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WEAK AND ALMOST SEMI REGULARITY IN A SPACE

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ABSTRACT

The concept of weak and almost regularity defined Singal and Arya in 1969[5] in a topological space. In this paper weak and almost semi regularity are introduced in a space defined by A.D. Alexandroff and some of their properties are investigated.

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1. INTRODUCTION

Topological spaces have been generalized in several ways. For example Mashhour *et al.* [4] omitted the intersection condition and then Das and Samanta [3] investigated a space without any structural conditions. Perhaps the first to introduce such a generalization was Alexandroff [1], who weakened the union requirements of a topological space. Though every generalization has it's own impact, the generalization by Alexandroff [1] occupies a prominent role in the literature. In this paper weak and almost semi regularity are introduced in a space defined by A.D.Alexandroff and some of their properties are investigated

2. PRELIMINARIES

Definition 1 [1]: A set X is called a space if in it is chosen a system of subsets F satisfying the following axioms

(i) The intersection of a countable number of sets from F is a set in F.
(ii) The union of a finite number of sets from F is a set in F.
(iii) The void set is a set in F.
(iv) The whole set X is a set in F.

Sets of F are called closed sets. Their complementary sets are called open. It is clear that instead of closed sets in the definition of a space, one may put open sets with subject to the conditions of countable summability, finite intersectability and the condition that X and the void set should be open. The collection of such open sets will sometimes be denoted by τ and the space by (X, τ). In general τ is not a topology. By a space we shall always mean an Alexandroff space.

Definition 2 [1]: With every $M \subset X$ we associate its closure cl(M) the intersection of all closed sets containing M and scl(M) the intersection of all semi closed sets containing M.

Note that cl(M) and scl(M) is not necessarily closed and semi closed respectively.

Definition 3[7]: A set N, a subset of X is said to be a semi neighborhood of a point x of X if and only if there exist a semi open set O containing x such that $O \subset N$.

Definition 4[7]: The semi interior of a set A in a space X is define as the union of all semi open sets contained in A and is denoted by s-int (A).

3. WEEKLY SEMI REGULAR AND ALMOST SEMI REGULAR SPACE

Definition 5: Two sets A, B in X are said weakly semi separated if there are two semi open sets U,V such that $A \subset U, B \subset V$ and $A \cap V = B \cap U = \Phi$.

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Definition 6: A subset A of a space X is called regularly semi open if it is the semi interior of it's own closure. A set A is said to be regularly semi closed if it is the semi closure of it's own interior.

It is evident that a set is regularly semi open iff it's complement is regularly semi closed.

Note 1: In a topological space a regular semi open set must be semi open. But this is not true in a space as shown by

Example 1: Let X=R-Q and τ ={X, ∅, G_i} where G_i runs over all countable subsets of R-Q. Then (X, τ) is a space but not a topological space .Clearly in this space for any α ∈X, cl{α}={α}.Let A be set of irrational numbers in [0,1]. Then A is uncountable and so A is not open that is not semi open. But s-int (cl(A))= s-in((A))= ∪{{α}: α ∈ A}=A. So A is regularly semi open but not semi open.

Definition 7: A space X is said to be weakly semi regular if for any weakly separated pair consisting of a regularly semi closed set A and a singleton {x}, there are semi open sets U, V such that A⊂U, x∈V, U∩V=Φ.

Definition 8: A space X is said to be almost semi regular if for any x ϵX and any a regularly closed set A not containing x, there are semi open sets U, V such that A⊂U, x∈V, U∩V=Φ.

Theorem 1: A topological space (X ,σ) is weakly semi regular if and only if for any point x ϵX and any regularly semi open set U such that σ –cl({x}) ⊂U, there is a semi open set V such that xϵV⊂σ –cl(V)⊂ U. Since the semi closure of a set in a space is not necessarily semi closed set, the characterization of weakly semi regularity in a space is somewhat different.

Theorem 2: A space X is weakly semi regular if and only if for each x ϵX and any regularly semi open set U such that x ϵ F⊂U, where F is semi closed set ,there is a semi open set V and a semi closed set F_1, such that xϵV ⊂ F_1 ⊂ U.

Proof: .Let X be weakly semi regular. Let x ϵX and U be a regularly semi open set such that x ϵF⊂U for some semi closed set F. Since U is the semi interior of its closure. U is the union of some semi open sets. So there is a semi open set V such that x ϵV ⊂U. Also X-U⊂X-F, where X-F is semi open .Hence {x} and the regularly semi closed set X-U are weakly semi separated .Then there are semi open sets U_1,V_1 such that xϵU_1, X-U⊂V_1 , U_1∩V_1=Φ.
Therefore x ϵU_1 X-V_1= F_1 U, where F_1 is semi closed.

Conversely let the given condition hold. Let x ϵX and F be a regularly semi closed set such that {x} and F are weakly semi separated. So there is a semi open set V_1 such that F⊂V_1 and x does not belongs to V_1, Therefore x ϵX-V1⊂X-F, where X-V_1 is semi closed and X-F is regularly semi open. Now by the given condition there is a semi open set U and a semi closed set F_1 such that x ϵU⊂F_1⊂X-F. Hence x ϵ U, F⊂X-F_1=V where U, V are semi open and U∩V= U∩(X-F_1) = Φ.

Theorem 3: A weakly semi regular T_1 space is almost semi regular.

Proof is simple and so omitted.

Theorem 4: For a space (X, τ) the following are equivalent.

(a) (X ,τ) is almost semi regular.

(b) For each point x ϵX and each regularly semi open set V containing x ,there is a regularly semi open set U and a semi closed set F such that x ϵU⊂F⊂V,

(c) For each point x ϵX and each semi neighbourhood M of x, there is a regularly semi open neighbourhood V of x and a semi closed set F such that x ϵV⊂F⊂s-int(cl(M)).

(d) For each point x ϵX and each semi neighbourhood M of x, there is a semi open neighbourhood V of x and a semi closed set F such that x ϵV⊂F⊂s- int (cl (M))..

(e) For every regularly semi closed set F and each point x not belong to F, there exist semi open sets U, V and semi closed sets F_1, F_2 such that x ϵU⊂F_1, F⊂V⊂F_2 and F_1∩F_2=Φ.

Proof.

(a)=>(b): Let x ϵ X and U be regularly semi open set containing x. Then U^C is regularly semi closed set not containing x. Therefore there exist semi open sets U_1, U_2 such that xϵU_1, U^c ⊂U_2, U_1 U_2 = Φ. Then x ϵU_1 s-int (cl(U_1)). cl(U_1) U_2^c U. Take s-int(cl(U_1)).=V and U_2^c=F. Then V is regularly open, F is semi closed that x ϵV F U.

(b)=>(c): The proof is obvious.

(c) =>(d): Since every regularly semi open set is the union of some semi open sets, the result follows.

(d) =>(e): Let F be regularly semi closed set and x does not belongs to F. Then F^c is a semi neighborhood of x. Therefore there is a semi open set V_1 and a semi closed set $F_1^{/}$ such that $x\in V_1\subset F_1\subset F^c$. Again since V_1 is also a semi neighborhood of x, there is a semi open set U and a semi closed set F_1 such that $x\in U\subset F_1\subset V_1$. Take $V=(F_1^{/})^c$ and $F_2=V_1^c$. Then $F\subset V\subset F_2$ where V is semi open and F_2 is semi closed and $F_1\cap F_2 \subset V_1\cap V_2=\Phi$.

(e) =>(a): The proof is obvious.

Theorem 5: Every regularly semi open subspace of an almost semi regular space is almost semi regular.

Proof is simple.

Definition 9: A set A in a space X is said to be almost semi bi compact if every semi open cover of A has a finite sub collection whose closures cover A.

Definition 10: Two sets A, B in X are said strongly semi separated if there are two semi open sets U, V such that $A \subset U$, $B \subset V$ and $V\cap U=\Phi$.

Theorem 6: In an almost semi regular space, every pair consisting of an almost semi bicompact set and a disjoined regularly semi closed set can be strongly semi separated.

Proof: Let (X, τ) be an almost semi regular space. Let A be an almost semi bi compact subset of X and B be a regularly semi closed set with $A\cap B=\Phi$. Since X is almost semi regular, for each $x\in A$, there are semi open sets U_x, V_x and semi closed sets E_x, F_x such that $x \in U_x\subset E_x$, $B\subset V_x \subset F_x$, $E_x \cap F_x = \Phi$. Now $\{U_x \cap A: x \in A\}$ is a relatively semi open cover of the almost semi bi compact set A and so there is a finite subfamily $\{U_{xi} \cap A: I = 1, 2,\ldots.n\}$ whose closures cover A. Since the closures of $U_{xi} \cap A$ in A.
$cl(U_{xi} \cap A) \cap A\subset cl(U_{xi})\cap A\subset cl(U_{xi})\subset E_{xi.}$ Hence $A\subset \cup\{E_{xi.}: i = 1, 2,..n\}$. Let $U= \{\cap V_{xi} :i=1,2,..n\}$, $V=X-\{\cap F_{xi} :i= 1,2,..n\}$. Then $A \subset \{\cup E_{xi.} :i=1,2,..n\} \subset \{\cup (F_{xi})^c: i = 1, 2,..n\}= X-\{\cap F_{xi}: i= 1, 2,..n\} = V$. and $B \subset U$. Also U and V are semi open sets and $U\cap V=\Phi$.

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