International Journal of Mathematical Archive-9(4), 2018, 61-65
Available online through www.ijma.info ISSN 2229 – 5046

μ-α-SEMI GENERALIZED CLOSED SETS IN GENERALIZED TOPOLOGICAL SPACES

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(Received On: 27-01-18; Revised & Accepted On: 09-03-18)

ABSTRACT

In this paper, I introduce a new class of sets in generalized topological spaces called μ-α-semi generalized closed sets. Also I investigate some of their basic properties and obtained some interesting theorems.

Keywords: *Generalized topological spaces, μ-α-semi closed sets,μ-α-semi generalized closed sets.*

1. INTRODUCTION

The concept of generalized topological spaces was introduced and investigated by A. Csaszar [1]. Many μ-closed sets like μ- semi closed sets, μ- pre closed sets etc., in generalized topological spaces are introduced by him. In this paper, I introduce a new class of sets in generalized topological spaces called μ-α-semi generalized closed sets. Also I investigate some of their basic properties and produced many interesting theorems.

2. PRELIMINARIES

Definition 2.1: [1] Let X be a nonempty set. A collection μ of subsets of X is a generalized topology (or briefly GT) on X if it satisfies the following:

1. $\emptyset, X \in \mu$ and
2. If $\{M_i : i \in I\} \subseteq \mu$, then $\cup_{i \in I} M_i \in \mu$.

If μ is a GT on X, then (X, μ) is called a generalized topological space (or briefly GTS) and the elements of μ are called μ -open sets and their complement are called μ -closed sets.

Definition 2.2: [1] Let (X, μ) be a GTS and let $A \subseteq X$. Then the μ-closure of A, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A.

Definition 2.3: [1] Let (X, μ) be a GTS and let $A \subseteq X$. Then the μ-interior of A, denoted by $i_\mu(A)$, is the union of all μ-open sets contained in A.

Definition 2.4: [1] Let (X, μ) be a GTS. A subset A of X is said to be

1. μ-semi-closed set if $i_\mu(c_\mu(A)) \subseteq A$
2. μ-pre-closed set if $c_\mu(i_\mu(A)) \subseteq A$
3. μ-α-closed set if $c_\mu(i_\mu(c_\mu(A))) \subseteq A$
4. μ-β-closed set if $i_\mu(c_\mu(i_\mu(A))) \subseteq A$
5. μ-regular-closed set if $A = c_\mu(i_\mu(A))$

Definition 2.5: [3**]** Let (X, μ) be a GTS. A subset A of X is said to be

1. μ-regular generalized closed set if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$, where U is μ-regular open in X
2. μ-generalized closed set if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$, where U is μ-open in X
3. μ-generalized-α- closed set if $\alpha c_\mu(A) \subseteq U$ whenever $A \subseteq U$, where U is μ-open in X

The complement of μ -semi closed (resp., μ - pre closed, μ-α-closed, μ-β-closed, μ-regular closed, μ-generalized closed) is said to be μ -semi open (resp., μ -pre open, μ-α-open, μ-β-open, μ- regular open, μ-generalized open) in X.

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3. μ-α-SEMI GENERALIZED CLOSED SETS

In this section I introduce μ-α-semi generalized closed sets in generalized topological spaces and studied some of their basic properties. Some interesting and important theorems are also obtained.

Definition 3.1: Let (X, μ) be a GTS. Then a non-empty subset A is said to be a μ-α-semi generalized closed set (briefly μ-α-SGCS) if $sc_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ- α -open in X.

Example 3.2: Let X = {a, b, c} and let μ = {Ø, {a}, {b}, {a, b}, X}.Then (X, μ) is a GTS. Now,

μ-αO(X) = {Ø, {a}, {b}, {a, b}, X}.

Then A = {a} is a μ-α - semi generalized closed set in (X, μ).

Theorem 3.3: Every μ-closed set in (X, μ) is a μ- α- semi generalized closed set in (X, μ) but not conversely in general.

Proof: Let A be a μ-closed set in (X, μ), then $c_\mu(A) = A$. Now let $A \subseteq U$ and U be μ-α-open in X. Then $sc_\mu(A) = A \cup i_\mu(c_\mu(A)) = A \cup i_\mu(A) \subseteq A \subseteq U$, by hypothesis. Therefore $sc_\mu(A) \subseteq U$. This implies, A is a μ- α- semi generalized closed set in (X, μ).

Example 3.4: Let X = {a, b, c, d} and let μ = {Ø, {b}, {d}, {b, d}, X}. Then (X, μ) is a GTS. Now,

μ-αO(X) = {Ø, {b}, {d}, {b, d}, X}.

Then A ={b} is a μ- α -semi generalized closed set in (X, μ). But, as $c_\mu(A) = c_\mu(\{b\}) = \{a, b, c\} \neq A$, A is not a μ-closed set in (X, μ).

Theorem 3.5: Every μ-semi closed set in (X, μ) is a μ-α- semi generalized closed set in (X, μ).

Proof: Let A be a μ-semi closed set in (X, μ). Then $i_\mu(c_\mu(A)) \subseteq A$. Now let $A \subseteq U$ and U be μ-α-open in X. Then $sc_\mu(A) = A \cup i_\mu(c_\mu(A)) \subseteq A \cup A = A \subseteq U$, by hypothesis. Therefore A is a μ- α- semi generalized closed set in (X, μ).

Remark 3.6: Every μ-α-semi generalized closed sets and μ- pre closed sets are independent in general in (X, μ).

Example 3.7: Let X = {a, b, c} and let μ = {Ø, {a}, {b}, {a, b}, X}.Then (X, μ) is a GTS. Now,

μ-αO(X) = {Ø, {a}, {b}, {a, b}, X}.

Then A = {a} is a μ-α-semi generalized closed set but not a μ-pre closed set as $c_\mu(i_\mu(A)) = c_\mu(i_\mu(\{a\})) = \{a, c\} \nsubseteq A$.

Example 3.8: Let X = {a, b, c} and let μ = {Ø, {a, b}, X}. Then (X, μ) is a GTS. Now let A = {a}. Then $c_\mu(i_\mu(A)) = c_\mu(i_\mu(\{a\})) = \emptyset \subseteq A$. Therefore A is a μ-pre closed set, but A is not a μ- α- semi generalized closed set as $A \subseteq U = \{a, b\}$, where U is a μ- α-open set and now,

μ-αO(X) = {Ø, {a, b}, X} and $sc_\mu(A) = X \nsubseteq \{a\} = U$.

Theorem 3.9: Every μ-α-closed set in (X, μ) is a μ- α- semi generalized closed set in (X, μ) but not conversely in general.

Proof: Let A be a μ-α-closed set in (X, μ).Then $c_\mu(i_\mu(c_\mu(A))) \subseteq A$. Now let $A \subseteq U$ and U be μ- α-open in X. Then $sc_\mu(A) = A \cup i_\mu(c_\mu(A)) \subseteq A \cup c_\mu(i_\mu(c_\mu(A))) \subseteq A \cup A = A \subseteq U$, by hypothesis. Therefore $sc_\mu(A) \subseteq U$. This implies, A is a μ- α- semi generalized closed set in (X,μ).

Example 3.10: Let X = {a, b, c} and let μ = {Ø, {a}, {b}, {c}, {a, c}, X}. Then (X, μ) is a GTS. Now,

μ-αO(X) = {Ø, {a}, {b}, {c}, {a, c}, X}.

Then A = {c} is a μ-α- semi generalized closed set in (X, μ). But, A is not a μ-α-closed set in X, as $c_\mu(i_\mu(c_\mu(A))) = c_\mu(i_\mu(c_\mu(\{c\}))) = \{a, c\} \nsubseteq A$.

Remark 3.11: A μ-β-closed set is not a μ- α- semi generalized closed set in (X, μ) in general.

Example 3.12: Let X = {a, b, c} and let μ = {Ø, {a, b}, X}. Then (X, μ) is a GTS. Let A = {a}. Then, $i_\mu(c_\mu(i_\mu(A))) = i_\mu(c_\mu(i_\mu(\{a\}))) = \emptyset \subseteq \{a\} = A$. Therefore A is a μ-β-closed set in (X, μ), but not a μ- α- semi generalized closed set as $A \subseteq U = \{a, b\}$, where U is a μ-α-open set and now,

μ-αO(X) = {Ø, {a, b}, X} and $sc_\mu(A) = X \nsubseteq \{a\} = U$.

Theorem 3.13: Every μ-regular closed set in (X, μ) is a μ- α- semi generalized closed set in (X, μ) but not conversely.

Proof: Let A be a μ-regular closed set in (X, μ). Then $A = c_\mu(i_\mu(A))$. Now let $A \subseteq U$ and U be μ- α-open in X. Then $sc_\mu(A) = A \cup i_\mu(c_\mu(A)) = A \cup i_\mu(c_\mu(c_\mu(i_\mu(A)))) = A \cup i_\mu(c_\mu(i_\mu(A))) = A \cup c_\mu(i_\mu(A)) = A \cup A = A \subseteq U$, by hypothesis. Therefore A is a μ-α- semi generalized closed set in X.

Example 3.14: Let X = {a, b, c, d} and let μ = {Ø, {a}, {c}, {b, d}, X}.Then (X, μ) is a GTS.

Now, μ-αO(X) = {Ø, {a}, {c}, {b, d}, X}. Then A = {c} is a μ- α- semi generalized closed set but not a μ-regular closed set as $c_\mu(i_\mu(A)) = c_\mu(i_\mu(\{c\})) = \{a, c\} \neq \{c\} = A$.

Remark 3.15: Every μ-α-semi generalized closed set and μ-generalized closed set in (X, μ) are independent to each other in general.

Example 3.16: Let X = {a, b, c, d} and let μ = {Ø, {a}, {b}, {a, b}, {a, c}, {b, c},{a, b, c}, X}.Then (X, μ) is a GTS. Now,

μ-αO(X) = {Ø, {a}, {b}, {a, b}, {a, c}, {b, c}, {a, b, c},X}.

Then A = {a} is a μ-α- semi generalized closed set in (X, μ), but not a μ-generalized closed set as $c_\mu(A) = c_\mu(\{a\}) = \{a, d\} \nsubseteq \{a, b, c\} = U$.

Example 3.17: Let X = {a, b, c} and let μ = {Ø, {a, b}, X}. Then (X, μ) is a GTS. Let A = {a, b}and U = X. Then A ⊆ U and $c_\mu(A) = c_\mu(\{a, b\}) = X \subseteq U$. Therefore A is a μ-generalized closed set, but not a μ-α- semi generalized closed set as A ⊆ U = {a, b}, where U is a μ-α-open set and now,

μ-αO(X) = {Ø, {a, b}, X} and $sc_\mu(A) = X \nsubseteq \{a, b\} = U$.

Remark 3.18: Every μ-α- semi generalized closed set and μ-generalized-α-closed set in (X, μ) are independent to each other in general.

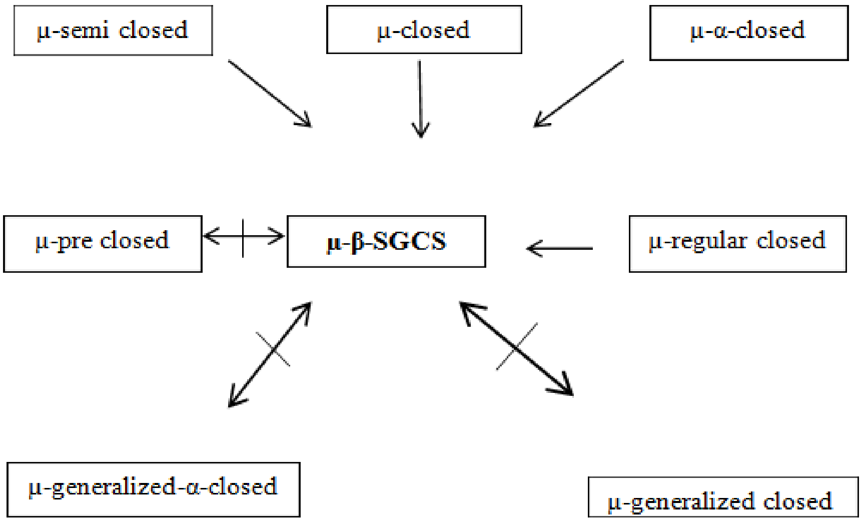
Example 3.19: Let X = {a, b, c, d} and let μ = {Ø, X, {a}, {b}, {a, b}, {a, b, c}}. Then (X, μ) is a GTS.

Now, μ-αO(X) = {Ø, {a}, {b}, {a, b}, {a, b, c}, X}. Then A = {b} is a μ-α- semi generalized closed set in (X, μ), but not a μ-generalized-α-closed set as $\alpha c_\mu(A) = \{b, c, d\} \nsubseteq \{a, b, c\} = U$ and A ⊆ U.

Example 3.20: Let X = {a, b, c} and let μ = {Ø, {a, b}, X}. Then (X, μ) is a GTS. Let A ={a} and U = X. Then A⊆ U and $\alpha c_\mu(A) = A \cup c_\mu(i_\mu(c_\mu(A))) = \{a\} \cup X = X \subseteq U$. Therefore A is a μ-generalized-α-closed set, but not a μ-α- semi generalized closed set as A ⊆ U = {a} where U is a μ-α-open set and now,

μ-αO(X) = {Ø, {a, b}, X}and $sc_\mu(A) = X \nsubseteq \{a\} = U$.

In the following diagram, we have provided relations between various types of μ-closed sets.



Remark 3.21: Union of any two μ-α-semi generalized closed sets in (X, μ) need not be a μ-α- semi generalized closed set in X.

Example 3.22: Let X = {a, b, c} and let μ = {Ø, {a}, {b}, {a, b}, X}.Then (X, μ) is a GTS. Now,

μ-αO(X) = {Ø, {a}, {b}, {a, b}, X}.

Then A = {a} and B = {b} are μ-α- semi generalized closed sets in (X, μ). But, A ∪ B ={a, b} is not a μ-α- semi generalized closed set as $sc_\mu(\{a ,b\}) = \{a, b\} \cup i_\mu(c_\mu(\{a, b\})) = \{a, b\} \cup X = X \nsubseteq \{a, b\} = U$ and A ∪ B ⊆U.

Theorem 3.23: If a subset A in X is a μ-α- semi generalized closed set in (X, μ), then $sc_\mu(A) - A$ contains no non-empty μ-α -closed set in X.

Proof: Let A be a μ-α- semi generalized closed set in (X, μ) and let $sc_\mu(A) – A$ contains a non-empty μ-α -closed set say F. That is, $F \subseteq sc_\mu(A) – A$. This implies, $F \subseteq sc_\mu(A) \cap A^c$ and hence $F \subseteq sc_\mu(A)$ & $F \subseteq A^c$ (1)

Now $F \subseteq A^c$ implies $A \subseteq F^c$. Since F^c is μ-α-open and A is a μ-α- semi generalized closed set, we have, $sc_\mu(A) \subseteq F^c$. This implies $F \subseteq (sc_\mu(A))^c$ (2)

From (1) & (2), $F \subseteq sc_\mu(A)$ and $F \subseteq (sc_\mu(A))^c$. This implies $F \subseteq sc_\mu(A) \cap (sc_\mu(A))^c = \emptyset$. Therefore F = Ø. Thus $sc_\mu(A) – A$ contains no non-emptyμ-α -closed set in X.

Theorem 3.24: Let (X, μ) be a GTS. Then for every $A \in sc_\mu(X)$ and for every set $B \subseteq X$, $A \subseteq B \subseteq sc_\mu(A)$ implies $B \in sc_\mu(X)$.

Proof: Let $B \subseteq U$ and U be a μ-α -open set in (X, μ). Then since $A \subseteq B$ and $A \subseteq U$. By hypothesis $B \subseteq sc_\mu(A)$. Therefore $sc_\mu(B) \subseteq sc_\mu(sc_\mu(A)) = sc_\mu(A) \subseteq U$ as A is a μ-α- semi generalized closed set of X. Hence $B \in sc_\mu(X)$.

Theorem 3.25: In a GTS X, for each $x \in X$, {x} is a μ-α-closed set or its complement X–{x} is a μ-α- semi generalized closed set in (X, μ).

Proof: Suppose that {x} is not a μ-α closed set in (X, μ). Then X – {x} is not a μ-α -open set in (X, μ). The only μ-α -open set containing X – {x} is X. Thus $X – \{x\} \subseteq X$ and so $sc_\mu(X-\{x\}) \subseteq sc_\mu(X) = X$. Therefore $sc_\mu(X-\{x\}) \subseteq X$ and so X – {x} is a μ-α- semi generalized closed set in (X, μ).

Theorem 3.26: If A is both a μ-α -open set and a μ-α- semi generalized closed set in (X, μ), then A is a μ-semi closed set in (X, μ).

Proof: Let A be μ-α -open and μ-α- semi generalized closed set in (X, μ). Then, $sc_\mu(A) \subseteq A$ as $A \subseteq A$. But always $A \subseteq sc_\mu(A)$. Therefore, $A = sc_\mu(A)$. Hence A is a μ-semi closed set in (X, μ).

Theorem 3.27: If $A \subseteq Y \subseteq X$ and A is a μ-α- semi generalized closed set in X then A is a μ-α- semi generalized closed set relative to Y.

Proof: Given that $A \subseteq Y \subseteq X$ and A is a μ-α- semi generalized closed set in X. Let $A \subseteq Y \cap U$, where U is a μ-α -open set in X. Since A is a μ-α- semi generalized closed set, $A \subseteq U$ implies $sc_\mu(A) \subseteq U$. This implies $Y \cap sc_\mu(A) \subseteq Y \cap U$ and $sc_\mu(A) \subseteq Y \cap U$. That is A is a μ-α- semi generalized closed set relative to Y.

Theorem 3.28: Let A be any μ-α- semi generalized closed set in (X, μ). Then A is a μ-semi closed set in (X, μ) iff $sc_\mu(A) – A$ is a μ-α closed set in X.

Proof: Necessity: Let A be a μ-semi closed set in (X, μ). Then $sc_\mu(A) = A$ and so, $sc_\mu(A) \cap A^c = A \cap A^c = \emptyset$. Therefore, $sc_\mu(A) \cap A^c = \emptyset$ and $sc_\mu(A) – A = \emptyset$, Therefore $sc_\mu(A) – A$ is a μ-α -closed set in (X,μ).

Sufficiency: Let $sc_\mu(A) – A$ be a μ-α- closed set and A be a μ-α -semi generalized closed set in X. Then by Theorem 3.23, $sc_\mu(A) – A$ does not contain any non-empty μ-α -closed set and hence $sc_\mu(A) – A = \emptyset$. That is $sc_\mu(A) = A$. Hence A is a μ-semi closed set in (X,μ).

Theorem 3.29: Every subset of X is a μ-α- semi generalized closed set in X iff every μ-α -open set is a μ-semi closed set in X.

Proof: Necessity: Let A be μ-α - open in X, and by hypothesis, A is a μ-α- semi generalized closed set in X. Hence by Theorem 3.26, A is a μ-semi closed set in X.

Sufficiency: Let $A \subseteq U$ where U is a μ-α -open set in X. Then by hypothesis, U is a μ-semi closed set. This implies $sc_\mu(U) = U$ and $sc_\mu(A) \subseteq sc_\mu(U) = U$. Hence $sc_\mu(A) \subseteq U$. Thus A is a μ-α- semi generalized closed set in X.

Theorem 3.30: Let A and B be μ-α- semi generalized closed set in (X, μ) such that $c_\mu(A) = sc_\mu(A)$ and $c_\mu(B) = sc_\mu(B)$, then $A \cup B$ is a μ-α- semi generalized closed set in X.

Proof: Let $A \cup B \subseteq U$, where U is a μ-α -open set. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are μ-α- semi generalized closed sets, $sc_\mu(A) \subseteq U$ and $sc_\mu(B) \subseteq U$. Now $sc_\mu(A \cup B) \subseteq c_\mu(A \cup B) = c_\mu(A) \cup c_\mu(B) = sc_\mu(A) \cup sc_\mu(B) \subseteq U \cup U = U$. Hence $A \cup B$ is aμ-β- semi generalized closed set in X.

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