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EDGE TRIMAGIC GRACEFUL LABELING OF SOME GRAPHS

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ABSTRACT

A (p, q) graph G is called edge trimagic total if there exists a bijection f: $V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, p+q\}$ such that for each edge xy in E(G) the value of $f(x) + f(xy) + f(y) = K_1$ or K_2 or K_3. G is called edge trimagic graceful if there exists a bijection f : $V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, p+q\}$ such that for each edge xy in E(G), $| f(x) – f(xy) + f(y) | = C_1$ or C_2 or C_3, where C_1, C_2 and C_3 are constants. In this paper, we proved that the Umbrella graph $U_{n,\,m}$, circular ladder graph CL(n) and the Dumbbell graph Db_n are edge trimagic graceful graphs.

Key words: *Graph, Labeling, Magic, Trimagic, Graceful.*

***AMS Subject Classification**: 05C78.*

1. INTRODUCTION

Let G be a simple undirected graph with n vertices. Let V(G) and E(G) denote the vertex set and the edge set of the graph G, respectively. Labeling of a graph G is an assignment f of labels to either the vertices or the edges or both subject to certain conditions. Graph labeling is an increasingly useful and important method of Mathematical models from a broad range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design, communication networks and data base management etc. Graph labeling was first introduced in1960's. In 1970, Kotzig and Rosa [1] defined, a magic labeling of graph G is a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge $uv \in E(G)$, f(u) + f(uv) + f(v) is a magic constant.

Rosa [1] introduced the β - valuations of a graph G with q edges is an injection f from the vertices of G to the set {0, 1, 2, …, q } such that, when each edge xy is assigned the label |f(x) – f(y)|, the resulting edge labels are distinct. Golomb [6] called such labeling as graceful. G. Marimuthu and M. Balakrishnan [4] introduced, super edge magic graceful labeling of graphs. In 2013, C. Jayasekaran, M. Regees and C. Davidraj introduced the edge trimagic total labeling of graphs [2]. A (p, q) graph G is called an edge magic graceful if there exists a bijection f: $V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge xy in E(G) the value of |f(x) + f(y) –f(xy)| = k, a constant. The graph G is said to be super edge magic graceful if V(G) = {1, 2, …, p}. An edge trimagic total labeling of a (p, q) graph G is a bijective function f: $V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge xy $\in$ E(G), the value of f(x) + f(xy) + f(y) is equal to any of the distinct constants k_1 or k_2 or k_3. A graph G is said to be edge trimagic total if it admits an edge trimagic total labeling [2]. An edge trimagic total labeling is called a super edge trimagic total labeling if G has the additional property that the vertices are labeled with smallest positive integers. The useful survey on graph labeling by J. A. Gallian (2017) can be found in [6].

The graph $F_n = P_n + K_1$ is called a fan [7] where P_n: $u_1u_2 \ldots u_n$ be a path and $V(K_1)$ = u. The Umbrella graph [7] $U_{n,\,m}$, m >1 is obtained from a fan F_n by passing the end vertex of the path P_m: $v_1v_2 \ldots v_m$ to the vertex of K_1 of the fan F_n. A Circular ladder [3, 5] CL(n) is the union of an outer cycle C_o: $u_1u_2u_3 \ldots u_nu_1$ and an inner cycle C_1: $v_1v_2v_3 \ldots v_nv_1$ with additional edges (u_iv_i), i = 1, 2, 3, ..., n called spokes. The graph obtained by joining two disjoint cycles $u_1u_2u_3 \ldots u_nu_1$ and $v_1v_2v_3 \ldots v_nv_1$ with an edge u_1v_1 is called dumbbell [7] graph Db_n.

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In this paper, we introduced edge trimagic graceful labelling of graphs and proved that the Umbrella $U_{n,m}$, circular ladder CL(n) and the Dumbbell Db_n are edge trimagic graceful graphs.

2. MAIN RESULTS

Theorem 2.1: The Umbrella $U_{n,m}$ admits an edge trimagic graceful labeling for all n.

Proof: Let $V(U_{n,m}) = \{u_i, v_i\ 1 \le i \le n\}$ be the vertex set and $E(U_{n,m}) = \{u_iu_{i+1}, v_iv_{i+1} / 1 \le i \le n-1\} \cup \{u_iv_1, / 1 \le i \le n\}$ be the edge set of the graph $U_{n,m}$. Then $U_{n,m}$ has n + m vertices and 2n + 2m –2 edges.

Case-1: n is odd and m is even

$f(u_iu_{i+1}) = 3n + m + i - 2, 1 \le i \le n-1$

Define a bijection $f: V \cup E \to \{1, 2, 3, ..., 3n + 2m - 2\}$ such that

$$f(u_i) = \begin{cases} m + \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ n + \frac{m+i}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ \frac{m+i}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

$$f(v_iv_{i+1}) = 2n + m + i, 1 \le i \le n-1$$

$$f(v_1u_i) = \begin{cases} n + m + \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ 2n + \frac{m+i}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

Now we prove this labeling is an edge trimagic graceful.

Consider the edges v_1u_i, $1 \le i \le n$.

For odd i, $\left| f(v_1) - f(v_1u_i) + f(u_i) \right| = \left| 1 - (n + m + \frac{i+1}{2}) + m + \frac{i+1}{2} \right| = \left| 1 - n \right| = C_1$

For even i, $\left| f(v_1) - f(v_1u_i) + f(u_i) \right| = \left| 1 - (2n + \frac{m+i}{2}) + n + \frac{m+i}{2} \right| = \left| 1 - n \right| = C_1$

Consider the edges u_iu_{i+1}, $1 \le i \le n$.

For odd i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| m + \frac{i+1}{2} - (3n + m + i - 2) + n + \frac{m+i+1}{2} \right| = \left| \frac{6+m-4n}{2} \right| = C_2$

For even i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| n + \frac{m+i}{2} - (3n + m + i - 2) + m + \frac{i+1}{2} \right| = \left| \frac{6+m-4n}{2} \right| = C_2$

Consider the edges v_iv_{i+1}, $1 \le i \le n$.

For odd i, $\left| f(v_i) - f(v_iv_{i+1}) + f(v_{i+1}) \right| = \left| \frac{i+1}{2} - (2n + m + i) + \frac{m+i-1}{2} \right| = \left| \frac{2-m-4n}{2} \right| = C_3$

For even i, $\left| f(v_i) - f(v_iv_{i+1}) + f(v_{i+1}) \right| = \left| \frac{m+i}{2} - (2n + m + i) + \frac{i+2}{2} \right| = \left| \frac{2-m-4n}{2} \right| = C_3$

Hence for each edge $uv \in E(U_{n,m})$, $\left| f(u) - f(uv) + f(v) \right|$ yields any one of the constants $C_1 = \left| 1 - n \right|$, $C_2 = \left| \frac{6+m-4n}{2} \right|$ and $C_3 = \left| \frac{2-m-4n}{2} \right|$. Therefore, the Umbrella graph $U_{n,m}$ admits an edge trimagic graceful labeling for odd n and even m.

Case-2: n is even and m is odd

Define a bijection $f: V \cup E \to \{1, 2, 3, ..., 3n + 2m - 2\}$ such that

$$f(u_i) = \begin{cases} m + \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ m + \frac{n+i}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ \frac{m+i+1}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

$$f(u_iu_{i+1}) = 2n + 2m + i - 1, 1 \le i \le n-1$$

$$f(v_iv_{i+1}) = 2n + m + i, \ 1 \le i \le n-1$$

$$f(v_1u_i) = \begin{cases} n + m + \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ m + n + \frac{n+i}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

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Now we prove this labeling is an edge trimagic graceful.

Consider the edges v_1u_i, $1 \le i \le n$.

For odd i, $|f(v_1) - f(v_1u_i) + f(u_i)| = |1 - (n + m + \frac{i+1}{2}) + m + \frac{i+1}{2}| = |1 - n| = C_1$

For even i, $|f(v_1) - f(v_1u_i) + f(u_i)| = |1 - (m + n + \frac{n+i}{2}) + m + \frac{n+i}{2}| = |1 - n| = C_1$

Consider the edges u_iu_{i+1}, $1 \le i \le n$.

For odd i, $|f(u_i) - f(u_iu_{i+1}) + f(u_{i+1})| = |m + \frac{i+1}{2} - (2n + 2m + i - 1) + n + \frac{i+1}{2}| = |2 - m - n|\, C_2$

For even i, $|f(u_i) - f(u_iu_{i+1}) + f(u_{i+1})| = |\frac{2n+i}{2} - (2n + 2m + i - 1) + m + \frac{i+1}{2}| = |2 - m - n| = C_2$

Consider the edges v_iv_{i+1}, $1 \le i \le n$.

For odd i, $|f(v_i) - f(v_iv_{i+1}) + f(v_i)| = |\frac{i+1}{2} - (2n + m + i) + \frac{m+i+1}{2}| = |\frac{3-m-4n}{2}| = C_3$

For even i, $|f(v_i) - f(v_iv_{i+1}) + f(v_i)| = |\frac{m+i+1}{2} - (2n + m + i) + \frac{i+2}{2}| = |\frac{3-m-4n}{2}| = C_3$

Hence for each edge $uv \in E(U_{n,m})$, $|f(u) - f(uv) + f(v)|$ yields any one of the constants $C_1 = |1 - n|$, $C_2 = |2 - m - n|$ and $C_3 = |\frac{3-m-4n}{2}|$. Therefore, the Umbrella graph $U_{n,m}$ admits an edge trimagic graceful labeling for even n and odd m.

Case-3: both n and m are odd

Define a bijection $f: V \cup E \to \{1, 2, 3, ..., 3n + 2m - 2\}$ such that

$$f(u_i) = \begin{cases} m + \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ m + \frac{n+i+1}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ \frac{m+i+1}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

$$f(u_iu_{i+1}) = 2n + 2m + i - 1, 1 \le i \le n - 1$$

$$f(v_iv_{i+1}) = 2n + m + i, 1 \le i \le n - 1$$

$$f(v_1u_i) = \begin{cases} n + m + \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ n + m + \frac{n+i+1}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

Now we prove this labeling is an edge trimagic graceful.

Consider the edges u_iv_1, $1 \le i \le n$.

For odd i, $|f(v_1) - f(v_1u_i) + f(u_i)| = |m + \frac{i+1}{2} - (n + m + \frac{i+1}{2}) + 1| = |1 - n| = C_1$

For even i, $|f(v_1) - f(v_1u_i) + f(u_i)| = |m + \frac{n+i+1}{2} - (n + m + \frac{n+i+1}{2}) + 1| = |1 - n| = C_1$

Consider the edges u_iu_{i+1}, $1 \le i \le n$.

For odd i, $|f(u_i) - f(u_iu_{i+1}) + f(u_{i+1})| = |m + \frac{i+1}{2} - (2n + 2m + i - 1) + m + \frac{n+i+2}{2}| = |\frac{5-3n}{2}|\, C_2$

For even i, $|f(u_i) - f(u_iu_{i+1}) + f(u_{i+1})| = |m + \frac{n+i+1}{2} - (2n + 2m + i - 1) + m + \frac{i+2}{2}| = |\frac{5-3n}{2}| = C_2$

Consider the edges v_iv_{i+1}, $1 \le i \le n$.

For odd i, $|f(v_i) - f(v_iv_{i+1}) + f(v_i)| = |\frac{i+1}{2} - (2n + m + i) + \frac{m+i+1}{2}| = |\frac{3-m-4n}{2}| = C_3$

For even i, $|f(v_i) - f(v_iv_{i+1}) + f(v_i)| = |\frac{m+i+1}{2} - (2n + m + i) + \frac{i+2}{2}| = |\frac{3-m-4n}{2}| = C_3$

Hence for each edge $uv \in E(U_{n,m})$, $|f(u) - f(uv) + f(v)|$ yields any one of the constants $C_1 = |1 - n|$, $C_2 = |\frac{5-3n}{2}|\,|$ and $C_3 = |\frac{3-m-4n}{2}|$. Therefore, the Umbrella graph $U_{n,m}$ admits an edge trimagic graceful labeling for both n and m are odd.

Case-4: both n and m are even

Define a bijection $f: V \cup E \to \{1, 2, 3, ..., 3n + 2m - 2\}$ such that

$$f(u_i) = \begin{cases} m + \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ m + \frac{n+i}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

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$$f(v_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n, \text{i is odd} \\ \frac{n+i}{2}, & 1 \le i \le n, \text{i is even} \end{cases}$$

$f(u_iu_{i+1}) = 3m+n+ i +1, 1 \le i \le n-1$

$f(v_iv_{i+1}) = 2n + m + i, 1 \le i \le n-1$

$$f(v_1u_i) = \begin{cases} n + m + \frac{i+1}{2}, & 1 \le i \le n, \text{i is odd} \\ n + m + \frac{n+i}{2}, & 1 \le i \le n, \text{i is even} \end{cases}$$

Now we prove this labeling is an edge trimagic graceful.

Consider the edges u_iv_1, $1 \le i \le n$.

For odd i, $\left| f(v_1) - f(v_1u_i) + f(u_i) \right| = \left| 1 - (n + m + \frac{i+1}{2}) + m + \frac{i+1}{2} \right| = \left| 1 - n \right| = C_1$

For even i, $\left| f(u_i) - f(v_1u_i) + f(u_i) \right| = \left| 1 - (n + m + \frac{n+i}{2}) + m + \frac{n+i}{2} \right| = \left| 1 - n \right| = C_1$

Consider the edges u_iu_{i+1}, $1 \le i \le n$.

For odd i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1} \right| = \left| m + \frac{i+1}{2} - (n + 3m + i + 1) + m + \frac{n+i+1}{2} \right| = \left| \frac{-2m-n}{2} \right| = C_2$

For even i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| m + \frac{n+i}{2} - (n + 3m + i + 1) + m + \frac{i+2}{2} \right| = \left| \frac{-2m-n}{2} \right| = C_2$

Consider the edges v_iv_{i+1}, $1 \le i \le n$.

For odd i, $\left| f(v_i) - f(v_iv_{i+1}) + f(v_i) \right| = \left| \frac{i+1}{2} - (2n + m + i) + \frac{m+i+1}{2} \right| = \left| \frac{2-m-4n}{2} \right| = C_3$

For even i, $\left| f(v_i) - f(v_iv_{i+1}) + f(v_i) \right| = \left| \frac{m+i}{2} - (2n + m + i) + \frac{i+2}{2} \right| = \left| \frac{2-m-4n}{2} \right| = C_3$

Hence for each edge $uv \in E(U_{n,m})$, $\left| f(u) - f(uv) + f(v) \right|$ yields any one of the constants $C_1 = \left| 1 - n \right|$, $C_2 = \left| \frac{-2m-n}{2} \right| \left| \right.$ and $C_3 = \left| \frac{2-m-4n}{2} \right|$. Therefore, the Umbrella graph $U_{n, m}$ admits an edge trimagic graceful labeling for both n and m are even.

Corollary 2.2: The Umbrella graph $U_{n, m}$ admits a super edge trimagic graceful labeling.

Proof: We proved that the Umbrella graph $U_{n, m}$ admits an edge trimagic graceful labeling. The labeling given in the proof of theorem 2.1, the vertices get labels 1, 2, .., n+m. Since the Umbrella graph $U_{n, m}$ has n+m vertices and these n+m vertices have labels 1, 2, ..., n+m for both odd and even n and m, $U_{n, m}$ is a super edge trimagic graceful.

Example 2.3: An edge trimagic graceful labeling of $U_{5, 4}$, $U_{6, 3}$, $U_{5, 3}$ and $U_{6, 4}$ are given in figure 1, 2, 3 and figure 4 respectively.

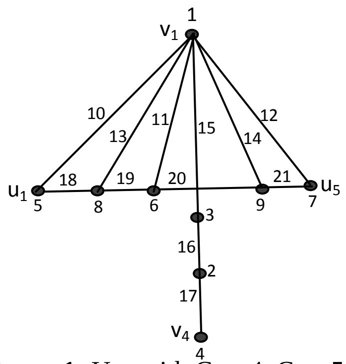

Figure-1: $U_{5, 4}$ with $C_1 = 4$, $C_2 = 5$ and $C_3 = 11$

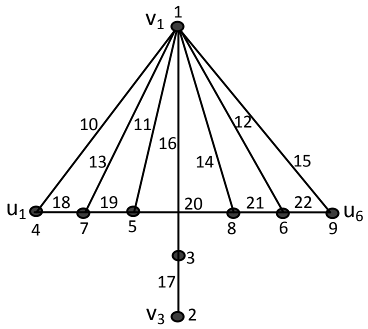

Figure-2: $U_{6, 3}$ with $C_1 = 5$, $C_2 = 7$ and $C_3 = 12$

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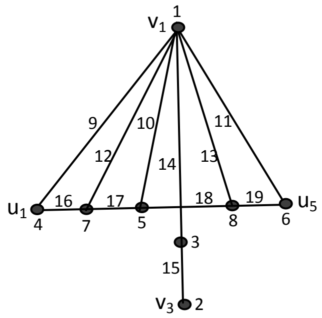

Figure-3: $U_{5,\,3}$ with $C_1 = 4$, $C_2 = 5$ and $C_3 = 10$

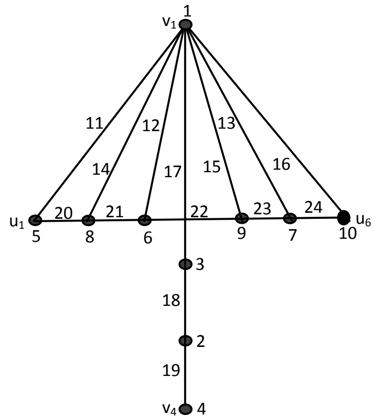

Figure-4: $U_{6,\,4}$ with $C_1 = 5$, $C_2 = 7$ and $C_3 = 13$

Theorem 2.4: The circular ladder CL(n) admits an edge graceful trimagic labeling for all n.

Proof: Let $V(CL(n)) = \{u_i, v_i \,/\, 1 \le i \le n\}$ be the vertex set and $E(CL(n)) = \{u_iu_{i+1}, v_iv_{i+1} \,/\, 1 \le i \le n-1\} \cup \{u_iv_i \,/\, 1 \le i \le n\}$ be the edge set of the graph CL(n). Then CL(n) has 2n vertices and 3n edges.

Case-1: n is odd

Define a bijection $f: V \cup E \to \{1, 2, 3, ..., 5n\}$ such that

$$f(u_i) = \begin{cases} n + \frac{n+i+2}{2} & , 1 \le i \le n-1, i \text{ is odd} \\ n + \frac{i+2}{2} & , 1 \le i \le n-1, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{i+1}{2} & , 1 \le i \le n, i \text{ is odd} \\ \frac{n+i+1}{2} & , 1 \le i \le n, i \text{ is even} \end{cases}$$

$f(u_n) = n+1$

$f(u_iu_{i+1}) = 4n + i + 2, 1 \le i \le n-2$

$f(u_nu_{n-1}) = 4n+1$

$f(u_1u_n) = 4n+2$

$f(v_iv_{i+1}) = 3n + i + 1, 1 \le i \le n-1$

$f(v_1v_n) = 3n + 1$

$f(u_iv_i) = 2n + i + 1, 1 \le i \le n-1$

and $f(u_nv_n) = 2n + 1$.

Now we prove this labeling is an edge trimagic graceful.

Consider the edges u_iu_{i+1}, $1 \le i \le n-2$.

For odd i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| n + \frac{n+i+2}{2} - (4n + i + 2) + n + \frac{i+3}{2} \right| = \left| \frac{1-3n}{2} \right| = C_1$

For even i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| n + \frac{i+2}{2} - (4n + i + 2) + n + \frac{n+i+3}{2} \right| = \left| \frac{1-3n}{2} \right| = C_1$

For the edge u_1u_n, $\left| f(u_1) - f(u_1u_n) + f(u_n) \right| = \left| n + \frac{n+3}{2} - (4n + 2) + n + 1 \right| = \left| \frac{1-3n}{2} \right| = C_1$

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For the edge $u_{n-1}u_n$, $|f(u_{n-1}) - f(u_{n-1}u_n) + f(u_n)| = |\, n + \frac{n+1}{2} - (4n+2) + n + 1| = |\frac{1-3n}{2}| = C_1$

Consider the edges u_iv_i, $1 \le i \le n-1$

For odd i, $|f(u_i) - f(u_iv_i) + f(v_i)| = |n + \frac{n+i+2}{2} - (2n+i+1) + \frac{i+1}{2}| = |\frac{1-n}{2}| = C_2$

For even i, $|f(u_i) - f(u_iv_i) + f(v_i)| = |n + \frac{i+2}{2} - (2n+i+1) + \frac{n+i+2}{2}| = |\frac{1-n}{2}| = C_2$

For the edge u_nv_n, $|f(u_n) - f(u_nv_n) + f(v_n)| = |\, n + 1 - (2n+1) + \frac{n+1}{2}| = |\frac{1-n}{2}| = C_2$

Consider the edges v_iv_{i+1}, $1 \le i \le n-1$.

For odd i, $|f(v_i) - f(v_iv_{i+1}) + f(v_i)| = |\frac{i+1}{2} - (3n+i+1) + \frac{n+i+2}{2}| = |\frac{1-5n}{2}| = C_3$

For even i, $|f(v_i) - f(v_iv_{i+1}) + f(v_i)| = |\frac{n+i+1}{2} - (3n+i+1) + \frac{i+2}{2}| = |\frac{1-5n}{2}| = C_3$

For the edge v_1v_n, $|f(v_1) - f(v_1v_n) + f(v_n)| = |\, 1 - (3n+1) + \frac{n+1}{2}| = |\frac{1-5n}{2}| = C_3$

Hence for each edge uv $\in E(CL(n))$, $|f(u) - f(uv) + f(v)|$ yields any one of the constants $C_1 = |\frac{1-3n}{2}|$, $C_2 = |\frac{1-n}{2}|$ and $C_3 = |\frac{1-5n}{2}|$. Therefore, the circular ladder CL(n) admits an edge trimagic graceful labeling for odd n.

Case-2: n is even

Define a bijection f: V $\cup$ E $\to$ {1, 2, 3, ..., 5n} such that

$$f(u_i) = \begin{cases} n + \frac{i+1}{2}, & 1 \le i \le n-1, \text{i is odd} \\ n + \frac{n+i}{2}, & 1 \le i \le n-1, \text{i is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n, \text{i is odd} \\ \frac{n+i}{2}, & 1 \le i \le n, \text{i is even} \end{cases}$$

$f(u_n) = 2n$

$f(u_iu_{i+1}) = 4n + i$, $1 \le i \le n-1$

$f(u_1u_n) = 5n$

$f(v_iv_{i+1}) = 2n + i$, $1 \le i \le n-1$

$f(v_1v_n) = 3n$

$f(u_iv_i) = 3n + i$, $1 \le i \le n-1$

and $f(u_nv_n) = 4n$

Now we prove this labeling is an edge trimagic graceful.

Consider the edges u_iu_{i+1}, $1 \le i \le n-2$.

For odd i, $|f(u_i) - f(u_iu_{i+1}) + f(u_{i+1})| = |n + \frac{i+2}{2} - (4n+i) + n + \frac{n+i+1}{2}| = |\frac{2-3n}{2}| = C_1$

For even i, $|f(u_i) - f(u_iu_{i+1}) + f(u_{i+1})| = |n + \frac{n+i}{2} - (4n+i) + n + \frac{i+2}{2}| = |\frac{2-3n}{2}| = C_1$

Consider the edges v_iv_{i+1}, $1 \le i \le n-1$.

For odd i, $|f(v_i) - f(v_iv_{i+1}) + f(v_i)| = |\frac{i+1}{2} - (2n+i) + \frac{n+i+1}{2}| = |\frac{2-3n}{2}| = C_1$

For even i, $|f(v_i) - f(v_iv_{i+1}) + f(v_i)| = |\frac{n+i}{2} - (2n+i) + \frac{i+2}{2}| = |\frac{2-3n}{2}| = C_1$

Consider the edges u_iv_i, $1 \le i \le n$,

For odd i, $|f(u_i) - f(u_iv_i) + f(v_i)| = |n + \frac{i+1}{2} - (3n+i) + \frac{i+1}{2}| = |1-2n| = C_2$

For even i, $|f(u_i) - f(u_iv_i) + f(v_i)| = |n + \frac{n+i}{2} - (3n+i) + \frac{n+i}{2}| = |-n| = C_3$

For the edge v_1v_n, $|f(v_1) - f(v_1v_n) + f(v_n)| = |\, 1 - 3n + \frac{2n}{2}| = |1-2n| = C_2$

For the edge u_1u_n, $|f(u_1) - f(u_1u_n) + f(u_n)| = |\frac{2n+2}{2} - (5n) + 2n| = |1-2n| = C_2$

For the edge u_nv_n, $|f(u_n) - f(u_nv_n) + f(v_n)| = |2n - (4n) + n| = |-n| = C_3$

Hence for each edge uv $\in E(CL(n))$, $|f(u) - f(uv) + f(v)|$ yields any one of the constants $C_1 = |\frac{2-3n}{2}|$, $C_2 = |1-2n|$ and $C_3 = |-n|$. Therefore, the circular ladder CL(n) admits an edge trimagic graceful labeling for even n.

Corollary 2.5: The circular ladder CL(n) admits a super edge trimagic graceful labeling.

Proof: We proved that the circular ladder CL(n) admits an edge trimagic graceful labeling. The labeling given in the proof of theorem 2.4, the vertices get labels 1, 2, ..., 2n. Since the circular ladder, CL(n) has 2n vertices and these 2n vertices have labels 1, 2, ..., 2n for both odd and even n, CL(n) is a super edge trimagic graceful.

Example 2.6: An edge trimagic graceful labeling of CL(5), CL(6) are given in figure 5, and figure 6 respectively.

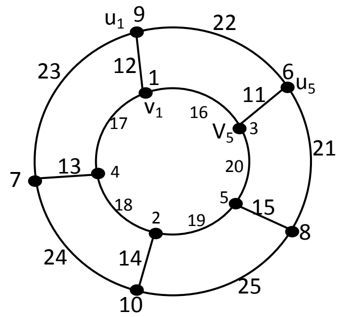

Figure-5: CL(5) with $C_1 = 7$, $C_2 = 2$ and $C_3 = 12$

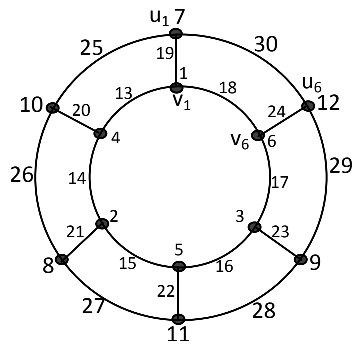

Figure-6: CL(6) with $C_1 = 8$, $C_2 = 11$ and $C_3 = 6$

Theorem 2.7: The Dumbbell Dbn admits an edge trimagic graceful labeling for all n.

Proof: Let $V(Db_n) = \{u_i, v_i \,/\, 1 \le i \le n\}$ be the vertex set and $E(Db_n) = \{u_iu_{i+1}, v_iv_{i+1} \,/\, 1 \le i \le n-1\} \cup \{u_1u_n, v_1v_n\} \cup \{u_1v_1\}$ be the edge set of the graph Db_n. Then Db_n has 2n vertices and 2n+1 edges.

Case-1: n is odd

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 4n+1\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n, \text{ i is odd} \\ \frac{n+i+1}{2}, & 1 \le i \le n, \text{ i is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{2n+i+1}{2}, & 1 \le i \le n, \text{ i is odd} \\ \frac{3n+i+1}{2}, & 1 \le i \le n, \text{ i is even} \end{cases}$$

$f(u_iu_{i+1}) = 2n + i + 1$

$f(v_iv_{i+1}) = 3n + i + 1$

$f(u_1u_n) = 2n + 1$

and $f(v_1v_n) = 3n + 1$

Now we prove this labeling is an edge trimagic graceful.

Consider the edges u_iu_{i+1}, $1 \le i \le n-1$

For odd i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| \frac{i+1}{2} - (2n - i - 1) + \frac{n+i}{2} + 1 \right| = \left| \frac{1-3n}{2} \right| = C_1$

For even i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| \frac{n+i+1}{2} - (2n - i - 1) + \frac{i+2}{2} \right| = \left| \frac{1-3n}{2} \right| = C_1$

Consider the edges v_iv_{i+1}, $1 \le i \le n-1$

For odd i, $\left| f(v_i) - f(v_iv_{i+1}) + f(v_{i+1}) \right| = \left| \frac{2n+i+1}{2} - (3n + i + 1) + \frac{3n+i+2}{2} \right| = \left| \frac{1-n}{2} \right| = C_2$

For even i, $\left| f(v_i) - f(v_iv_{i+1}) + f(v_{i+1}) \right| = \left| \frac{3n+i+1}{2} - (3n + i + 1) + n + \frac{i+2}{2} \right| = \left| \frac{1-n}{2} \right| = C_2$

Consider the edge u_1u_n, $\left| f(u_1) - f(u_1u_n) + f(u_n) \right| = \left| 1 - (2n - 1) + \frac{n+1}{2} \right| = \left| \frac{1-3n}{2} \right| = C_1$

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Consider the edge v_1v_n, $\left| f(v_1) - f(v_1v_n) + f(v_n) \right| = \left| n + 1 - (3n+1) + \frac{3n+1}{2} \right| = \left| \frac{1-n}{2} \right| = C_2$

Consider the edge u_1v_1, $\left| f(u_1) - f(u_1v_1) + f(v_1) \right| = \left| 1 - (4n-1) + n + 1 \right| = \left| 1 - 3n \right| = C_3$

Hence for each edge $uv \in E(Db_n)$, $\left| f(u) - f(uv) + f(v) \right|$ yields any one of the constants $C_1 = \left| \frac{1-3n}{2} \right|$, $C_2 = \left| \frac{1-n}{2} \right|$ and $C_3 = \left| 1 - 3n \right|$. Therefore, the Dumbbell graph Db_n admits an edge trimagic graceful labeling for odd n.

Case-2: n is even

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 4n+1\}$ such that

$$f(u_i) = \begin{cases} n + \frac{i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ \frac{i}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{n+i+1}{2}, & 1 \le i \le n, i \text{ is odd} \\ \frac{3n+i}{2}, & 1 \le i \le n, i \text{ is even} \end{cases}$$

$f(u_iu_{i+1}) = 2n + i$

$f(v_iv_{i+1}) = 3n + i$

$f(u_1u_n) = 3n$

and $f(v_1v_n) = 4n$.

Now we prove this labeling is an edge trimagic graceful.

Consider the edges u_iu_{i+1}, $1 \le i \le n-1$

For odd i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| n + \frac{i+1}{2} - (2n+i) + \frac{i+1}{2} \right| = \left| 1 - n \right| = C_1$ (say)

For even i, $\left| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \right| = \left| \frac{i}{2} - (2n+i) + n + \frac{i+2}{2} \right| = \left| 1 - n \right| = C_1$.

Consider the edges v_iv_{i+1}, $1 \le i \le n-1$

For odd i, $\left| f(v_i) - f(v_iv_{i+1}) + f(v_{i+1}) \right| = \left| \frac{n+i+1}{2} - (3n+i) + \frac{3n+i+1}{2} \right| = \left| 1 - n \right| = C_1$

For even i, $\left| f(v_i) - f(v_iv_{i+1}) + f(v_{i+1}) \right| = \left| \frac{3n+i}{2} - (3n+i) + \frac{n+i+2}{2} \right| = \left| 1 - n \right| = C_1$

Consider the edges u_1u_n, $\left| f(u_1) - f(u_1u_n) + f(u_n) \right| = \left| n + 1 - 3n + \frac{n}{2} \right| = \left| \frac{2-3n}{2} \right| = C_2$.

Consider the edges v_1v_n, $\left| f(v_1) - f(v_1v_n) + f(v_n) \right| = \left| \frac{n}{2} + 1 - 4n + 2n \right| = \left| \frac{2-3n}{2} \right| = C_2$.

Consider the edges u_1v_1, $\left| f(u_1) - f(u_1v_1) + f(v_1) \right| = \left| n - 1 - (4n-1) + \frac{n+2}{2} \right| = \left| \frac{2-5n}{2} \right| = C_3$

Hence for each edge $uv \in E(Db_n)$, $\left| f(u) - f(uv) + f(v) \right|$ yields any one of the constants $C_1 = \left| 1 - n \right|$, $C_2 = \left| \frac{2-3n}{2} \right|$ and $C_3 = \left| \frac{2-5n}{2} \right|$. Therefore, the Dumbbell graph Db_n admits an edge trimagic graceful labeling for even n.

Corollary 2.8: The Dumbbell graph Db_n admits a super edge trimagic graceful labeling.

Proof: We proved that the graph Db_n admits an edge trimagic graceful labeling. The labeling given in the proof of theorem 2.7, the vertices get labels 1, 2, .., 3n+3. Since the Dumbbell graph Db_n has 2n vertices and these 2n vertices have labels 1, 2, ..., 2n for both odd and even n, Db_n is a super edge trimagic graceful.

Example 2.9: An edge trimagic graceful labeling of Db_5 and Db_6 are given in figure 7 and figure 8 respectively.

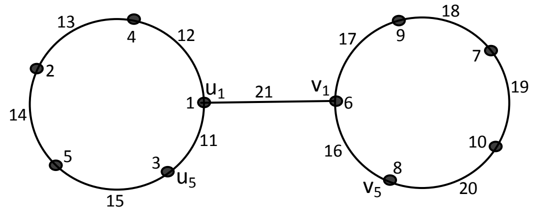

Figure-7: Db_5 with $C_1 = 7$, $C_2 = 2$ and $C_3 = 14$.

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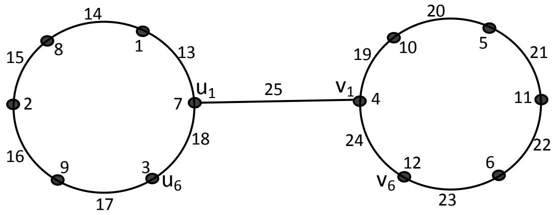

Figure-8: Db_6 with $C_1 = 5$, $C_2 = 8$ and $C_3 = 14$.

3. CONCLUSION

In this paper, we proved that the Umbrella graph $U_{n, m}$, circular ladder graph CL(n) and the Dumbbell graph Db_n are edge trimagic graceful and super edge trimagic graceful. In future, we can construct many trimagic graceful graphs using these ideas.

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