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Q-FUZZY SOFT RING

S. MAHALAKSHMI[1] AND A. SOLAIRAJU*[2]

[1]Research Scholar (PT) in Mathematics, Department of Mathematics,
Mother Teresa Women's University, Kodaikanal - 624 102, India.

[2]Department of Mathematics, Jamal Mohamed College (A), Trichy, India.

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ABSTRACT

Solairaju and Nagarajan [2009] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. Sarala and Suganya [2014] presented some properties of fuzzy soft groups. Further Sarala and Suganya [2014] introduced on normal fuzzy soft groups. In this paper, we study Q-fuzzy soft ring theory by using fuzzy soft sets and studied some of algebraic properties. In this paper, the study of Q- fuzzy soft ring by combining soft set theory. The notions of Q-fuzzy soft ring as defined and several related properties and structural characteristics are investigated some related properties. Then the definition of Q-fuzzy soft ring and the theorem of homomorphic image and homomorphic pre-image are given.

INTRODUCTION

The concept of soft sets was introduced by Molodtsov [1999], soft sets theory has been extensively studied by many authors. It is well known that the concept of fuzzy sets, introduced by Zadeh [1965], has been extensively applied to many scientific fields. Rosenfeld [1971] applied the concept to the theory of groupoids and groups. In Ahmat amd kharal [2009] have already introduced the definition of fuzzy soft set and studied some of their basic properties. Zhiming Zhang [2012] studied intuitionistic fuzzy soft rings. Onar *et al.* [2012] discussed fuzzy soft gamma ring. Solairaju and Nagarajan [2008] analyzed Q-fuzzy left R-subgroups of near rings with respect to T-norm.

SECTION 2 – DEFINITIONS AND PRELIMINARIES

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

Definition 2.1: Suppose that U is an initial universe set and E is a set of parameters, let $P(U)$ denotes the power set of U. A pair (F, E) is called a ***soft set*** over U where F is a mapping given by $F: E \to P(U)$. Clearly, a soft set is a mapping from parameters to $P(U)$, and it is not a set, but a parameterized family of subsets of the Universe.

Definition 2.2: Let U be an initial Universe set and E be the set of parameters. Let $A \subset E$. A pair (F, A) is called **fuzzy soft set** over U where F is a mapping given by $F: A \to I^U$, where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.3: Let X be a group and (F, A) be a soft set over X. Then (F, A) is said to be a **soft group** over X iff $F(a) < X$, for each $a \in A$.

Definition 2.4: Let X be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is said to be a fuzzy soft group over X iff for each $a \in A$ and $x, y \in X$,

(i) $f_a(x\,.\,y) \geq T\,(f_a(x), f_a(y)$

(ii) $f_a(x^{-1}) \geq f_a(x)$

Thus f_a is a fuzzy subgroup for each $a \in A$.

Definition 2.5: Let (f, A) be a soft set over a ring R. Then (f, A) is said to be a **soft ring** over R if and only if $f(a)$ is sub ring of R for each $a \in A$.

Corresponding Author: A. Solairaju[2]*
[2]Department of Mathematics, Jamal Mohamed College (A), Trichy, India.

Definition 2.6: Let R be a soft ring. A fuzzy set $'\mu'$ in R is called **fuzzy soft ring** in R if
(i) $\mu\ ((x+y)) \geq T\{\mu(x), \mu(y)\}$
(ii) $\mu\ (-x) \geq\ \mu\ (x)$ and
(iii) $((xy)) \geq T\{\mu(x), \mu(y)\}$, for all $x, y \in R$.

Definition 2.7: Let (φ, Ψ): $X\ \rightarrow Y$ is a fuzzy soft function, if φ is a homomorphism from $x \rightarrow y$ then (φ, Ψ) is said to be **fuzzy soft homomorphism**. if φ is a isomorphism from $X \rightarrow Y$ and Ψ is 1-1 mapping from A on to B then (φ, Ψ) is said to be **fuzzy soft isomorphism**.

SECTION 3 – SOME PROPERTIES ON Q-FUZZY SOFT RINGS

Definition 3.1: Let R be a soft ring. A fuzzy set μ in R is called ***Q- fuzzy soft ring*** in R if
(i) $\mu((x+y)\ , q) \geq\ T\{\mu(x, q), \mu(y, q)\}$
(ii) $\mu\ (-x, q) \geq \mu(x, q)$ and
(iii) $\mu((xy)\ , q) \geq\ T\{\mu(x, q), \mu(y, q)\}$, for all $x, y \in R$. & $q \in Q$

Theorem 3.2: Every imaginable Q- fuzzy soft ring μ is a Q fuzzy soft ring of R.

Proof: Assume that μ is imaginable Q- fuzzy soft ring of R, then we have
$\mu\ ((x+y), q) \geq T\{\mu(x, q), \mu(y, q)\}$
$\mu\ (-x, q)\ \geq\ \mu\ (x, q)$ and
$\mu((xy), q) \geq T\{\mu(x, q), \mu(y, q)\}$, for all $x, y \in\ R.$ & $q \in Q$

Since μ is imaginable, we have

$$\begin{aligned} min\{\mu(x,q)\ , \mu(y,q)\} &= T\{min\{\mu(x,q), \mu(y,q)\}, min\{x,q)\ , \mu(y,q)\ \}\} \\ &\leq T\ \{\mu(x,q)\ , \mu(y,q)\} \\ &\leq min\ \{\mu(x,q)\ , \mu(y,q)\} \end{aligned}$$

and so

$$T\{\mu(x,q)\ , \mu(y,q)\} = min\ \{\ \mu(x,q)\ , \mu(y,q)\}$$

It follows that

$$\begin{aligned} \mu\ ((x+y), q) &\geq T\{\mu(x,q)\ , \mu(y,q)\} \\ &=\ min\ \{\mu(x,q)\ , \mu(y,q)\} \text{ for all } x, y \in R, q \in Q \end{aligned}$$

Hence μ is a Q-fuzzy soft ring of R.

Theorem 3.3: If μ is Q-fuzzy soft ring R and θ is an endomorphism of R, then $\mu[\theta]$ is a Q- Fuzzy soft ring of R

Proof: For any $x, y \in R$, we have

(FSR1)

$$\begin{aligned} \text{(i) } \mu[\theta]((x+y), q)) &= \mu(\theta((x+y), q)) \\ &= \mu(\theta(x, q\ ), \theta(y, q)) \\ &\geq T\ \{\mu(\theta(x,q)), \mu(\theta(y,q))\} \\ &\geq T\ \{\mu[\theta]\ (x,q), \mu[\theta](y,q)\} \end{aligned}$$

(FSR2)

$$\begin{aligned} \text{(ii) } \mu[\theta](-x, q) &=\ \mu\ (\theta(-x, q)) \\ &\geq \mu\ (\theta(x,q)) \\ &\geq\ \mu[\theta](x,q) \end{aligned}$$

(FSR3)

$$\begin{aligned} \text{(iii) } \mu[\theta]((xy), q)) &= \mu(\theta((xy), q)) \\ &=\ \mu\ ((\theta x, q\ ), (y, q)) \\ &\geq T\ \{\mu(\theta x, q)\ , \mu\ (\theta y, q\ )\} \\ &\geq\ T\ \{\mu\theta(x,q)\ , \mu\theta(y, q\ )\} \\ &\geq T\ \{\mu\ [\theta]\ (x,q), \mu[\theta](y,q)\} \end{aligned}$$

Hence $\mu\ [\theta]$ is a Q-fuzzy soft ring of R.

Theorem 3.5: Let R and R' be two rings and θ: $R \rightarrow\ R'$ be a soft homomorphism. If μ and f_a is a Q-fuzzy soft ring of R then the pre-image $\theta^{-1}(f_a)$ Q-fuzzy soft ring of R.

Proof: Assume that f_a is a Q-fuzzy soft ring of R'. Let $x, y\ \in\ R$ & $q \in Q$

(FSR1)

(i) $\mu_{\theta^{-1}[f_a]}((x+y),q)) = \mu_{f_a}(\theta(x+y),q))$
$= \mu_{f_a}((\theta x,q),(\theta y,q))$
$= T\left\{\mu_{f_a}\left(\theta(x,q),\mu_{f_a}(\theta(y,q))\right)\right\}$
$\geq T\{\mu_{\theta^{-1}[f_a]}(x,q),\mu_{\theta^{-1}[f_a]}(y,q)\}$

(FSR2)

(ii) $\mu_{\theta^{-1}[f_a]}(-x,q) = \mu_{f_a}(\theta(-x,q))$
$\geq \mu_{f_a}(\theta(x,q))$
$\geq \mu_{\theta^{-1}[f_a]}(x,q)$

(FSR3)

(iii) $\mu_{\theta^{-1}[f_a]}((xy),q) = \mu_{f_a}(\theta(xy),q)$
$= \mu_{f_a}((\theta x,q),(\theta y,q))$
$\geq T\left\{\mu_{f_a}\left(\theta(x,q),\mu_{f_a}(\theta(y,q))\right)\right\}$
$\geq T\{\mu_{\theta^{-1}[f_a]}(x,q),\mu_{\theta^{-1}[f_a]}(y,q)\}$

Hence $\theta^{-1}(f_a)$ is a Q-fuzzy soft ring of R.

SECTION 4 – OTHER PROPERTIES ON Q-FUZZY SOFT RING

Theorem 4.1: Let $\theta: R \to R'$ be an epimorphism and f_a be fuzzy soft set in R'. If $\theta[f_a]$ is q-fuzzy soft ring of R' then f_a is Q-fuzzy soft ring of R.

Proof: Let $x, y \in R$, Then there exist $a, b \in R$ such that $\theta\ (a) = x, \theta(b) = y$. It follows that

(FSR1)

(i) $\mu_{\theta[f_a]}((x+y),q)) = \mu_{f_a}(\theta(x+y),q))$
$= \mu_{f_a}((\theta x,q),(\theta y,q))$
$\geq T\{\mu_{f_a}(\theta(x,q)),\mu_{f_a}(\theta(y,q))\}$
$\geq T\{\mu_{\theta[f_a]}(x,q),\mu_{\theta[f_a]}(y,q)\}$

(FSR2)

(ii) $\mu_{\theta[f_a]}(-x,q) = \mu_{[f_a]}(\theta(-x,q))$
$\geq \mu_{f_a}(\theta(x,q))$
$\geq \mu_{\theta[f_a]}(x,q)$

(iii) $\mu_{\theta[f_a]}((xy),q) = \mu_{f_a}(\theta(xy),q)$
$= \mu_{f_a}((\theta x,q),(\theta y,q))$
$\geq T\{\mu_{f_a}(\theta(x,q)),\mu_{f_a}(\theta(y,q))\}$
$\geq T\{\mu_{\theta[f_a]}(x,q),\mu_{\theta[f_a]}(y,q)\}$

Hence $\theta[f_a]$ is a Q-fuzzy soft ring of R

Theorem 4.2: Onto homomorphic image of a Q-fuzzy soft ring with the **sup** property is Q-fuzzy soft ring of R.

Proof: Let $f: R \to R'$ be an onto homomorphism of Q fuzzy soft rings and let μ be a **sup** property of Q-fuzzy soft ring of R.

Let $x^1, y^1 \in R^1$, and $x_0 \in f^1(x^1),\ y_0 \in f^1(y^1)$be such that

$$\mu(x_0,q) = \sup_{(h,q) \in f^1(x^1)} \mu(h,q)$$

and

$$\mu(y_0,q) = \sup_{(h,q) \in f^1(y^1)} \mu(h,q)$$

Respectively, then we can deduce that

(FSR1)

(i) $\mu^f((x^1+y^1),q) = \sup_{(z,q) \in f^1((x^1+y^1),q)} \mu(z,q)$
$\geq T\{\mu(x_0,q),\mu(y_0,q)\}$
$= T\left\{\sup_{(h,q) \in f^1(x^1,q)} \mu(h,q),\ \sup_{(h,q) \in f^1(y^1,q)} \mu(h,q)\right\}$
$= \min\{\mu^f(x^1,q),\mu^f(y^1,q)\}$

(FSR2)

(ii) $\mu^f(-x^1, q) = \underset{(z,q) \in f^1(-x^1,q)}{sup} \mu(z,q)$

$\geq \mu(x_0, q)$

$\geq \underset{(h,q) \in f^1(x^1,q)}{sup} \mu(h,q)$

$= \mu^f(x^1, q)$

(FSR3)

(iii) $\mu^f\big((x^1y^1), q\big) = \underset{(z,q) \in f^1((x^1y^1),q)}{sup} \mu(z,q)$

$\geq T\{\mu(x_0, q), \mu(y_0, q)\}$

$= T\left\{\underset{(h,q) \in f^1(x^1,q)}{sup} \mu(h,q), \underset{(h,q) \in f^1(y^1,q)}{sup} \mu(h,q)\right\}$

$= \min\{\mu^f(x^1, q), \mu^f(y^1, q)\}$

Hence μ^f is a Q-fuzzy soft ring of R^1

Theorem 4.3: Let T be a continuous t-norm and Let f be a soft homomorphism on R. If μ is Q-fuzzy soft of R, then μ^f is Q-fuzzy soft ring of $f(R)$.

Proof: Let $A_1 = f^{-1}(y_1, q), A_2 = f^{-1}(y_2, q)$ and $A_{12} = f^{-1}((y_1 + y_2), q)$ where $y_1, y_2 \in f(R), q \in Q$

Consider the set

$A_1 + A_2 = \{x \in R/(x.q) = (a_1, q) + (a_2, q)\}$ for some $(a_1, q) \in A_1$ and $(a_2, q) \in A_2$

If $(x, q) \in A_1 + A_2$, then $(x, q) = (x_1, q) + (x_2, q)$ for some $(x_1, q) \in A_1$ and $(x_2, q) \in A_2$ so that we have

$f(x, q) = f(x_1, q) + f(x_2, q)$

$= y_1 + y_2$

Since $(x, q) \in f^{-1}\big((y_1, q) + (y_2, q)\big) = A_{12}$. Thus $A_1 + A_2 \in A_{12}$

It follows that

(FSR1)

(i) $\mu^f((y_1 + y_2), q\ ) = sup\{\mu(x, q)/(x, q) \in f^{-1}(y_1 + y_2, q)\}$

$= sup\ \{\mu(x, q)\ /\ (x, q) \in A_{12}\}$

$\geq sup\ \{\mu(x, q)/\ (x, q) \in A_1 + A_2\}$

$\geq sup\ \{\mu((x_1, q) + (x_2, q))/(x_1, q) \in A_1$ and $(x_2, q) \in A_2\}$

$\geq sup\ \{S(\mu(x_1, q), \mu(x_2, q))/(x_1, q) \in A_1$ and $(x_2, q) \in A_2\}$

Since T is continuous. For every $\varepsilon > 0$, we see that if

$sup\ \{\mu(x_1, q)\ /\ (x_1, q) \in A_1\} + (x_1{}^*, q)\ \} \leq \delta$ and

$sup\ \{\mu(x_2, q)\ /\ (x_2, q) \in A_2\} + (x_2{}^*, q)\ \} \leq \delta$

$T\{sup\{\mu(x_1, q)/\ (x_1, q) \in A_1\}$,

$sup\ \{\ \mu(x_2, q)\ /\ (x_2, q) \in A_2\} + T((x_1{}^*, q), (x_2{}^*, q) \leq \varepsilon$

Choose $(a_1, q) \in A_1$ and $(a_2, q) \in A_2$ such that

$sup\{\mu(x_1, q)\ /\ (x_1, q) \in A_1\} + \mu(a_1, q) \leq \delta$ and

$sup\{\mu(x_2, q)\ /\ (x_2, q) \in A_2\} + \mu(a_2, q) \leq \delta$

Then we have

$T\{sup\{\mu(x_1, q)/(x_1, q) \in A_1\}, sup\{\mu(x_2, q)/(x_2, q)\ \in A_2\} + T(\mu(a_1, q), \mu(ma_2, q)\ \leq \varepsilon$

Consequently, we have

$\mu^f((y_1 + y_2), q\ ) \geq sup\{\ T\big(\mu(x_1, q), \mu(x_2, q)\big)\ /\ (x_1, q) \in A_1, (x_2, q) \in A_2\}$

$\geq T(sup\{\mu(x_1, q)\ /\ (x_1, q) \in A_1\}, sup\{\mu(x_2, q)\ /\ (x_2, q) \in A_2\}$

Similarly we can show $\mu^f(-x, q\ ) \geq\ \mu^f(x, q)$ and $\mu^f(xy, q) \geq T\ \{(\mu^f(x, q)\ , \mu^f(y, q)\}$

Hence μ^f is Q-fuzzy soft ring of $f(R)$.

Theorem 4.4: Let μ be a Q-fuzzy soft ring R and let μ^* be a Q fuzzy set in N defined by $\mu^*(x,q) = \mu(x,q) + 1 - \mu(0,q)$ for all $x \in N$. Then μ^* is a normal Q-fuzzy subgroup of R

Proof: For any $x, y \in R$ and $q \in Q$ we have

(*FSR1)*

$$\begin{aligned}\mu^*((x+y),q) &= \mu((x+y),q) + 1 - \mu(0,q)\\ &\geq T(\mu(x,q),\mu(y,q)) + 1 - \mu(0,q))\\ &\geq T(\mu(x,q) + 1 - \mu(0,q), (\mu(y,q) + 1 - \mu(0,q))\\ &= T(\mu^*(mx,q),\ \mu^*(my,q)).\end{aligned}$$

(FSR2)

$$\begin{aligned}\mu^*(-x,q) &= \mu(-x,q) + 1 - \mu(0,q)\\ &\geq \mu(x,q) + 1 - \mu(0,q)\\ &= \mu(x,q)\end{aligned}$$

(FSR3)

$$\begin{aligned}\mu^*((xy),q) &= \mu((xy),q) + 1 - \mu(0,q)\\ &\geq T(\mu(x,q),\mu(y,q)) + 1 - \mu(0,q))\\ &\geq T(\mu(x,q) + 1 - \mu(0,q), (\mu(y,q) + 1 - \mu(0,q))\\ &= T(\mu^*(mx,q),\ \mu^*(my,q)).\end{aligned}$$

CONCLUSION

In this chapter, we investigate the notion of Q-fuzzy soft ring. This work focused on Q-fuzzy soft rings of fuzzy soft rings. To extend this work one could study the properties of fuzzy soft sets in other algebraic structure.

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