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BRIDGE GRAPH RELATED FAMILIES OF E-CORDIAL GRAPHS

MUKUND V.BAPAT*

Mukund V. Bapat, Hindale, Devgad, Sindhudurg, Maharashtra India: 416630.

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ABSTRACT

The two copies of graph G(p,q) are joined by t paths on n-points each. These paths are bridges between the two graphs. We represent the family by $G(tP_n)$. The paths are attached at the same fixed point on G. We discuss E-cordiality of $C_3(Pn)$, $K_4(P_n)$, $S_4(P_n)$ (shel graph S_4). We show that under certain conditions these graphs are E-cordial.

Key words: *graph, E-cordial, shel graph, C_3, C_4.*

Subject Classification: *05C78.*

1. INTRODUCTION

In 1997 Yilmaz and Cahit introduced weaker version of edge graceful labeling E-cordial labeling [4] Let G be a (p,q). f:E→{0,1}Define f on V by $f(v) = \sum\{f(vu)(vu)\in E(G)\}(mod\ 2)$. The function f is called as E-cordial labeling if $|vf(0)-vf(1)| \leq 1$ and $|ef(0)-ef(1)| \leq 1$ where $v_f(i)$ is the number of vertices labeled with i =0, 1. And $e_f(i)$ is the number of edges labeled with i= 0, 1, We follow the convention that $v_f(0, 1) = (a, b)$ for $v_f(0) = a$ and $vf(1) = b$ further $e_f(0,1) = (x, y)$ for $e_f(0)=x$ and $e_f(1) = y$. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit prove that trees T_n are E-cordial iff for n not congruent to 2(mod 4), K_n are E-cordial iff n not congruent to 2(mod 4), Fans F_n are E-cordial iff for n not congruent to 1(mod 4). Yilmaz and Cahit observe that A graph on n vertices cannot be E-cordial if n is congruent to 2 (mod 4). One should refer Dynamic survey of graph labeling by Joe Gallian [2] for more results on E-cordial graphs. The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary[3] and Dynamic survey of graph labeling by Joe Gillian [2]. The families we discuss are obtained by taking two copies of graph G and join them by t paths of equal length. The paths are attached at the same fixed point on G. we represent these families by G(tPn). We take t = 2 and G = C_3, S_4, K_4, C_4.

2. PRELIMINARIES

2.1 Fusion of vertex. Let G be a (p, q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has at least p-1vertices and q-1 edges. [5].

3. THEOREMS PROVED

3.1 Theorem. G= $C_3(2P_n)$ is e-cordial iff n is odd number.

Proof: The ordinary labeling of G is shown below.

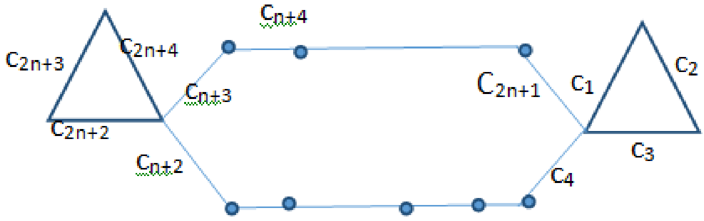


Figure-3.1: ordinary labeling of $C_3(2P_n)$

Corresponding Author: Mukund V.Bapat*
Mukund V. Bapat, Hindale, Devgad, Sindhudurg, Maharashtra India: 416630.

The graph has 2n+2 vertices and 2n+4 edges.

Define f: $E(G)\rightarrow\{0, 1\}$ as follows:
$f(c_i) = 0$ for i=1,3,5,.. and $i \leq n$.
$f(c_i) = 1$ for i =2,4,6,…and $i \leq n-1$.
$f(c_i) =1$ for I = n+1 ,n+2,…, $n+\frac{n+3}{2}$.
$f(c_i) = 0$ for all other $i > \frac{3n+3}{2}$. This gives vertex and label number distribution as $v_f(0,1) = (n+1,n+1)$ and $e_f(0,1) = (n+2,n+2)$.

3.2 Theorem. G= $C_4(2P_n)$ is e-cordial iff n is even number.

Proof: G has 2n + 4 vertices and 2n+6 edges. Define f: $V(E)\rightarrow\{0,1\}$ as follows.

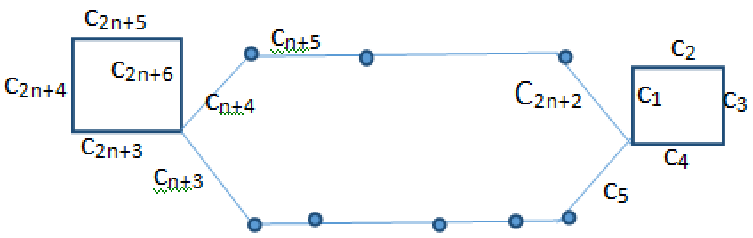


Figure-3.2: ordinary labeling of $C_4(2P_n)$

$f(c_i) = 0$ for i=1,3,5,..and $i\leq n+1$
$f(c_i) = 1$ for i =2,4,6,…and $i\leq n$
$f(c_i) =1$ for i=n+2, n+3,..,n+x+4.
$f(c_i) = 0$ for rest of i. The label number distribution is $v_f(0,1) = (n+2, n+2)$ and $e_f(0,1) = (n+3,n+3)$ as required.

3.3 Theorem. G= $S_4(2P_n)$ is e-cordial iff n is even number.

Proof: G has 2n + 4 vertices and 2n+8 edges. Define f: $V(E)\rightarrow\{0,1\}$ as follows.

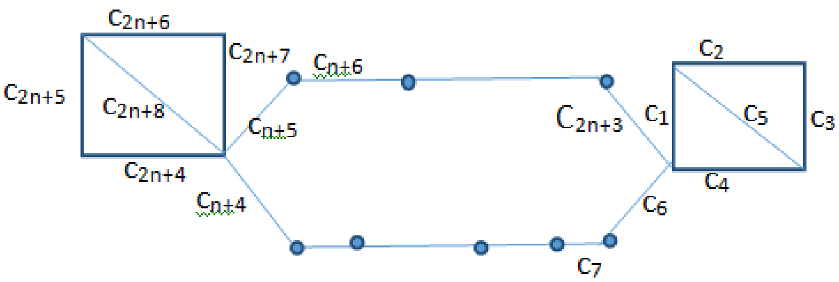


Figure-3.3: ordinary labeling of $S_4(2P_n)$

$f(c_i) =1$ for i=1,2,3,4,5
$f(c_{5+i}) = 0$ for i =2p+1 and p = 0, 1, ,, x-1.
$f(c_i) =1$ for i=7,9,..,n+3
$f(c_{n+3+i}) =1$ for i=1, 2, 3,..x
$f(c_i) = 0$ for all rest of i.

The label number distribution is $v_f(0, 1) = (n+2, n+2)$ and $e_f(0, 1) = (n+4, n+4)$ as required. In the above graph two paths P_n are fused at vertex of degree 2 on S_4. If we change the vertex of fusion on S_4 from degree two vertex to degree 3 vertex then also the same labeling function as above will do. This is shown in ordinary labeling in diagram below and with example in fig.3.5.

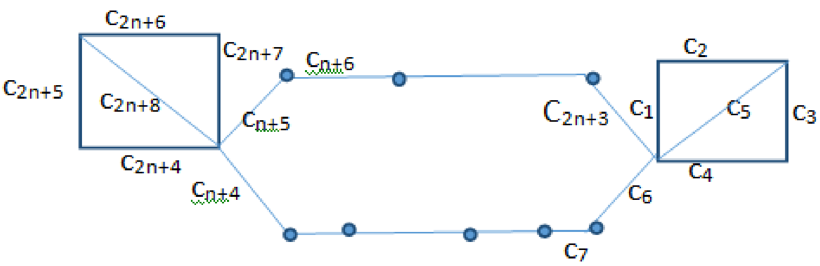


Figure-3.4: ordinary labeling of $S_4(2P_n)$

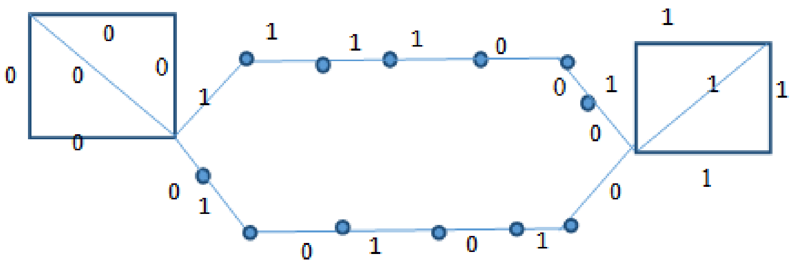


Figure-3.5: labeling of $S_4(2P_8)$

3.4 Theorem. G= $K_4(2P_n)$ is e-cordial iff n is even number.

Proof: G has 2n + 4 vertices and 2n+10 edges. Define f: V(E)→{0,1} as follows.

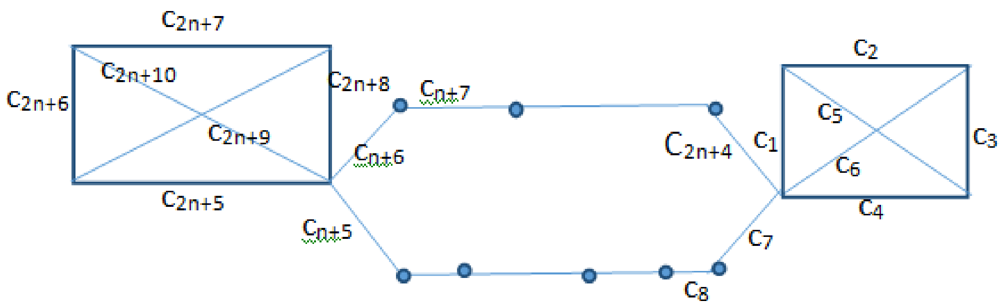


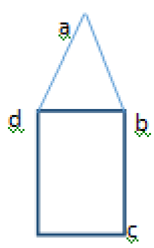
Figure-3.6: ordinary labeling of $k_4(2P_n)$

$f(c_i)$ =1 for i=1,2,3,4,5,6
$f(c_{6+2i})$ = 1 for i =1, 2, …, $(\frac{n}{2}-1)$
$f(c_{6+2i+1})$ =1 for i=0, 1 ,2, ..,$(\frac{n}{2}-1)$
$f(c_{n+5+i})$ =1 for i=1, 2, 3,..x
$f(c_i)$ = 0 for all rest of i.

The label number distribution is $v_f(0,1)$ = (n+2, n+2) and $e_f(0,1)$ = (n+5, n+5) as required.

3.5 Theorem. G= $H(2P_n)$ is e-cordial iff n is odd number where H is house graph on C_4

Proof: G has 2n + 6 vertices and 2n+10 edges. From figure below it follows that we can fuse the paths at three different points, ‘a’, ‘b’ and ‘c’ on H.



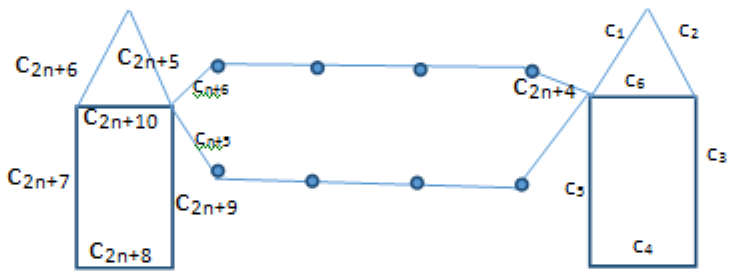


Figure-3.6: House graph H with vertices ‘a’, ‘b’, and ‘c’ **Figure-3.7:** ordinary labeling of $H(2P_n)$

Define f: V(E)→{0,1} as follows.
$f(c_1)$ = 1,
$f(c_2)$ = 0,
$f(c_3)$ = 0,
$f(c_4)$ = 0,
$f(c_5)$ = 1,
$f(c_6)$ = 1
$f(c_{6+i})$=0 for i=1, 3, 5,.., $\frac{n-1}{2}$
$f(c_{7+i})$ = 1 for i =1, 3, …, $\frac{n-1}{2}$
$f(c_{n+5+i})$=1 for i =1, 2, .., (x+3)

$f(c_i)=0$ for all rest of i. In the above graph we have taken double paths from vertex 'a' on one copy of H to vertex 'a' of other copy. In fact we need not restrict ourselves to build bridges from same vertex to same vertex. Above labeling will work to build bridges between cross vertices such as 'a' to 'b', 'b' to 'd' etc. Since there are multiple bridges between two vertices by deleting any bridge or edge there of will not add to the components of graph which is contrary to the established sense of bridge. Conclusions: A new family of graphs is obtained when two copies of given graph are joined by two paths of same length at the same vertex. We have shown that for G = C_3, C_4, S_4 (shel graph) and K_4 the graphs $G(2P_m)$ are E-cordial. We also noticed that for house graph H the two paths need not be between same point on H. We can choose cross points still the cordiality will not affect. This is advantageous.

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