Q-FUZZY SOFT RING

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ABSTRACT

Solairaju and Nagarajan [2009] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. Sarala and Suganya [2014] presented some properties of fuzzy soft groups. Further Sarala and Suganya [2014] introduced normal fuzzy soft groups. In this paper, we study Q-fuzzy soft ring theory by using fuzzy soft sets and studied some of algebraic properties. In this paper, the study of Q-fuzzy soft ring by combining soft set theory. The notions of Q-fuzzy soft ring as defined and several related properties and structural characteristics are investigated some related properties. Then the definition of Q-fuzzy soft ring and the theorem of homomorphic pre-image are given.

INTRODUCTION

The concept of soft sets was introduced by Molodtsov [1999], soft sets theory has been extensively studied by many authors. It is well known that the concept of fuzzy sets, introduced by Zadeh [1965], has been extensively applied to many scientific fields. Rosenfeld [1971] applied the concept to the theory of groupoids and groups. In Ahmat and kharal [2009] have already introduced the definition of fuzzy soft set and studied some of their basic properties. Zhiming Zhang [2012] studied intuitionistic fuzzy soft rings. Onar et al. [2012] discussed fuzzy soft gamma ring. Solairaju and Nagarajan [2008] analyzed Q-fuzzy left R-subgroups of near rings with respect to T-norm.

SECTION 2 – DEFINITIONS AND PRELIMINARIES

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

Definition 2.1: Suppose that U is an initial universe set and E is a set of parameters, let P(U) denotes the power set of U. A pair (F, E) is called a soft set over U where F is a mapping given by F: E → P(U). Clearly, a soft set is a mapping from parameters to P(U), and it is not a set, but a parameterized family of subsets of the Universe.

Definition 2.2: Let U be an initial Universe set and E be the set of parameters. Let A ⊂ E. A pair (F, A) is called fuzzy soft set over U where F is a mapping given by F: A → I(U), where I(U) denotes the collection of all fuzzy subsets of U.

Definition 2.3: Let X be a group and (F, A) be a soft set over X. Then (F, A) is said to be a soft group over X iff F(a) < X, for each a ∈ A.

Definition 2.4: Let X be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is said to be a fuzzy soft group over X iff for each a ∈ A and x, y ∈ X,

(i) f_a(x - y) ≥ T (f_a(x), f_a(y))
(ii) f_a(x - a) ≥ f_a(x)

Thus f_a is a fuzzy subgroup for each a ∈ A.

Definition 2.5: Let (f, A) be a soft set over a ring R. Then (f, A) is said to be a soft ring over R if and only if f(a) is sub ring of R for each a ∈ A.
Definition 2.6: Let \( R \) be a soft ring. A fuzzy set ‘\( \mu \)’ in \( R \) is called fuzzy soft ring in \( R \) if

(i) \( \mu ((x + y)) \geq T[\mu(x), \mu(y)] \)
(ii) \( \mu (-x) \geq \mu (x) \) and
(iii) \( (xy)) \geq T[\mu(x), \mu(y)] \), for all \( x, y \in R \).

Definition 2.7: Let \( (\varphi, \Psi) : X \to Y \) is a fuzzy soft function, if \( \varphi \) is a homomorphism from \( x \to y \) then \((\varphi, \Psi)\) is said to be fuzzy soft homomorphism. if \( \varphi \) is an isomorphism from \( X \to Y \) and \( \Psi \) is 1-1 mapping from \( A \) on to \( B \) then \((\varphi, \Psi)\) is said to be fuzzy soft isomorphism.

SECTION 3 – SOME PROPERTIES ON Q-FUZZY SOFT RINGS

Definition 3.1: Let \( R \) be a soft ring. A fuzzy set \( \mu \) in \( R \) is called Q-fuzzy soft ring in \( R \) if

(i) \( \mu ((x + y), q) \geq T[\mu(x), \mu(y), q] \)
(ii) \( \mu (-x, q) \geq \mu (x, q) \) and
(iii) \( \mu ((xy), q) \geq T[\mu(x, q), \mu(y, q)] \), for all \( x, y \in R \& q \in Q \)

Theorem 3.2: Every imaginable Q-fuzzy soft ring \( \mu \) is a Q fuzzy soft ring of \( R \).

Proof: Assume that \( \mu \) is an imaginable Q-fuzzy soft ring of \( R \), then we have

\[
\begin{align*}
\mu ((x + y), q) & \geq T[\mu(x, q), \mu(y, q)] \\
\mu (-x, q) & \geq \mu (x, q) \\
\mu ((xy), q) & \geq T[\mu(x, q), \mu(y, q)]
\end{align*}
\]

Since \( \mu \) is imaginable, we have

\[
\begin{align*}
\min \{\mu(x, q), \mu(y, q)\} & = T[\min \{\mu(x, q), \mu(y, q)\}, \min \{x, q\}, \min \{y, q\}] \\
& \leq T[\mu(x, q), \mu(y, q)] \\
& \leq \min \{\mu(x, q), \mu(y, q)\}
\end{align*}
\]

and so

\[
T[\mu(x, q), \mu(y, q)] = \min \{\mu(x, q), \mu(y, q)\}
\]

It follows that

\[
\begin{align*}
\mu ((x + y), q) & \geq T[\mu(x, q), \mu(y, q)] \\
& = \min \{\mu(x, q), \mu(y, q)\} \text{ for all } x, y \in R, q \in Q
\end{align*}
\]

Hence \( \mu \) is a Q-fuzzy soft ring of \( R \).

Theorem 3.3: If \( \mu \) is Q-fuzzy soft ring \( R \) and \( \theta \) is an endomorphism of \( R \), then \( \mu[\theta] \) is a Q-Fuzzy soft ring of \( R \)

Proof: For any \( x, y \in R \), we have

(FSR1)

(i) \( \mu[\theta][(x + y), q] = \mu(\theta((x + y), q)) = \mu(\theta(x, q), \theta(y, q)) \geq T[\mu(\theta(x, q)), \mu(\theta(y, q))] \geq T[\mu[\theta](x, q), \mu[\theta](y, q)] \)

(FSR2)

(ii) \( \mu[\theta](-x, q) = \mu(\theta(-x, q)) \geq \mu(\theta(x, q)) \geq \mu[\theta](x, q) \)

(FSR3)

(iii) \( \mu[\theta][(xy), q] = \mu(\theta((xy), q)) = \mu((\theta(x, q), (y, q)) \geq T[\mu(\theta(x, q), \mu(\theta(y, q))] \geq T[\mu(\theta(x, q)), \mu(\theta(y, q))] \geq T[\mu[\theta](x, q), \mu[\theta](y, q)] \)

Hence \( \mu[\theta] \) is a Q-fuzzy soft ring of \( R \).

Theorem 3.5: Let \( R \) and \( R' \) be two rings and \( \theta: R \to R' \) be a soft homomorphism. If \( \mu \) and \( f_a \) is a Q-fuzzy soft ring of \( R \) then the pre-image \( \theta^{-1}(f_a) \) Q-fuzzy soft ring of \( R \).

Proof: Assume that \( f_a \) is a Q-fuzzy soft ring of \( R' \). Let \( x, y \in R \& q \in Q \)
Theorem 4.2: Hence R.

SECTION 4 – OTHER PROPERTIES ON Q-FUZZY SOFT RING

Theorem 4.1: Let $\theta : R \rightarrow R'$ be an epimorphism and $f_a$ be fuzzy soft set in $R'$. If $\theta[f_a]$ is q-fuzzy soft ring of $R'$ then $f_a$ is Q-fuzzy soft ring of $R$.

Proof: Let $x, y \in R$, Then there exist $a, b \in R$ such that $\theta (a) = x, \theta (b) = y$. It follows that

\[(FSR1)\]
\[\mu_{\theta^{-1}[f_a]}((x+y),q) = \mu_{f_a}(\theta(x+y),q) \]
\[= \mu_{f_a}(\theta(x,q),\theta(y,q)) \]
\[\geq T\left\{\mu_{f_a}(\theta(x,q),\mu_{f_a}(\theta(y,q))\right\} \]
\[\geq T\left\{\mu_{\theta^{-1}[f_a]}(x,q),\mu_{\theta^{-1}[f_a]}(y,q)\right\} \]

\[\mu_{\theta^{-1}[f_a]}(-x,q) = \mu_{f_a}(\theta(-x),q) \]
\[\geq \mu_{f_a}(\theta(x,q)) \]
\[\geq \mu_{\theta^{-1}[f_a]}(x,q) \]

\[\mu_{\theta^{-1}[f_a]}((xy),q) = \mu_{f_a}(\theta(xy),q) \]
\[= \mu_{f_a}(\theta(x,q),\theta(y,q)) \]
\[\geq T\left\{\mu_{f_a}(\theta(x,q),\mu_{f_a}(\theta(y,q))\right\} \]
\[\geq T\left\{\mu_{\theta^{-1}[f_a]}(x,q),\mu_{\theta^{-1}[f_a]}(y,q)\right\} \]

Hence $\theta^{-1}(f_a)$ is a Q-fuzzy soft ring of $R$.

**Theorem 4.2:** Onto homomorphic image of a Q-fuzzy soft ring with the sup property is Q-fuzzy soft ring of $R$.

Proof: Let $f : R \rightarrow R'$ be an onto homomorphism of Q fuzzy soft rings and let $\mu$ be a sup property of Q-fuzzy soft ring of $R$.

Let $x^1, y^1 \in R^1$, and $x_0 \in f^1(x^1)$, $y_0 \in f^1(y^1)$ be such that

\[\mu(x_0, q) = (h, q) \in f^1(x^1) \mu(h, q)\]

and

\[\mu(y_0, q) = (h, q) \in f^1(y^1) \mu(h, q)\]

Respectively, then we can deduce that

\[(FSR)\]
\[\mu^f((x^1 + y^1),q) = (z, q) \in f^1((x^1 + y^1),q) \mu(z, q) \]
\[\geq T\left\{\mu(x_0, q),\mu(y_0, q)\right\} \]
\[\sup \mu^f((x^1 + y^1),q) \]
\[\sup \mu^f((x^1 + y^1),q) \mu(h, q) \]
\[\sup \min \{\mu^f((x^1, q),\mu^f((y^1, q)\mu(h, q)\} \]
Consequently, we have

\[ \sup \{ \mu(1, 1) : (1, 1) \in A \} \leq \delta \quad \text{and} \quad \sup \{ \mu(x, q) : (x, q) \in A \} \leq \delta \]

Thus, we have

\[ \sup \{ \mu(x, q) : (x, q) \in A \} \leq \delta \]

Then we have

\[ \sup \{ \mu(x, q) : (x, q) \in A \} \leq \delta \]

Consequently, we have

\[ \mu^f(1, 1) \geq \sup \{ \mu(x, q) : (x, q) \in A \} \]

Similarly, we can show \( \mu^f(-x, q) \geq \mu^f(x, q) \) and \( \mu^f(xy, q) \geq T(\mu^f(x, q), \mu^f(y, q)) \)

Hence \( \mu^f \) is Q-fuzzy soft ring of \( f(R) \).
Theorem 4.4: Let $\mu$ be a Q-fuzzy soft ring $R$ and let $\mu^*$ be a Q fuzzy set in $\mathcal{N}$ defined by $\mu^*(x, q) = \mu(x, q) + 1 - \mu(0, q)$ for all $x \in \mathcal{N}$. Then $\mu^*$ is a normal Q-fuzzy subgroup of $R$

Proof: For any $x, y \in R$ and $q \in Q$ we have
\[(FSR1)\]
$$\mu^*((x + y), q) = \mu((x + y), q) + 1 - \mu(0, q)$$
$$\geq T(\mu(x, q), \mu(y, q)) + 1 - \mu(0, q))$$
$$\geq T(\mu(x, q) + 1 - \mu(0, q), (\mu(y, q) + 1 - \mu(0, q))$$
$$= T(\mu^*(mx, q), \mu^*(my, q)).$$

\[(FSR2)\]
$$\mu^*(-x, q) = \mu(-x, q) + 1 - \mu(0, q)$$
$$\geq \mu(x, q) + 1 - \mu(0, q)$$
$$= \mu(x, q)$$

\[(FSR3)\]
$$\mu^*(xy, q) = \mu((xy), q) + 1 - \mu(0, q)$$
$$\geq T(\mu(x, q), \mu(y, q)) + 1 - \mu(0, q))$$
$$\geq T(\mu(x, q) + 1 - \mu(0, q), (\mu(y, q) + 1 - \mu(0, q))$$
$$= T(\mu^*(mx, q), \mu^*(my, q)).$$

CONCLUSION

In this chapter, we investigate the notion of Q-fuzzy soft ring. This work focused on Q-fuzzy soft rings of fuzzy soft rings. To extend this work one could study the properties of fuzzy soft sets in other algebraic structure.

REFERENCES

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