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STOCHASTIC ANALYSIS OF RELIABILITY OF POWER CABLES USED AS A TWO UNIT COLD STANDBY SYSTEM IN METRO RAILWAYS

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ABSTRACT

Today's mass-rapid-transportation in the cities around the world centres dependable and failure-resistant metro network which uses power cables for energising the metro system substations to further feed motor drives etc.. The present study of the metro network system necessitates carrying out reliability and profit analysis of this power energizing system through power cables. These cables are laid in loops for feeding supply to substations from each station. Initially, one cable is in operation and the other cable of same voltage capacity is kept as cold standby. If one cable fails then the other cable is made operative with very short time delay. Inspection has been carried out at each failure to check whether the system is repairable or irreparable. The data depicting the real failure situations have been collected and various measures of system effectiveness are obtained using semi-Markov processes and regenerative point technique. The Graphical analysis has also been made for several cases which enable to draw various important conclusions with respect to profit analysis of the present system.

Keywords: *Power Cables, system effectiveness, profit analysis, semi-Markov processes, regenerative point technique.*

1. INTRODUCTION

Predominantly reliability study results in taking recourse for effective utilization and timely maintenance of different industrial engineering systems. A lot of work on reliability and profit analysis has been carried out on different types of standby redundant systems by various researchers including [1-4]. B. Parashar and G. Taneja [5] analyzed a PLC system based upon real data collected from industry. Z. Zhang *et al.* [6] and G. Taneja [8] discussed aspects of maintenance and variation in demand of different systems. B. Parashar & A. Naithani [7&10] studied a system of ID fans in a thermal plant wherein different conditions making the system work at full/reduced capacity discussed. Electricity is heart-throb of modern day human life. Flow of Electrical Energy is through medium namely conductors. Cables are insulation covered conductors, thus, become energy transportation nerves and to maintain the system to work round the clock efficiently, it becomes imperative that the failure whether due to cable manufacturing defects or some system malfunctioning, gets reduced to lowest level [9]. However, to assess and predict the failure or breakdown, mathematical modelling tools come handy. The probability analysis is thought of to critically delve into the problem of "FAILURE" and achieve near reality solution. The reduction in break-downs to a minimum level contributes to higher availability of the Electrical System in considered Metro trains and transportation becomes smooth and public at large gets motivated with better confidence. Consequently, the time loss in reworking and maintaining the system to bring back to life is monetarily rewarded.

In the metros network, various power cables with rated capacities of 220, 132, 66, 33, 25 KV are used in supplying power for the functioning of the system. The failure data of eight years have been collected for the power cable with the capacity of 33KV. These cables are laid in loops for feeding supply to substation at each station. So to avoid the damage of cables due to various factors that include manufacturing defects, external defects etc., maintenance of the power cables are of prime concern as it has direct impact on the functioning of the metro railways system. The aim is to analyse the reliability of the system by considering a two unit cold standby identical parallel power cable of 33KV capacity. The following measures of the system effectiveness are analyzed by making use of semi-Markov processes and regenerative point technique:

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- Mean Time to System Failure
- The Steady State Availability Analysis
- Busy period of the repairman for Inspection, repair and replacements of the power cable at $t = 0$
- Expected number of visits by the Repairman at $t = 0$
- Expected number of replacements
- Expected profit incurred to the system

2. SYMBOLS AND NOTATIONS

O	Operative state of the power cable
O_{cs}	Cold standby unit of the power cable
λ	Constant failure rate of the operative power cable
p	Probability of failure (repairable) of the unit
q	Probability of replacement (irreparable) of the unit
r	Probability of the unit found Ok
F_{ui}	Unit is under inspection in case of failure
F_r	Unit is under repair in case of major failure
F_{rp}	Unit is under replacement
F_{Ui}	Continuation of inspection of failure from its previous state.
F_R	Continuation of repair of failure from its previous state.
F_{RP}	Continuation of replacement from its previous state.
F_{wi}	Failed unit waiting for inspection
γ	Constant rate of inspection
α	Constant rate of repairable failure
β	Constant rate of replacement failure
$g(t), G(t)$	p.d.f. and c.d.f. of repair time of unit having major failure
$h(t), H(t)$	p.d.f. and c.d.f. of replacement time of unit having major failure
$h_1(t), H_1(t)$	p.d.f. and c.d.f. of inspection time of unit having failure
$w(t), W(t)$	p.d.f. and c.d.f. of waiting time of a failed unit
$q_{ij}(t), Q_{ij}(t)$	p.d.f. and c.d.f. of first passage time from a regenerative state i to j or to a failed state j without visiting any other regenerative state in $(0, t]$
p_{ij}	Transition probability from regenerative state i to regenerative state j
$M_i(t)$	Probability that system up initially in regenerative state i is up at time t without passing through any other regenerative state
m_{ij}	Contribution to mean sojourn time in regenerative state i before transiting to regenerative state j without visiting to any other state
$\mu_i(t)$	Mean sojourn time in regenerative state before transiting to any other state
©, $\otimes$	Symbols for Laplace and Laplace Stieltje's convolution
*, *	Symbols for Laplace and Laplace Stieltje's transforms
C_0	Revenue per unit up time
C_1	Represent cost per unit up time for which the repairman is busy for repair
C_2	Cost per unit up time for which the repairman is busy for replacement
C_3	Cost per unit up time for which the repairman is busy for inspection
C_4	Cost per visit of the repairman
C_5	Cost per unit replacement
A_0	Steady state availability of the system

B_0	Busy period of the repairman for repair at $t=0$
BR_0	Busy period of the repairman for replacement at $t=0$
BR_{i0}	Busy period of the repairman for inspection at $t=0$
V_0	Repairman expected number of visits at $t=0$
R_0	Expected number of replacements

3. DATA SUMMARY

The following values have been obtained from the collected data:

- Estimated value of failure rate (λ) =.000015 per hour
- Estimated value of repair rate (α) =.067per hour
- Estimated value of replacement rate (β) =.002 per hour
- Estimated value of inspection rate (γ) =1 per hour
- Probability of repairable failure (p) =.69
- Probability of replaceable failure (q) =.16
- Probability of unit found ok (r) =.15
- Expected cost of Revenue up time (C_0) = 30000
- Expected cost of repairman during repair (C_1) = 3000
- Expected cost of repairman during replacement (C_2) = 250
- Expected cost of repairman per visit during inspection (C_3) = 600 per hour
- Expected cost of repairman per visit (C_4) = 500 per hour
- Expected cost of cable replacement (C_5) = 150000

(All costs are in INR)

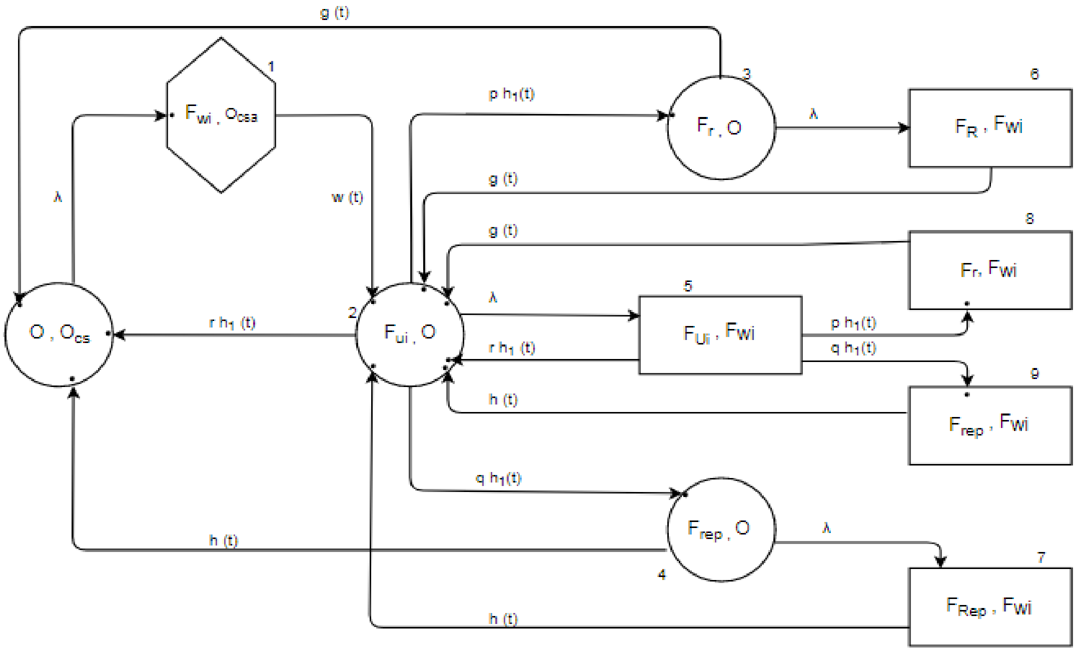
4. MODEL DESCRIPTION AND ASSUMPTIONS

The following assumptions have been used in probabilistic model:

1) Initially the system is operative at full capacity with two power cables, one is operative and other one is used as cold standby
2) The two power cables are identical and constitute a parallel system.
3) If one unit is failed, standby unit takes some time to become operative.
4) As the unit fails, it is undertaken for inspection.
5) The failure rate of cold standby unit is same as that of operative unit.
6) Failure, repair and inspection times are assumed to follow an exponential and general time distribution respectively.
7) The repairman is available for inspection and is common for repair as well as for replacement.
8) The repaired unit works as good as new one.
9) The system will be in the failed state when both units are not working.
10) All the random variables are independent.

5. TRANSITION DIAGRAM

A state transition diagram showing all the possible states of the system under consideration as shown in the Fig.1. The epochs of entry into states 0, 1, 2, 3, 4, 8, 9 are regeneration points and thus these are called regenerative states. The states 0, 2, 3, 4 are up states where as 5, 6, 7, 8, 9 are failed states. State 1 is down state.



• Regeneration Point

Down State

Operative State for the system

Failed State for the system

Figure-1: State Transition Diagram

6. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The steady-state transition probabilities are

$$dQ_{01}(t) = \lambda e^{-\lambda t} dt,$$
$$dQ_{12}(t) = w(t)dt,$$
$$dQ_{20}(t) = re^{-\lambda t} h_1(t)dt,$$
$$dQ_{23}(t) = pe^{-\lambda t} h_1(t)dt,$$
$$dQ_{24}(t) = qe^{-\lambda t} h_1(t)dt,$$
$$dQ_{25}(t) = \lambda e^{-\lambda t}\, \bar{H}_1(t)dt,$$
$$dQ_{22}^{5}(t) = (\lambda e^{-\lambda t} \copyright \mathrm{r})\mathrm{h}_1(t)dt,$$
$$dQ_{28}^{5}(t) = p(1-e^{-\lambda t})\mathrm{h}_1(t)dt,$$
$$dQ_{29}^{5}(t) = q(1-e^{-\lambda t})h_1(t)dt,$$
$$dQ_{30}(t) = e^{-\lambda t}\mathrm{g}(t)dt,$$
$$dQ_{36}(t) = \lambda e^{-\lambda t}\, \bar{G}(t)dt,$$
$$dQ_{32}^{6}(t) = (\lambda e^{-\lambda t} \copyright 1)g(t)dt,$$
$$dQ_{40}(t) = e^{-\lambda t}h(t)dt,$$
$$dQ_{47}(t) = \lambda e^{-\lambda t}\, \bar{H}(t)dt,$$
$$dQ_{42}^{7}(t) = (\lambda e^{-\lambda t} \copyright 1)\mathrm{h}(t)dt,$$
$$dQ_{82}(t) = g(t)dt,$$
$$dQ_{92}(t) = h(t)dt \qquad (1)$$

The non-zero elements $p_{ij} = \lim_{s\to 0} q_{ij}^*(s)$ can be obtained as

$$\begin{aligned}
p_{01} &= 1,\\
p_{12} &= 1,\\
p_{20} &= rh_1^*(\lambda),\\
p_{23} &= ph_1^*(\lambda),\\
p_{24} &= qh_1^*(\lambda),\\
p_{22}^5 &= r[1-h_1^*(\lambda)],\\
p_{25} &= [1-h_1^*(\lambda)],\\
p_{28}^5 &= p[1-h^*(\lambda)],\\
p_{29}^5 &= q[1-h_1^*(\lambda)],\\
p_{32}^6 &= [1-g_1^*(\lambda)],\\
p_{30} &= g^*(\lambda),\\
p_{36} &= [1-g^*(\lambda)],\\
p_{40} &= h^*(\lambda),\\
p_{42}^7 &= [1-h^*(\lambda)],\\
p_{47} &= [1-h^*(\lambda)],\\
p_{82} &= 1,\\
p_{92} &= 1.
\end{aligned} \tag{2}$$

From the above steady-state transition probabilities, it can be verified that

$$\begin{aligned}
&p_{01} = p_{12} = p_{82} = p_{92} = 1,\\
&p_{20} + p_{23} + p_{24} + p_{25} = p_{20} + p_{23} + p_{24} + p_{22}^5 + p_{28}^5 + p_{29}^5 = 1,\\
&p_{30} + p_{36} = p_{30} + p_{32}^6 = p_{40} + p_{47} = p_{40} + p_{42}^7 = 1.
\end{aligned} \tag{3}$$

The mean sojourn time (μ_i) in the regenerative state ‘i’ is

$$\begin{aligned}
\mu_0 &= \frac{1}{\lambda},\\
\mu_1 &= \frac{1-h_1^*(\lambda)}{\lambda},\\
\mu_2 &= \int_0^\infty tw(t)dt.\\
\mu_3 &= \frac{1-g^*(\lambda)}{\lambda},\\
\mu_4 &= \frac{1-h^*(\lambda)}{\lambda},\\
\mu_8 &= \int_0^\infty tg(t)dt,\\
\mu_9 &= \int_0^\infty th(t)dt,
\end{aligned} \tag{4}$$

The unconditional mean time taken by the system to transit for any regenerative state j when it is counted from the epoch of entrance into state i is mathematically stated as

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*'}(0). \tag{5}$$

Thus,

$$m_{01} = \mu_0,$$
$$m_{12} = \mu_2,$$
$$m_{20} + m_{23} + m_{24} + m_{25} = \mu_4,$$
$$m_{30} + m_{36} = \mu_3,$$
$$m_{30} + m_{32}^6 = m_{82} = \mu_8,$$
$$m_{40} + m_{47} = m_{40} + m_{42}^7 = m_{92} = \mu_9,$$
$$m_{20} + m_{23} + m_{24} + m_{22}^5 + m_{28}^5 + m_{29}^5 = \kappa_1. \tag{6}$$

Where,

$$\kappa_1 = \int_0^\infty \bar{H}_1(t)dt = \int_0^\infty th_1(t)dt,$$
$$\mu_2 = \int_0^\infty \bar{W}(t)dt = \int_0^\infty tw(t)dt,$$
$$\mu_8 = \int_0^\infty \bar{G}(t)dt = \int_0^\infty tg(t)dt,$$
$$\mu_9 = \int_0^\infty \bar{H}(t)dt = \int_0^\infty th(t)dt.$$

7. MEASURES OF SYSTEM EFFECTIVENESS

7.1. Mean Time to System Failure (MTSF). Taking failed state of the system as absorbing state, Let $\phi_i(t)$, be the cumulative distribution function of first passage time from i^{th} state to a failed state where i =0,1,2,3,4,5,6. MTSF of the system can be determined by the following recursive relations for $\phi_i(t)$

$$\phi_0(t) = Q_{01}(t) \otimes \phi_1(t),$$
$$\phi_1(t) = Q_{12}(t) \otimes \phi_2(t),$$
$$\phi_2(t) = Q_{23}(t) \otimes \phi_3(t) + Q_{24}(t) \otimes \phi_4(t) + Q_{20}(t) \otimes \phi_0(t) + Q_{25}(t).$$
$$\phi_3(t) = Q_{30}(t) \otimes \phi_0(t) + Q_{36}(t).$$
$$\phi_4(t) = Q_{40}(t) \otimes \phi_0(t) + Q_{47}(t). \tag{7}$$

Taking Laplace-Stieltjes Transform (L.S.T.) of the above relations given by (7) on both the sides and solving them for $\phi_0^{**}(s)$, we obtain

$\phi_0^{**}(s) = \dfrac{N_0(s)}{D_0(s)}$, $\phi_0^{**}(s)$ represents the Laplace-Stieltjes Transform of $\phi_0(s)$.

The mean time to system failure (MTSF) for the present system starts from the state '0' is

$$MTSF = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{D_0^{'}(0) - N_0^{'}(0)}{D_0(0)} = \frac{N}{D},$$

Where,

$$N = \mu_0 + \mu_2 + \mu_4 + p_{23}\mu_3 + p_{24}\mu_9,$$
$$D = 1 - p_{20} - p_{23}p_{30} - p_{24}p_{40}. \tag{8}$$

7.2. Availability Analysis

Let $A_i(t)$ be the probability that the system is working at instant time t, given that the system entered regenerative state i at $t=0$. Then,

$$\begin{aligned}
&A_0(t) = M_0(t) + q_{01}(t)© A_1(t),\\
&A_1(t) = q_{12}(t)© A_2(t),\\
&A_2(t) = M_2(t) + q_{20}(t)© \mathrm{A}_0(t) + q_{23}(t)© A_3(t) + q_{24}(t)© \mathrm{A}_4(t) + q_{22}^5(t)© \mathrm{A}_2(t) + q_{28}^5(\mathrm{t})© \mathrm{A}_8\\
&\qquad + q_{29}^5(t)© \mathrm{A}_9(t),\\
&A_3(t) = M_3(t) + q_{32}^6(t)© \mathrm{A}_2(t) + q_{30}(t)© A_0(t),\\
&A_4(t) = M_4(t) + q_{42}^7(t)© \mathrm{A}_2(t) + q_{40}(t)© A_0(t),\\
&A_8(t) = q_{82}(t)© \mathrm{A}_2(t),\\
&A_9(t) = q_{92}(t)© \mathrm{A}_2(t).
\end{aligned} \tag{9}$$

Where,

$$M_0(t) = e^{-\lambda t},\ M_2(t) = e^{-\lambda t}\ \bar{H}_1(t),\ M_3(t) = e^{-\lambda t}\ \bar{G}(t),\ M_4(t) = e^{-\lambda t}\ \bar{H}(t).$$

Taking Laplace transforms of above equations and solving them for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)},$$

The availability A_0 in steady-state of the present system is define as

$$A_0 = \lim_{s\to 0} s.\frac{N_1(s)}{D_1(s)} = \frac{N_1(0)}{D_1^{'}(0)} = \frac{N_1}{D_1},$$

Where,

$$\begin{aligned}
N_1 &= \mu_0(1 - p_{25} - p_{32}^6 p_{23} - p_{24} p_{42}^7) + \mu_1 + \mu_3 p_{23} + \mu_4 p_{24},\\
D_1 &= (\mu_0 + \mu_2)(p_{20} + p_{30} p_{23} + p_{40} p_{24}) + \kappa_1 + \mu_8(p_{23} + p_{28}^5) + \mu_9(p_{24} + p_{29}^5).
\end{aligned} \tag{10}$$

7.3. Busy Period Analysis of Repairman (Repair only)

The total fraction of the time $B_0^*(s)$ for which the system is under repair of the repairman, is calculated by the following recursive relation

$$\begin{aligned}
&B_0(t) = q_{01}(t)© B_1(t),\\
&B_1(t) = q_{12}(t)© B_2(t),\\
&B_2(t) = q_{20}(t)© B_0(t) + q_{23}(t)© B_3(t) + q_{24}(t)© B_4(t) + q_{22}^5(t)© \mathrm{B}_2(t) + q_{28}^5(\mathrm{t})© \mathrm{B}_8(t)\\
&\qquad + q_{29}^5(t)© \mathrm{B}_9(t),\\
&B_3(t) = W_3(t) + q_{32}^6(t)© \mathrm{B}_2(t) + q_{30}(t)© B_0(t),\\
&B_4(t) = q_{42}^7(t)© \mathrm{B}_2(t) + q_{40}(t)© B_0(t),\\
&B_8(t) = W_8(t) + q_{82}(t)© B_2(t),\\
&B_9(t) = q_{92}(t)© B_2(t),
\end{aligned} \tag{11}$$

Where,

$$W_3(t) = \bar{G}(t), W_8(t) = \bar{G}(t).$$

Taking Laplace transforms of above equations and solving them for $B_0^*(s)$, we get

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)},$$

In steady-state, the busy period of the repairman is given by,

$$B_0 = \lim_{s\to 0} s.B_0^*(s) = \frac{N_2}{D_1},$$

Where,

$$N_2 = \mu_8(p_{01}p_{12}p_{23} + p_{01}p_{12}p_{28}^5), \tag{12}$$

and $D_1(s)$ is already calculated.

7.4. Busy Period Analysis of Repairman (Replacement only)

$$BR_0(t) = q_{01}(t)© BR_1(t),$$
$$BR_1(t) = q_{12}(t)© BR_2(t),$$
$$BR_2(t) = q_{20}(t)© BR_0(t) + q_{23}(t)© BR_3(t) + q_{24}(t)© BR_4(t) + q_{22}^5(t)© \mathrm{BR}_2(t) + q_{28}^5(\mathrm{t})© \mathrm{BR}_8(t)$$
$$+ q_{29}^5(t)© \mathrm{BR}_9(t),$$
$$BR_3(t) = q_{32}^6(t)© \mathrm{BR}_2(t) + q_{30}(t)© BR_0(t),$$
$$BR_4(t) = W_4(t) + q_{42}^7(t)© \mathrm{BR}_2(t) + q_{40}(t)© BR_0(t),$$
$$BR_8(t) = q_{82}(t)© BR_2(t),$$
$$BR_9(t) = W_9(t) + q_{92}(t)© BR_2(t), \tag{13}$$

Where,

$$W_4(t) = \bar{H}(t), W_9(t) = \bar{H}(t).$$

Taking Laplace transforms of above equations and solving them for $BR_0^*(s)$, we get

$$BR_0^*(s) = \frac{N_3(s)}{D_1(s)},$$

In steady-state, the busy period of the repairman is given by,

$$BR_0 = \lim_{s\to 0} s.BR_0^*(s) = \frac{N_3}{D_1},$$

Where,

$$N_3 = (p_{24} + p_{29}^5)(p_{01}p_{12}\mu_9), \tag{14}$$

and $D_1(s)$ is already specified.

7.5. Busy Period Analysis of Repairman (Inspection time only)

$$BR_{i0}(t) = q_{01}(t)© BR_{i1}(t),$$
$$BR_{i1}(t) = q_{12}(t)© BR_{i2}(t),$$
$$BR_{i2}(t) = W_2(t) + q_{20}(t)© BR_{i0}(t) + q_{23}(t)© BR_{i3}(t) + q_{24}(t)© BR_{i4}(t) + q_{22}^5(t)© \mathrm{BR}_{\mathrm{i2}}(t)$$
$$+ q_{28}^5(\mathrm{t})© \mathrm{BR}_{\mathrm{i8}}(t) + q_{29}^5(t)© \mathrm{BR}_{\mathrm{i9}}(t),$$
$$BR_{i3}(t) = q_{32}^6(t)© \mathrm{BR}_{\mathrm{i2}}(t) + q_{30}(t)© BR_{i0}(t),$$
$$BR_{i4}(t) = q_{42}^7(t)© \mathrm{BR}_{\mathrm{i2}}(t) + q_{40}(t)© BR_{i0}(t),$$
$$BR_{i8}(t) = q_{82}(t)© BR_{i2}(t),$$
$$BR_{i9}(t) = q_{92}(t)© BR_{i2}(t). \tag{15}$$

Where,

$$W_2(t) = \bar{H}_1(t).$$

Taking Laplace transforms of above equations and solving them for $BR_0^*(s)$, we get

$$BR_{i0}^*(s) = \frac{N_4(s)}{D_1(s)},$$

In steady-state, the busy period of the repairman is given by,

$$BR_{i0} = \lim_{s\to 0} s.BR_{i0}^*(s) = \frac{N_4}{D_1},$$

Where,

$$N_4 = \kappa_1 p_{01} p_{12}, \tag{16}$$

and $D_1(s)$ is already specified.

7.6. Expected Number of Visits by the Repairman

$$\begin{aligned}
V_0(t) &= Q_{01}(t) \otimes [1 + V_1(t)],\\
V_1(t) &= Q_{12}(t) \otimes V_2(t),\\
V_2(t) &= Q_{23}(t) \otimes V_3(t) + Q_{24}(t) \otimes V_4(t) + Q_{22}^5(t) \otimes V_2(t) + Q_{28}^5(t) \otimes V_8(t)\\
&\quad + Q_{29}^5(t) \otimes V_9(t) + Q_{20}(t) \otimes V_0(t),\\
V_3(t) &= Q_{32}^6(t) \otimes V_2(t) + Q_{30}(t) \otimes V_0(t),\\
V_4(t) &= Q_{42}^7(t) \otimes V_2(t) + Q_{40}(t) \otimes V_0(t),\\
V_8(t) &= Q_{82}(t) \otimes V_2(t),\\
V_9(t) &= Q_{92}(t) \otimes V_2(t).
\end{aligned} \tag{17}$$

Taking Laplace-Stieltjes Transforms (L.S.T.) of above equations on both the sides and solving them for $V_0^{**}(s)$, we get

$$V_0^{**}(s) = \frac{N_5(s)}{D_1(s)},$$

In steady-state, the expected number of visits per unit time by the repairman is given by,

$$V_0 = \lim_{s\to 0} s.V_0^{**}(s) = \frac{N_5}{D_1},$$

Where,

$$N_5 = p_{01}(p_{92}p_{29}^5 - p_{82}p_{28}^5 + (1 - p_{22}^5) - p_{23}p_{32}^6 - p_{24}p_{42}^7),$$

and $D_1(s)$ is already specified. (18)

7.7. Expected Number of Replacements.

$$\begin{aligned}
R_0(t) &= Q_{01}(t) \otimes R_1(t),\\
R_1(t) &= Q_{12}(t) \otimes R_2(t),\\
R_2(t) &= Q_{23}(t) \otimes R_3(t) + Q_{24}(t) \otimes [1 + R_4(t)] + Q_{22}^5(t) \otimes R_2(t) + Q_{28}^5(t) \otimes R_8(t)\\
&\quad + Q_{29}^5(t) \otimes [1 + R_9(t)] + Q_{20}(t) \otimes R_0(t),\\
R_3(t) &= Q_{32}^6(t) \otimes R_2(t) + Q_{30}(t) \otimes R_0(t),\\
R_4(t) &= Q_{42}^7(t) \otimes R_2(t) + Q_{40}(t) \otimes R_0(t),\\
R_8(t) &= Q_{82}(t) \otimes R_2(t),\\
R_9(t) &= Q_{92}(t) \otimes R_2(t).
\end{aligned} \tag{19}$$

Taking Laplace-Stieltjes Transforms (L.S.T.) of above equations on both the sides and solving them for $R_0^{**}(s)$, we get

$$R_0^{**}(s) = \frac{N_6(s)}{D_1(s)},$$

In steady-state, the expected number of replacements is given by,

$$R_0 = \lim_{s\to 0} s.R_0^{**}(s) = \frac{N_6}{D_1},$$

Where,

$$N_6 = (p_{24} + p_{29}^5)p_{01}p_{12}, \tag{20}$$

and $D_1(s)$ is already specified.

8. PROFIT ANALYSIS

At steady state, the expected profit P can be calculated by expected total revenue in $(0,t]$ minus expected total costs of repair, replacement and inspection in $(0,t]$ minus expected cost of visits by repairman in $(0,t]$ minus the cost of number of replacements in $(0,t]$. Hence, the overall profit in $(0,t]$ is given by

$$P = C_0A_0 - C_1B_0 - C_2BR_0 - C_3BR_{i0} - C_4V_0 - C_5R_0, \tag{21}$$

9. PARTICULAR CASE

For the particular case, the inspection, repair and replacement rate are assumed to be exponentially distributed, let us take $g(t) = \alpha e^{-\alpha t}; h(t) = \beta e^{-\beta t}; h_1(t) = \gamma e^{-\gamma t}$

Using the values, as estimated in section 3, of various probabilities and repairable/replaceable rates the following measures of system effectiveness are obtained as:

Mean Time to System Failure: 49433849.5338 hours

Availability of the system A_0 : .9999760

Expected busy period of the repairman for repairable failure B_0 : .0001545

Expected busy period of the repairman for replaceable failure BR_0 : .001200

Expected busy period of the repairman for inspection of a failure BR_{i0} : .000155

Expected number of visits by the repairman V_0 : .000015

Expected number of replacement R_0 : .0000024

The Graphical Analysis

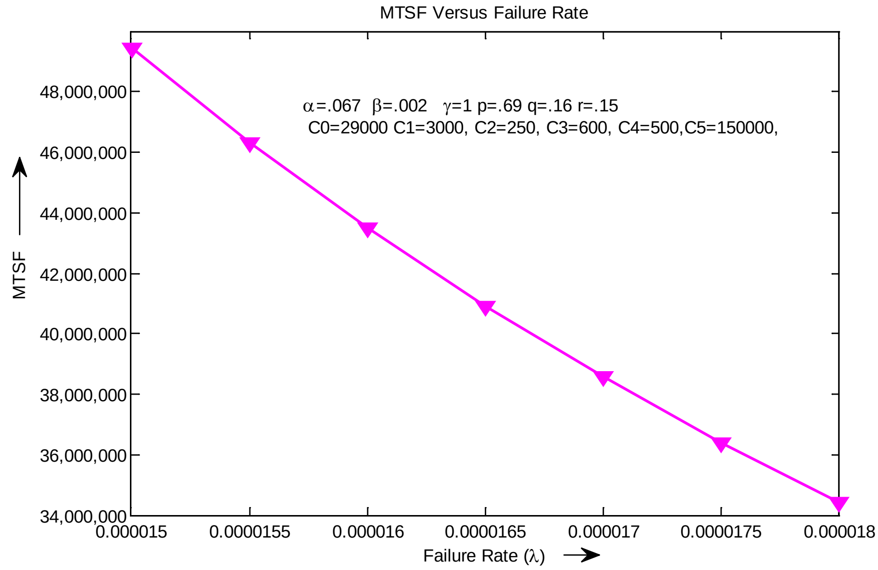


Figure-2: MTSF versus Failure rate

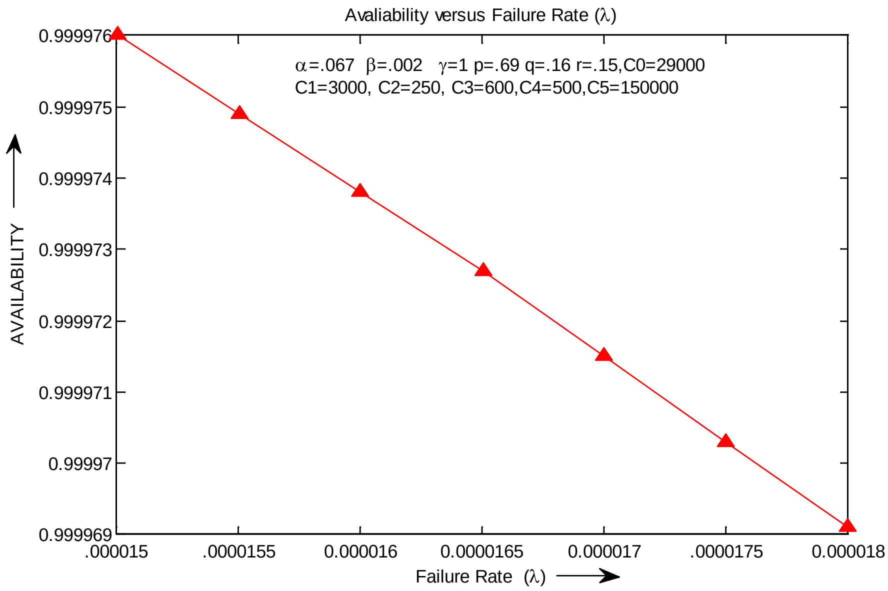


Figure-3: Availability versus Failure rate

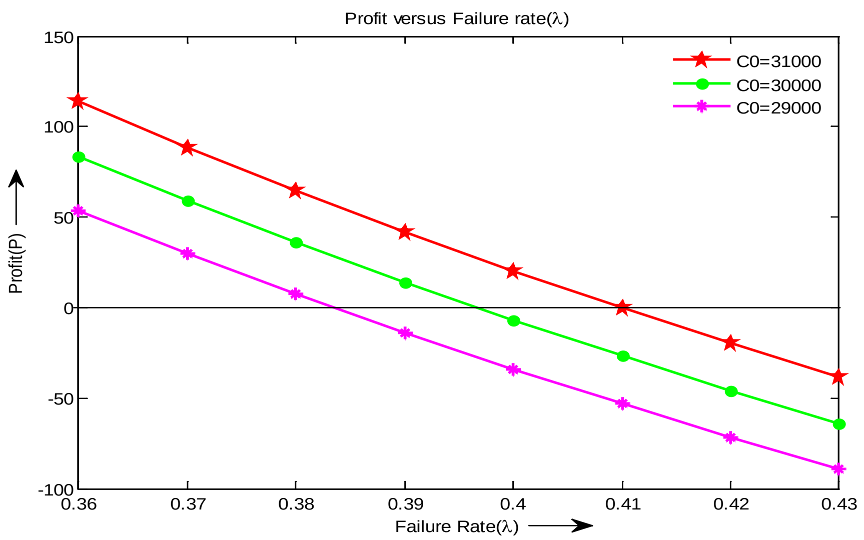


Figure-4: Profit (P) Versus Failure rate λ for different values of the revenue per unit up time (C_0)

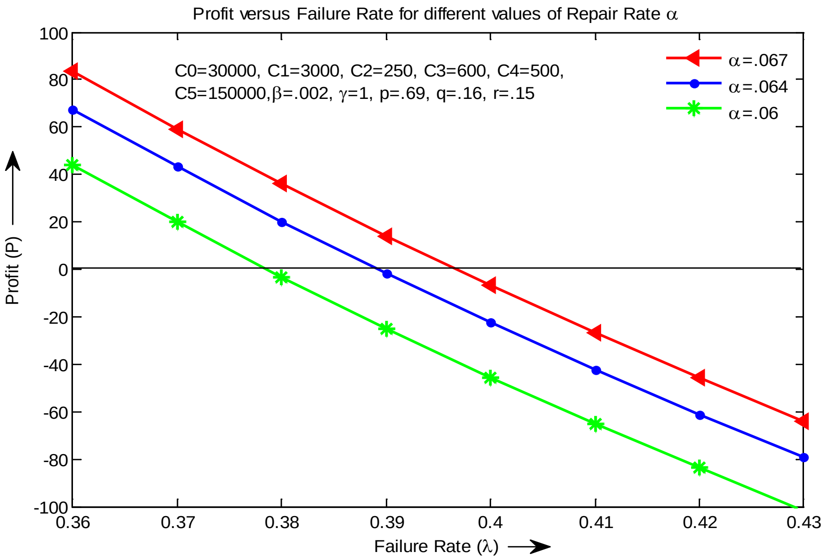


Figure-5: Profit (P) versus Failure rate λ for different values of Repair Rate (α)

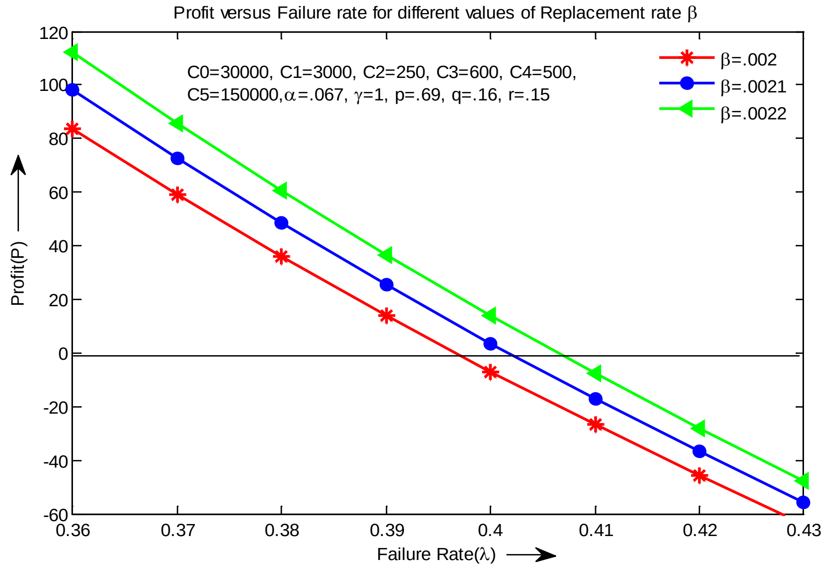


Figure-6: Profit (P) versus Failure rate λ for different values of Replacement Rate (β)

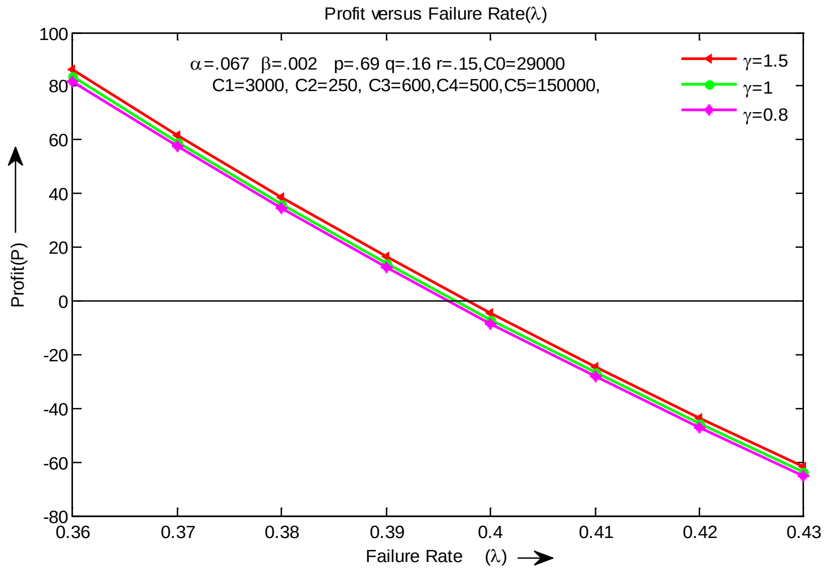


Figure-7: Profit (P) versus Failure rate λ for different values of Inspection Rate (γ)

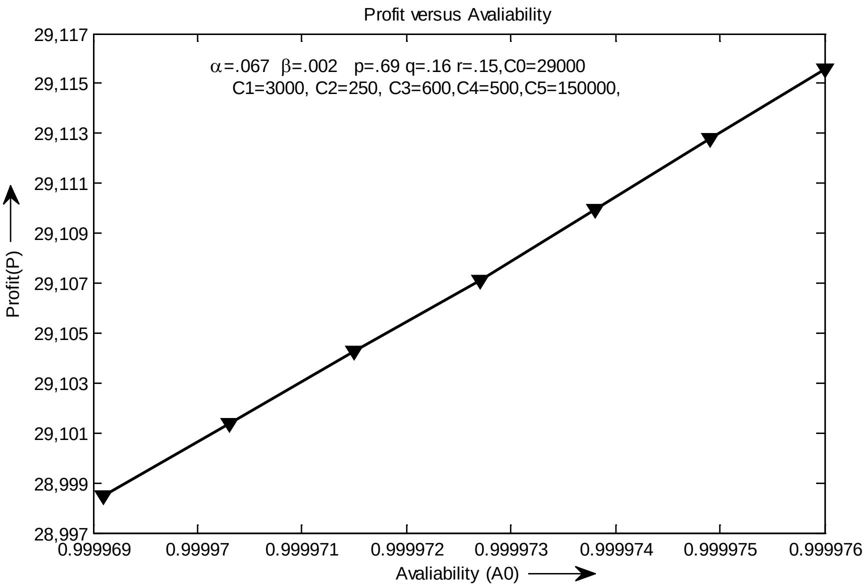


Figure-8: Profit (P) versus Availability (A_0)

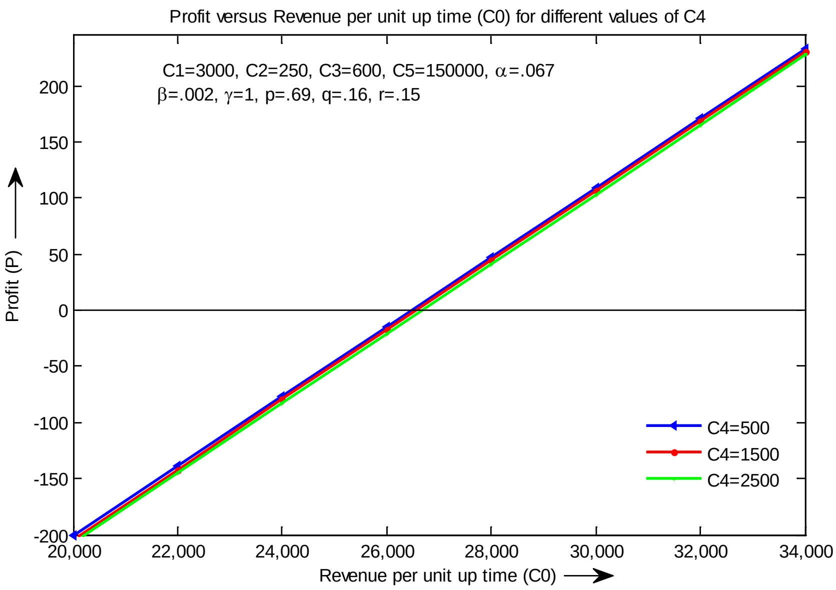


Figure-9: Profit (P) versus Revenue per unit up time for different values of cost (C4)

The following interpretations have been made from the graphs.

i. Figure 2 depicts the decrease of MTSF with respect to the increase of failure rate λ .
ii. Figure 3 displays the Availability of the system decreases with increase in the failure rate λ .
iii. Figure 4, 5, 6, 7 showcases that the profit decreases with increase in the failure rate λ , repair rate/ revenue per unit up time (C_0)/replacement rate/inspection rate.
iv. Figure 8 shows that the profit increases when the availability of the system increases.
v. Figure 9 reveals that the profit increases with respect to increase in the revenue per unit up time (C_0) for different values of cost per visit of the repairman (C_4) and decreases with increase in (C_4).

CONCLUSION

To make the system effectively dependable and near failure-proof, variant measures have been thoughtfully considered and accordingly the calculations made to compute system mean time failure, predict reliability and availability in the concerned system. Graphical representations have also been made for mark-up of cut-off points. This will directly result in modifying the system with a view to achieve greater profitability and concurrently minimize the losses. This present study will fortify our efforts in target optimization of the existing system.

REFERENCES

1. H. Mine and H. Kawai, "Repair priority effect on avail ability of two-unit system," *IEEE Transactions on Reliability*, vol. 28, no. 4, pp. 325–326, 1979.
2. K. Murari and V. Goyal, "Reliability system with two types of repair facilities," *Microelectronics Reliability*, vol. 23, no. 6, pp. 1015–1025, 1983.
3. R. K. Tuteja, R. T. Arora, and G. Taneja, "Analysis of a two-unit system with partial failures and three types of repairs," *Reliability Engineering & System Safety*, vol. 33, no. 2, pp. 199–214, 1991.
4. P. Chandrasekhar, R. Natarajan, and V. S. Yadavalli, "A study on a two unit standby system with Erlangian repair time," *Asia-Pacific Journal of Operational Research*, vol. 21, no. 3, pp.271–277, 2004.
5. B. Parashar and G. Taneja, "Reliability and profit evaluation of a PLC hot standby system based on a master-slave concept and two types of repair facilities," *IEEE Transactions on Reliability*, vol. 56, no. 3, pp. 534–539, 2007.
6. Z. Zhang, W. Gao, Y. Zhou, and Z. Zhiqiang, "Reliability modeling and maintenance optimization of the Diesel system in Locomotives," *Maintenance and Reliability*, vol. 14, no. 4, pp. 302–311, 2012.
7. B. Parashar, A. Naithani, P.K. Bhatia, "Analysis of a 3-Unit Induced Draft Fan System with One Warm Standby" *International Journal of Engineering Science and Technology* (IJEST), vol. 4, no.11, pp4620-4628, November 2012.
8. G. Taneja and R. Malhotra, "Cost-benefit analysis of a single unit system with scheduled maintenance and variation in demand," *Journal of Mathematics and Statistics*, vol. 9, no. 3, pp. 155–160, 2013.
9. Institution of Railway Signal Engineers minor Railways section guideline on Train Detection Bonding and Cables, ref no. TC11, issue no. 1.0, March 2013.
10. Anjali Naithani, Bhupender Parashar, P.K. Bhatia and Gulshan Taneja, "Probabilistic analysis of a 3-unit induced draft fan system with one warm standby with priority to repair of the unit in working state," *International Journal of System Assurance Engineering and Management*, ISSN 0975-6809, 2017.

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