}(A) \subseteq A$ as $A \subseteq A$. But always $A \subseteq sc_{\mu}(A)$. Therefore, $A = sc_{\mu}(A)$. Hence A is a $\mu$-semi closed set in $(X, \mu)$.

Theorem 3.27: If $A \subseteq Y \subseteq X$ and A is a $\mu$-$\alpha$-semi generalized closed set in X then A is a $\mu$-$\alpha$-semi generalized closed set relative to Y.

Proof: Given that $A \subseteq Y \subseteq X$ and A is a $\mu$-$\alpha$-semi generalized closed set in X. Let $A \subseteq Y \cap U$, where U is a $\mu$-$\alpha$-open set in X. Since A is a $\mu$-$\alpha$-semi generalized closed set, $A \subseteq U$ implies $sc_{\mu}(A) \subseteq U$. This implies $Y \cap sc_{\mu}(A) \subseteq Y \cap U$ and $sc_{\mu}(A) \subseteq Y \cap U$. That is A is a $\mu$-$\alpha$-semi generalized closed set relative to Y.

Theorem 3.28: Let A be any $\mu$-$\alpha$-semi generalized closed set in $(X, \mu)$. Then A is a $\mu$-semi closed set in $(X, \mu)$ iff $sc_{\mu}(A) - A$ is a $\mu$-$\alpha$-closed set in X.

Proof: Necessity: Let A be a $\mu$-semi closed set in $(X, \mu)$. Then $sc_{\mu}(A) = A$ and so, $sc_{\mu}(A) \cap A^c = A \cap A^c = \emptyset$. Therefore, $sc_{\mu}(A) \cap A^c = \emptyset$ and $sc_{\mu}(A) - A = \emptyset$, Therefore $sc_{\mu}(A) - A$ is a $\mu$-$\alpha$-closed set in $(X, \mu)$.

Sufficiency: Let $sc_{\mu}(A) - A$ be a $\mu$-$\alpha$-closed set and A be a $\mu$-$\alpha$-semi generalized closed set in X. Then by Theorem 3.23, $sc_{\mu}(A) - A$ does not contain any non-empty $\mu$-$\alpha$-closed set and hence $sc_{\mu}(A) - A = \emptyset$. That is $sc_{\mu}(A) = A$. Hence A is a $\mu$-semi closed set in $(X, \mu)$.

Theorem 3.29: Every subset of X is a $\mu$-$\alpha$-semi generalized closed set in X iff every $\mu$-$\alpha$-open set is a $\mu$-semi closed set in X.

Proof: Necessity: Let A be $\mu$-$\alpha$-open in X, and by hypothesis, A is a $\mu$-$\alpha$-semi generalized closed set in X. Hence by Theorem 3.26, A is a $\mu$-semi closed set in X.

Sufficiency: Let $A \subseteq U$ where U is a $\mu$-$\alpha$-open set in X. Then by hypothesis, U is a $\mu$-semi closed set. This implies $sc_{\mu}(U) = U$ and $sc_{\mu}(A) \subseteq sc_{\mu}(U) = U$. Hence $sc_{\mu}(A) \subseteq U$. Thus A is a $\mu$-$\alpha$-semi generalized closed set in X.

Theorem 3.30: Let A and B be $\mu$-$\alpha$-semi generalized closed set in $(X, \mu)$ such that $c_{\mu}(A) = sc_{\mu}(A)$ and $c_{\mu}(B) = sc_{\mu}(B)$, then $A \cup B$ is a $\mu$-$\alpha$-semi generalized closed set in X.

Proof: Let $A \cup B \subseteq U$, where U is a $\mu$-$\alpha$-open set. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $\mu$-$\alpha$-semi generalized closed sets, $sc_{\mu}(A) \subseteq U$ and $sc_{\mu}(B) \subseteq U$. Now $sc_{\mu}(A \cup B) \subseteq c_{\mu}(A \cup B) = c_{\mu}(A) \cup c_{\mu}(B) = sc_{\mu}(A) \cup sc_{\mu}(B) \subseteq U \cup U = U$. Hence $A \cup B$ is a $\mu$-$\beta$-semi generalized closed set in X.
4. REFERENCES