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(k,1)-CONTRA HARMONIC MEAN LABELING OF GRAPHS

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ABSTRACT

Let G =(V,E) be a graph with p vertices and q edges. Let f : V(G) $\rightarrow$ {0,1,2,…, k+(q-1)} be an injective function such that the induced edge labeling f(e = uv) is defined by f(e) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ is a bijection from E to {k, k+1, k+2,…,k+(q-1)}.Then f has a (k,1)- contra harmonic mean labeling. Any graph which admits a (k,1)- contra harmonic mean labeling is called a (k,1)-contra harmonic mean graph. In this paper we investigate the (k,1)- Contra Harmonic mean labeling for some path related graphs.

Keywords: *Contra Harmonic mean labeling, (k, 1) - Contra Harmonic mean labeling.*

1. INTRODUCTION

Let G = (V, E) be a finite, simple, undirected graph with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notation we follow Harray [4].We will give a brief summary of definition and other information which are useful for the present investigation.

A graph G = (V,E) with p vertices and q edges is called a Contra Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0,1,...,q in such a way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels . Here f is called a Contra Harmonic mean labeling of G.

S. Somasundaram and R. Ponraj introduced mean labeling of graphs and investigated mean labeling for some standard graph in [6]. These labeling patterns motivated us to introduced Contra Harmonic mean labeling [5]. In this paper we prove that some path related graphs admits (k,1)-Contra Harmonic mean labeling where k is any positive integer greater than or equal to 1.

Definition 1.1: Let G = (V, E) be a graph with p vertices and q edges . Let f : V(G) $\rightarrow$ {0,1, 2,…, k+(q-1)} be an injective function such that the induced edge labeling f(e = uv) is defined by f(e) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ is a bijection from E to {k, k+1, k+2,…,k+(q-1)}. Then f has a (k,1)- contra harmonic mean labeling. Any graph which admits a (k,1)- contra harmonic mean labeling is called a (k,1)-contra harmonic mean graph. Here k is a positive integer greater than or equal to 1.

Definition 1.2: A Triangular snake T_n is obtained from a path $u_1 \ldots u_n$ by joining u_i and to a vertex v_i for $1 \leq i \leq n - 1$.

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Definition 1.3: A Quadrilateral snake Q_n is obtained from a path $u_1 \ldots u_n$ by joining u_i and u_{i+1} to new vertices v_i, w_i, $1 \le i \le n-1$.

Definition 1.4: A middle graph M(G) of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

2. MAIN RESULTS

Theorem 2.1: A path has a (k, 1)-Contra Harmonic mean labeling for all k.

Proof: Let $u_1 \ldots u_n$ be the vertices of the path P_n.

We define an injective function $f: V(P_n) \to \{0, 1, 2, \ldots, k+(q-1)\}$ as follows

$f(u_1)=k-1$, $f(u_i)=k+i-2$, $2 \le i \le n$

The distinct edge labeling are as follows

$f(u_i u_{i+1}) = i-1+k$, $1 \le i \le n-1$

Hence, f is a (k, 1) -Contra Harmonic mean labeling of G.

Thus, the path admits a (k, 1)-Contra Harmonic mean graph for all k.

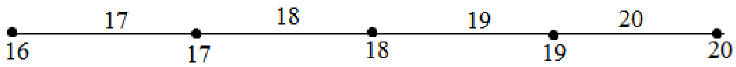

Figure-1: (17, 1) Contra Harmonic mean labeling of P_5

Theorem 2.2: A comb is a (k, 1) - Contra Harmonic mean graph for all k.

Proof: Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the path and let v_i be the pendant vertices attached to each u_i, $1 \le i \le n$.

Let G be a comb graph $P_n \odot K_1$.

We define $f: V(G) \to \{0, 1, 2, \ldots, k+(q-1)\}$ as follows

$f(u_i)=2i-3+k$, $1 \le i \le n$

$f(v_i)=2i-2+k$, $1 \le i \le n$

The distinct edge labeling are as follows

$f(u_i u_{i+1})=2i-1+k$, $1 \le i \le n-1$

$f(u_i v_i)=2i-2+k$, $1 \le i \le n$

Hence, f is a (k, 1)-Contra Harmonic mean labeling of G.

Then, the comb is a (k, 1)-Contra Harmonic mean graph for all k.

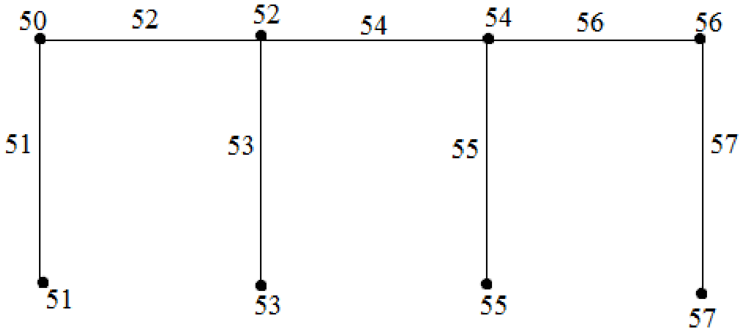

Figure-2: (51, 1)-Contra Harmonic mean labeling of $P_4 \odot K_1$

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Theorem 2.3: $P_n \odot K_{1,2}$ admits (k,1)-Contra Harmonic mean labeling for all k.

Proof: Let P_n be the path $u_1, u_2, u_3, \ldots, u_n$ and v_i , w_i be the vertices of $K_{1,2}$ which are attached to the vertex u_i of P_n, $1 \leq i \leq n$

Let G =$P_n \odot K_{1,2}$

We define f: V(G)$\rightarrow${0, 1, 2…,k+(q-1)} as

$f(u_i)=3i-3+k, 1 \leq i \leq n$
$f(v_i)=3i-4+k, 1 \leq i \leq n$
$f(x_i)=3i-2+k, 1 \leq i \leq n$

The distinct edge labeling are as follows

$f(u_iu_{i+1})=3i-1+k, 1 \leq i \leq n-1$
$f(u_iv_i)=3i-3+k, 1 \leq i \leq n$
$f(u_ix_i)=3i-2+k, 1 \leq i \leq n$

Hence, the function f is a (k,1)-Contra Harmonic mean labeling of G.

Thus, $P_n \odot K_{1,2}$ is a (k-1)-Contra Harmonic mean graph for all k.

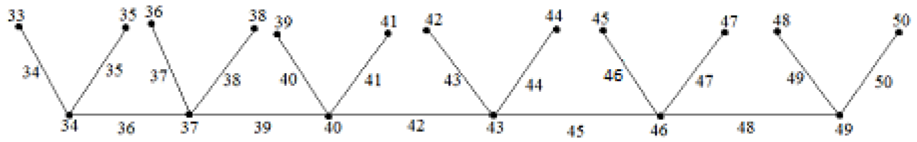

Figure-3: (34, 1)-Contra Harmonic mean labeling of $P_6 \odot K_{1,2}$

Theorem 2.4: $P_n \odot K_{1,3}$ admits (k,1)-Contra Harmonic mean labeling for all k.

Proof: Let P_n be the path with vertices $u_1, u_2, u_3, \ldots, u_n$ and v_i, w_i, z_i be the vertices of $K_{1,3}$ which are joined to the vertices u_i of the path P_n,$1 \leq i \leq n$.

Let G = $P_n \odot K_{1,3}$.

Let f : V(G) $\rightarrow${0, 1, 2,…,k+(q-1)} be defined by

$f(u_i)=4i-4+k, 1 \leq i \leq n$
$f(v_i)=4i-4+k-1, 1 \leq i \leq n$
$f(w_i)=4i-3+k, 1 \leq i \leq n$
$f(x_i)=4i-2+k, 1 \leq i \leq n$

The distinct edge labeling are as follows

$f(u_iu_{i+1})=4i-1+k, 1 \leq i \leq n-1$
$f(u_iv_i)=4i-4+k, 1 \leq i \leq n$
$f(u_iw_i)=4i-3+k, 1 \leq i \leq n$
$f(u_ix_i) = 4i-2+k, 1 \leq i \leq n$

Hence, the function f is a (k, 1)-Contra Harmonic mean labeling of G.

Then, $P_n \odot K_{1,3}$ is a (k-1)-Contra Harmonic mean graph.

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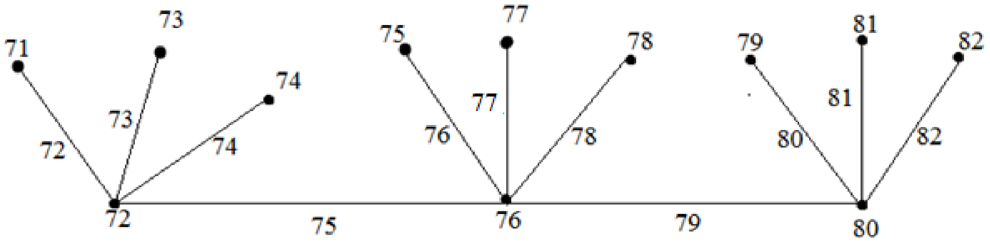

Figure-4: (72, 1)- Contra Harmonic mean labeling of $P_3 \odot K_{1,3}$

Theorem 2.5: A Ladder is a (k, 1) - Contra Harmonic mean graph for all k.

Proof: Let G = $P_2 x P_n$ be a ladder graph. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of G.

Let us define f: V(G) $\rightarrow \{0, 1, 2 \ldots, k+(q-1)\}$ as follows

$f(u_i)=3i-3+k,\ 1 \le i \le n$

$f(v_i) = k+3i-4$, if i is odd

$f(v_i) = k+3i-5$ if *i* is even

The distinct edge labeling are as follows

$f(u_i u_{i+1})=3i-1+k,\ 1 \le i \le n-1$

$f(v_i v_{i+1})=3i-2+k,\ 1 \le i \le n-1$

$f(u_i v_i)=3i-3+k,\ 1 \le i \le n$

Hence, the function f is a (k, 1)- Contra Harmonic mean labeling of G.

Then, $P_2 x P_n$ is a (k-1)-Contra Harmonic mean graph for all k.

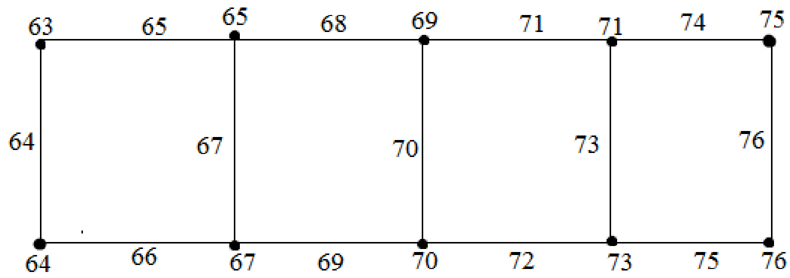

Figure-5: (64,1) – Contra Harmonic mean labeling of $P_2 x P_5$

Theorem 2.6: $(P_n \odot K_1) \odot K_{1,2}$ admits (k,1)-Contra Harmonic mean graph for all k.

Proof: Let G = $(P_n \odot K_1) \odot K_{1,2}$, where P_n is a path with vertices $u_1, u_2, u_3, \ldots, u_n$. Let v_i be a vertex adjacent to u_i, $1 \le i \le n$.

The resultant graph is $(P_n \odot K_1)$. Let x_i, w_i, z_i be the vertices of i^{th} copy of $K_{1,2}$ with z_i the central vertex. Identify the vertex z_i with v_i we get the resultant graph G.

That is, G is a graph obtained by attaching the central vertex of $K_{1,2}$ at each pendent vertex of a comb.

Let us define f: V (G) $\rightarrow \{0, 1, 2 \ldots . k+(q-1)\}$ as

$f(u_i)=4i-3+k,\ 1 \le i \le n$

$f(v_i)=4i-4+k,\ 1 \le i \le n$

$f(w_i)=4i-5+k,\ 1 \le i \le n$

$f(x_i)=4i-2+k,\ 1 \le i \le n$

Then the distinct edge labeling are

$f(u_i u_{i+1})= 4i-1+k,\ 1 \le i \le n-1$

$f(u_i v_i)=4i-3+k,\ 1 \le i \le n$

$f(v_i w_i)=4i-4+k,\ 1 \le i \le n$

$f(v_i x_i)=4i-2+k,\ 1 \le i \le n$

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Then f provides a (k, 1) – Contra Harmonic mean labeling of G.

Hence $(P_n \odot K_1) \odot K_{1,2}$ is a (k-1) Contra Harmonic mean graph.

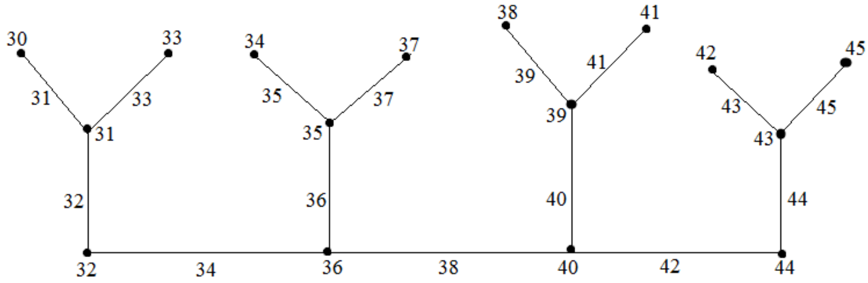

Figure-6: (31,1)- Contra Harmonic mean labeling of $(P_4 \odot K_1) \odot K_{1,2}$

Theorem: 2.7: Any Triangular snake is a (k, 1)-Contra Harmonic mean graph for all k.

Proof: Let G = T_n, where T_n is a Triangular snake obtained from a path $u_1, u_2, u_3, \ldots, u_n$ by joining u_i to v_{i+1} to a new vertex v_i for $1 \le i \le n-1$.

Let us define f: V(G) $\to$ {0, 1, 2..., k+(q-1)} as follows

$f(u_i) = 3i-4+k,\ 1 \le i \le n$

$f(v_i))=3i-2+k,\ 1 \le i \le n-1$

The distinct edge labeling are as follows

$f(u_iu_{i+1})=3i-2+k,\ 1 \le i \le n-1$

$f(u_iv_i)=3i-3+k,\ 1 \le i \le n-1$

$f(v_iu_{i+1})=3i-1+k,\ 1 \le i \le n-1$

Then f is a (k, 1)- Contra Harmonic mean labeling of G.

Hence, any Triangular snake is a (k, 1)-Contra Harmonic mean Graph.

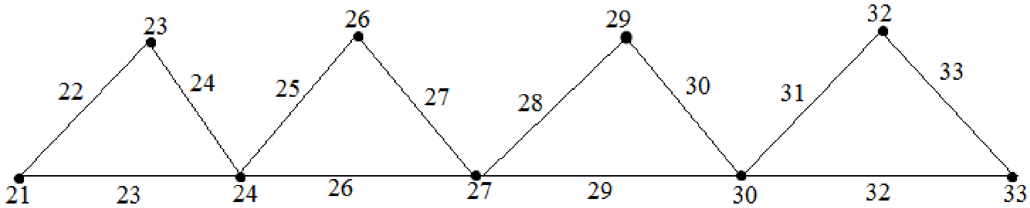

Figure-7: (22, 1) – Contra Harmonic mean labeling of T_4

Theorem 2.1:8: Any Quadrilateral Snake is a (k, 1) – Contra Harmonic mean graph for all k.

Proof: Let G be a Quadrilateral snake obtained from a path $u_1, u_2, u_3, \ldots, u_n$ by joining u_i and u_{i+1} to new vertices v_i and w_i respectively and joining the vertices v_i and w_i $1 \le i \le n-1$

Let us define f: V(G) $\to$ {0, 1, 2…,k+(q-1) } as follows

$f(u_i) =4i-5+k,\ 1 \le i \le n$

$f(v_i)= 4i-4+k,\ 1 \le i \le n-1$

$f(w_i)= 4i-3+k,\ 1 \le i \le n-1$

Then the distinct edge labeling are as follows

$f(u_iu_{i+1})=4i-2+k,\ 1 \le i \le n-1$

$f(u_iv_i)=4i-4+k,\ 1 \le i \le n-1$

$f(v_iw_i)=4i-3+k,\ 1 \le i \le n-1$

$f(w_iu_{i+1})=4i-1+k,\ 1 \le i \le n-1$

Hence, f is a (k, 1)-Contra Harmonic mean labeling for G.

Thus any Quadrilateral snake is a (k, 1) – Contra Harmonic mean graph.

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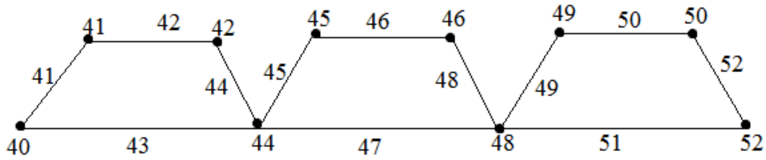

Figure-8: (41, 1) – Contra Harmonic mean labeling of Q_3

Theorem: 2.9: The middle graph of path P_n (n≥3) is a (k, 1) – Contra Harmonic mean graph for all k.

Proof: Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1}, 1 \le i \le n-1\}$ be the vertex set and edge set of the path P_n. Then $V(G) = \{v_1, v_2, \ldots, v_n, e_1, e_2, e_3, \ldots, e_n\}$ and $E(G) = \{v_i e_i, e_i v_{i+1}, 1 \le i \le n-1\} \cup \{e_i e_{i+1}, 1 \le i \le n-2\}$.

Let us define $f: V(G) \to \{0, 1, 2 \ldots, k+(q-1)\}$ by

$f(e_i) = 3i-3+k, 1 \le i \le n$

$f(v_i) = 3i-5+k, 1 \le i \le n+1$

Then the distinct edge labels are

$f(e_i e_{i+1}) = 3i-1+k, 1 \le i \le n-1$

$f(e_i v_i) = 3i-3+k, 1 \le i \le n$, $f(e_i v_{i+1}) = 3i-2+k, 1 \le i \le n$

Clearly, f provides a (k,1)- contra Harmonic mean labeling for G.

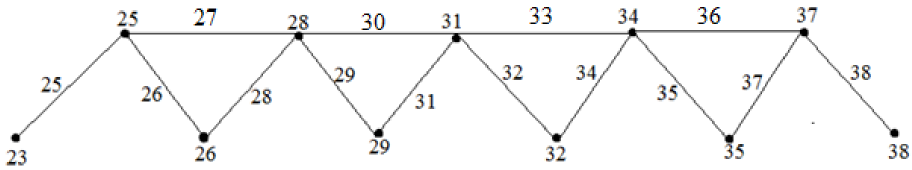

Figure-9: (25, 1) – Contra Harmonic mean labeling of $M(P_5)$

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