

## MHD FLOW OVER A MOVING INFINITE VERTICAL POROUS PLATE WITH UNIFORM HEAT FLUX IN THE PRESENCE OF THERMAL RADIATION

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### ABSTRACT

*MHD Flow Over a Moving Infinite Vertical Porous Plate With Uniform Heat Flux In The Presence Of Thermal Radiation With reference to all critical parameters that appear in the field equations were was studied in this paper. In most of the literature, the flow parameters were neither examined nor commented in detail in any of the above investigators. This is a point of great concern and is if academic interest to study. Also, not much of prime importance was given to the bounding surface and factors influencing the skin friction. Therefore, an attempt has been made to study the influence of such critical parameters that appear in the field equations and their effects on various flow entities. It is noticed that as the frequency of excitation decreases the velocity decreases. Further, as we move away from the boundary of the surface the velocity decreases. The effect of the frequency of excitation and for a constant pore size has been illustrated. In this case, it is noticed that as the frequency of excitation decreases the velocity decreases. Further, as we move away from the boundary of the surface the velocity decreases. it is reported that, as the frequency of excitation decreases the velocity also decreases. In addition to the above, as we move far away from the boundary of the surface the velocity decreases. Also, it is has been reported that, as the pore size of the boundary surface decreases, the velocity also decreases. Further, as we away from the bounding surface the velocity decreases. For different values frequency of excitation, the profiles of skin friction are noticed to be linear and of course with negative slope. It is noticed that as the frequency of excitation decreases, the skin friction on boundary surface found to be increasing.*

**Key words:** *Impulsively started vertical plate, MHD flow, Heat and Mass, Radiation transfer, Heat flux.*

### NOMENCLATURE

$C_p$	: Specific heat at constant pressure
$g$	: Acceleration due to gravity
$G_r$	: Thermal Grashoff number
$\kappa$	: Thermal conductivity of the fluid
$P_r$	: Prandtl number
$p$	: Pressure
$q_r$	: Radiative heat flux in the y-direction
$k$	: Thermal diffusivity
$K$	: Porosity
$M$	: Magnetic field
$N$	: Radiation parameter
$t$	: Time
$T$	: Temperature of the fluid near the plate
$T_w$	: Temperature of the plate
$T_\infty$	: Temperature of the fluid far away from the plate
$u$	: Velocity of the fluid in the x-direction

$u_0$	: Velocity of the fluid plate
$U$	: Dimensionless velocity
$y$	: Coordinate axis normal to the plate
$y'$	: Dimensionless coordinate axis normal to the plate
$k^*$	: Mean absorption coefficient
$\alpha$	: Thermal diffusivity
$\beta$	: Volumetric coefficient of thermal expansion
$\mu$	: Coefficient of viscosity
$\nu$	: Kinematic viscosity
$\rho$	: Density
$\sigma$	: Stefan-Boltzmann constant
$\tau$	: Dimensionless skin-friction
$\theta$	: Dimensionless temperature

## INTRODUCTION

Radiative convective flow in several industrial and environmental situations occurs more frequently in many situations. The applications are more found in, fossil fuel combustion and in cooling chambers and more so in energy processes, astrophysical flows, solar power technology and space vehicle re-entry. An important role in manufacturing sectors for the design of highly precision equipment in radiative heat transfer is found to have several applications: generally, nuclear power plants, gas turbines and propulsion devices for air craft, missiles and space vehicles are few such examples.

Stokes [1] initially, studied the problem of viscous incompressible fluid past an impulsively started infinite horizontal plate which moves in its own plane. Subsequently, Brinkman [2], examined the viscous force imparted by a flowing fluid in a dense swarm of particles. Later, Stewartson [3] studied an analytical solution for a viscous flow past an impulsively started semi-infinite horizontal plate. Subsequently, the case of two dimensional steady state flow of an incompressible fluid with parallel rigid porous walls, with the flow being influenced by uniform suction or injection was investigated by Berman [4]. Later, Macy [6] and Mori [5] studied the flow between two vertical plates, where the plates are electrically non-conducting and under the assumption that the wall temperature influences linearly in the direction of the flow and existence of heat source in the vertical channel. Subsequently, the flow in the renal tubules as viscous flow through circular tube of uniform cross section with a permeable boundary by prescribing their radial velocity at the wall as exponentially decreasing function as axial distance was studied by Macy [6]. Hall [7] examined similar such problem by using finite differences method of a mixed explicit and implicit time for the stability of the solution. The effects of radioactive heat transfer of free convection regimes in an enclosed with specialized applications which occurs geophysics and geothermal reservoirs was analysed by Chang *et al.* [8]. Thereafter, Mahajan *et al.* [9] analysed, the influence of viscous heat dissipating effect in natural convective flows. Later, the thermal and radiation effects of an optionally thin gray gas bounded by a stationary vertical plate was examined by Soundalgekar and Thaker [10]. By applying Rossland's approximation, Hossain *et al.* [11] examined the radiation effects on a mixed convection along a vertical plate with a uniform surface temperature. Subsequently, the effects of thermal radiation and convective flow past a moving infinite vertical plate was discussed and presented by Raptis and Perdakis [12]. Thereafter, the effects of thermal radiation on the flow past a semi-infinite vertical isothermal plate with uniform heat flux in the presence of transversely applied magnetic field was analysed by Antony Raj *et al.* [13].

The nature of velocity with reference to all critical parameters that appear in the field equations were neither examined nor commented in detail in any of the above investigators. This is a point of great concern and is of academic interest to study. Also, not much of prime importance was given to the bounding surface and factors influencing the skin friction. Therefore, an attempt has been made to study the influence of such critical parameters that appear in the field equations and their effects on various flow entities.

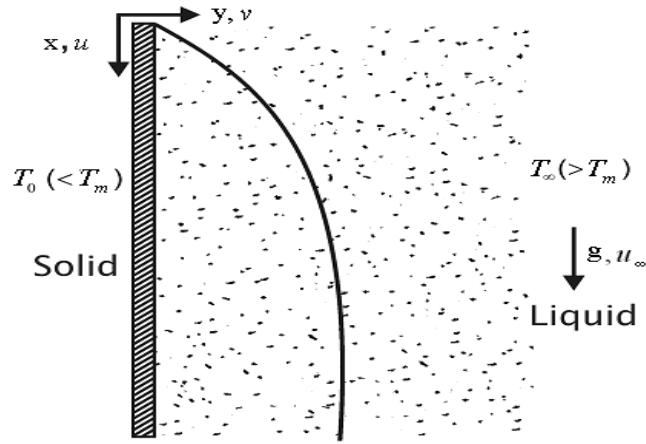
## MATHEMATICAL FORMULATION

Flow of an incompressible viscous radiating fluid past an impulsively started infinite vertical plate with uniform heat flux is considered. The x-axis is taken along the plate in the vertical direction and the y-axis is taken normal to the plate. The Flow geometry is as shown below

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Schematic representation of the problem

Initially, the plate and fluid are at the same temperature in a stationary condition. The plate is given an impulsive motion in the vertical direction against the gravitational field with constant velocity  $u_0$  when  $t > 0$ . At the same time, the heat is supplied from the plate to the fluid at uniform rate. The fluid exhibits the properties of grey, absorbing-emitting radiation but a non-scattering medium. Then by usual Bossiness's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

In view of Rossland approximation  $q_r$  is given by:

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y} \quad (3)$$

While, the initial and boundary conditions are:

$$\left. \begin{aligned} t' \leq 0 : u = 0, T = T_\infty \text{ for all } y' \\ t' > 0 : u = u_0, \frac{\partial T}{\partial y} = -\frac{q}{k} \text{ at } y' = 0 \\ u = 0, T \rightarrow T_\infty \text{ as } y' \rightarrow \infty \end{aligned} \right\} \quad (4)$$

Under the assumption that, the temperature differences within the flow are sufficiently small such that  $T^4$  may be expressed as a linear function of the temperature. This is accomplished by expanding  $T^4$  in a Taylor series about  $T_\infty$  and neglecting higher order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (5)$$

By using equations (4) and (5), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} \quad (6)$$

On introducing the following non-dimensional quantities

$$\left. \begin{aligned} U = \frac{u}{u_0}, t = \frac{t' u_0^2}{\nu}, y = \frac{y' u_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ (7) \\ Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, Pr = \frac{\mu C_p}{k}, N = \frac{\kappa^* k}{4\sigma T_\infty^3} \end{aligned} \right\}$$

By considering the magnetic intensity as  $M$  and the permeability of the boundary as  $K$  Eqs. (1) to (6), can be re defined as:

$$\frac{\partial U}{\partial t} = Gr\theta + \frac{\partial^2 U}{\partial y^2} + \left(M + \frac{1}{K}\right)u \quad (8)$$

$$3N \Pr \frac{\partial \theta}{\partial t} = (3N + \Pr) \frac{\partial^2 \theta}{\partial y^2} \quad (9)$$

The initial and boundary conditions in non-dimensionless form are

$$\left. \begin{aligned} u = 0, \theta = 0, \text{ for all } y, t \leq 0 \\ t > 0 : u = 1, \frac{\partial \theta}{\partial y} = -1 \text{ at } y = 0 \\ u = 0, \theta \rightarrow 0, \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (10)$$

### Methodology for solution

We assume that the solutions for Eqn (8) and Eqn (9) in form of:

$$u(x, t) = u_0(y)e^{i\omega t} \quad (11)$$

$$\theta(y, t) = \theta_0(y)e^{i\omega t} \quad (12)$$

Under the modified initial and boundary conditions:

$$\left. \begin{aligned} u_0 = 0, \theta_0 = 0, \text{ for all } y, t \leq 0 \\ t > 0 : u_0 = e^{-i\omega t}, \frac{d\theta_0}{dy} = e^{-i\omega t} \text{ at } y = 0 \\ u_0 = 0, \theta_0 \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (13)$$

Using Eqns (11), (12) and (13) in Eqns (8) and (9)

$$u(y, t) = \frac{Gr}{R_1} (\exp(-m_2 y) - \exp(-m_1 y)) + \exp(-m_2 y) \quad (14)$$

$$\theta(y, t) = \frac{\exp(-m_1 y)}{m_1} \quad (15)$$

The expression for the skin friction is:

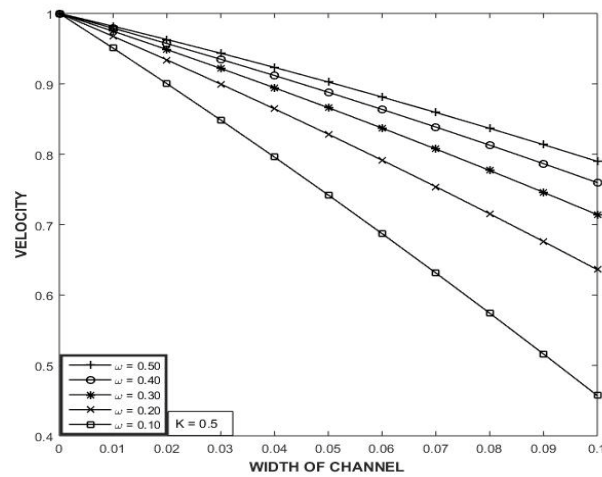
$$\left[ \frac{\partial u}{\partial y} \right]_{y=0} = \frac{Gr}{R_1} [m_1 - m_2] - m_2 \quad (16)$$

Where

$$m_1 = \sqrt{\frac{3N \Pr i\omega}{3N + \Pr}}, m_2 = \sqrt{\left(i\omega - M - \frac{1}{K}\right)}, R_1 = m_1 \left( m_1^2 - \left(i\omega - M - \frac{1}{K}\right) \right)$$

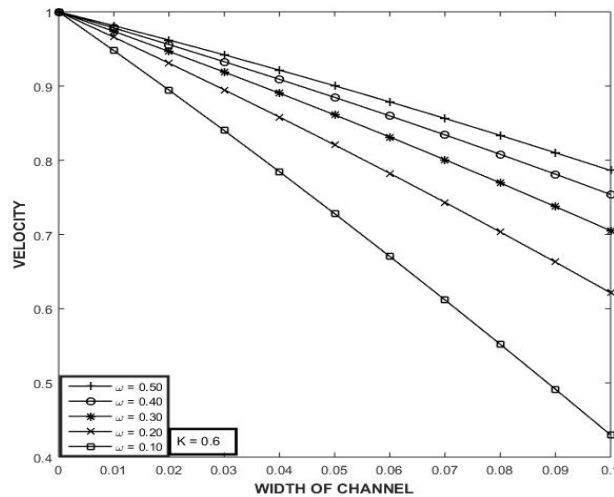
### RESULTS AND DISCUSSION

1. The influence of frequency of excitation for size 0.5 on the velocity field has been illustrated in Fig – 1. In this case, it is noticed that as the frequency of excitation decreases the velocity decreases. Further, as we move away from the boundary of the surface the velocity decreases.



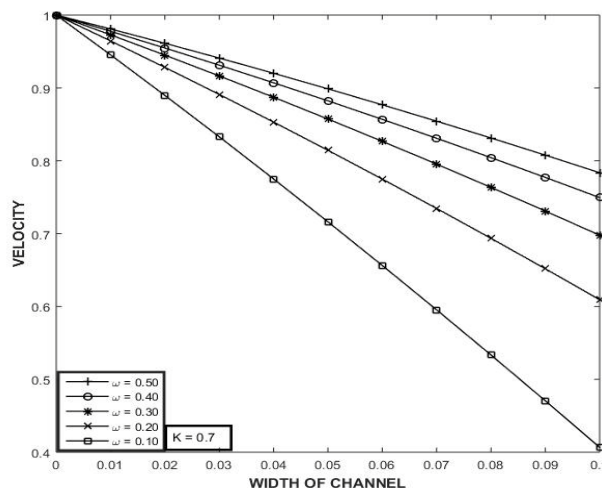
**Figure-1:** Influence of frequency of excitation on velocity

- Fig - 2 illustrates the influence the frequency of excitation for size 0.6. The effect of the frequency of excitation and for a constant pore size has been illustrated. In this case, it is noticed that as the frequency of excitation decreases the velocity decreases. Further, as we move away from the boundary of the surface the velocity decreases.



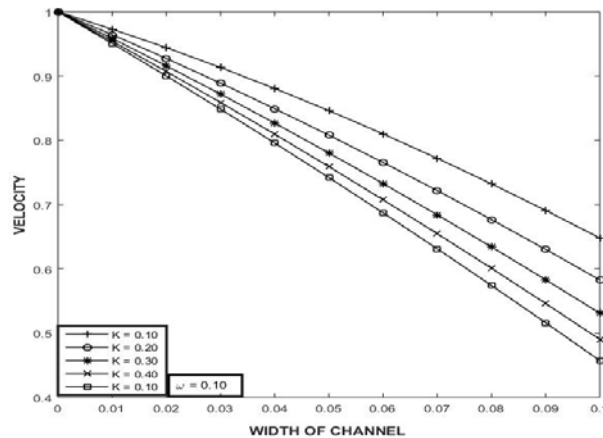
**Figure-2:** Influence of frequency of excitation on velocity

- The influence the frequency of excitation for pore size of 0.7 is shown in Fig – 3. In this case, it is reported that, as the frequency of excitation decreases the velocity also decreases. In addition to the above, as we move far away from the boundary of the surface the velocity decreases.



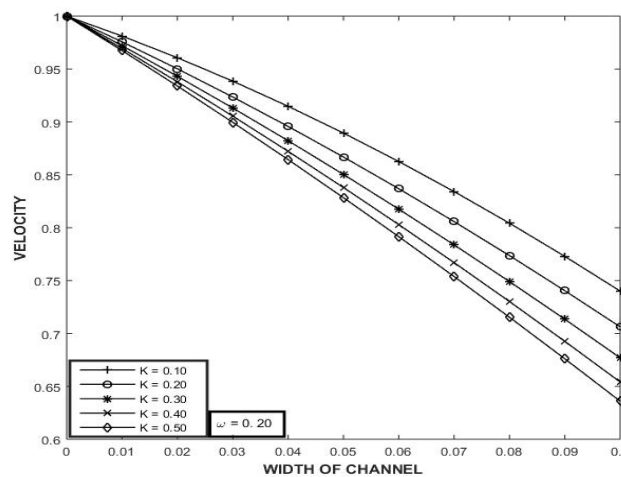
**Figure-3:** Effect of frequency of excitation on velocity

4. Fig – 4 exhibits the consolidated effect of porosity and frequency of excitation on the velocity profiles. In this case, it is reported that as the pore size of the boundary size of the boundary decreases, then the velocity decreases. Further, as we move away from the bounding surface the velocity decreases.



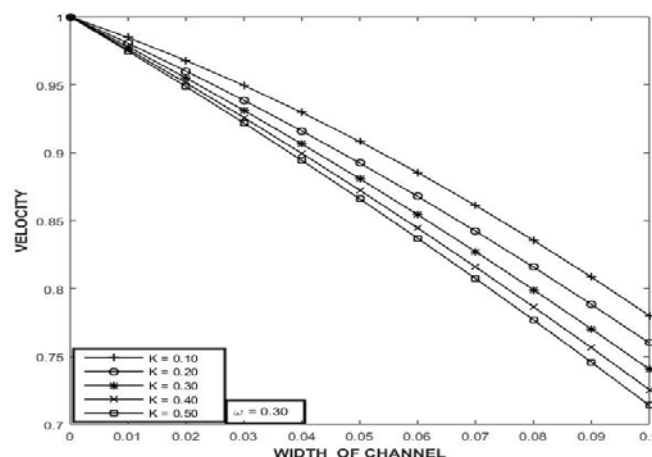
**Figure-4:** Effect of frequency of excitation on velocity

5. The influence of porosity for a fixed frequency of excitation over the velocity profiles has been depicted in Fig- 5. In this situation, it is has been reported that, as the pore size of the boundary surface decreases, the velocity also decreases. Further, as we away from the bounding surface the velocity decreases.



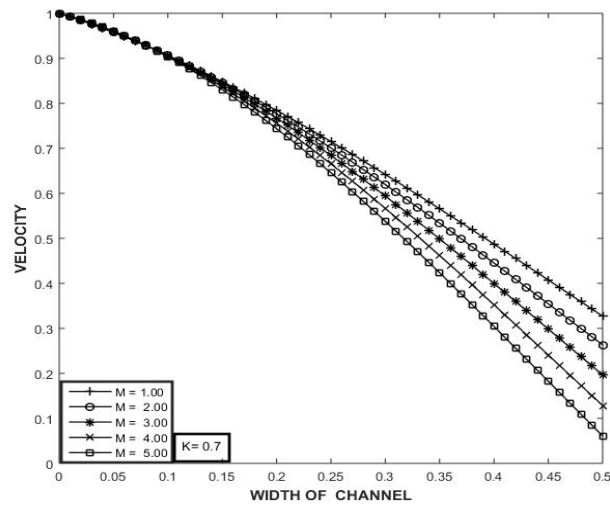
**Figure-5:** Influence of frequency of excitation on velocity

6. The influence of porosity for a fixed frequency of excitation over the velocity profiles is shown in Fig – 6. In this case, it is reported that as the pore size of the boundary surface size decreases, the velocity also decreases. Further, as we move away from the bounding surface the velocity also decreases.



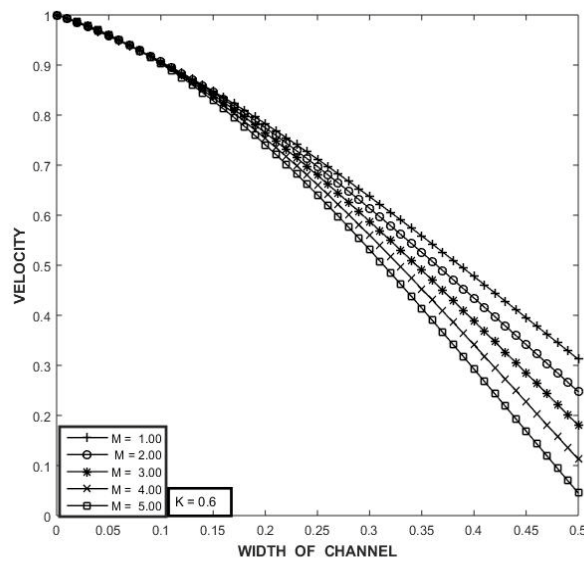
**Figure-6:** Effect of frequency of excitation on velocity

7. Fig – 7 illustrates the influence of magnetic field on the velocity field when the pore size of the bounding surface is 0.7. In this case, it is reported that as magnetic intensity increases the velocity decreases. Also as we move away from the bounding surface, the velocity decreases.



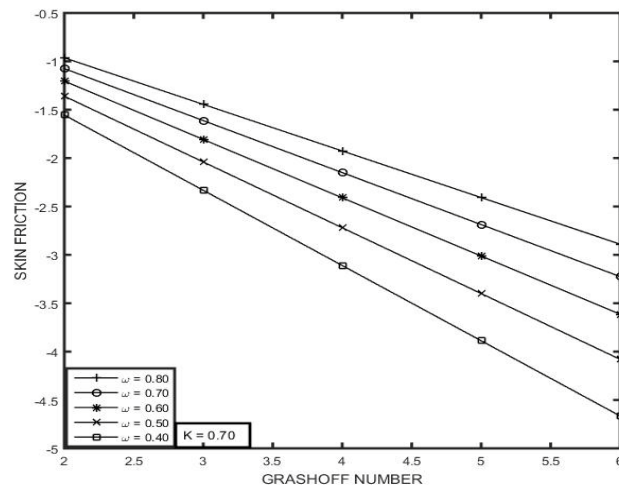
**Figure-7:** Influence of Magnetic field on velocity

8. The influence of magnetic field on the velocity when the pore size is 0.8 is illustrated in Fig-8. In this case, it is reported that as the magnetic intensity increases, the velocity decreases and also as we move away from the bounding surface, the velocity diminishes gradually.



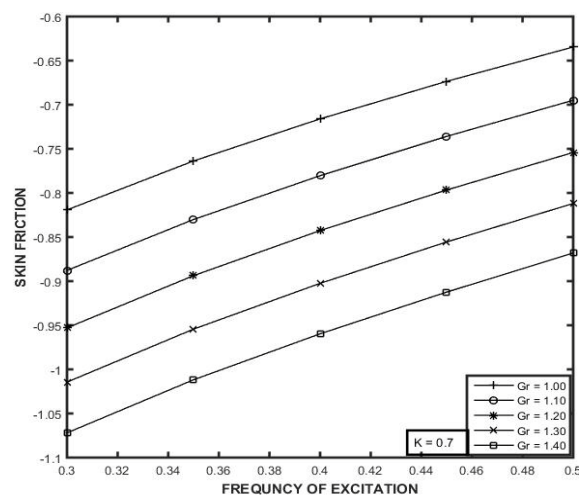
**Figure-8:** Effect of Magnetic field on velocity

9. The influence of Grashoff number with respect to the frequency of excitation has been illustrated in fig 9. For different values frequency of excitation, the profiles of skin friction are noticed to be linear and of course with negative slope. It is noticed that as the frequency of excitation decreases, the skin friction on boundary surface found to be increasing.



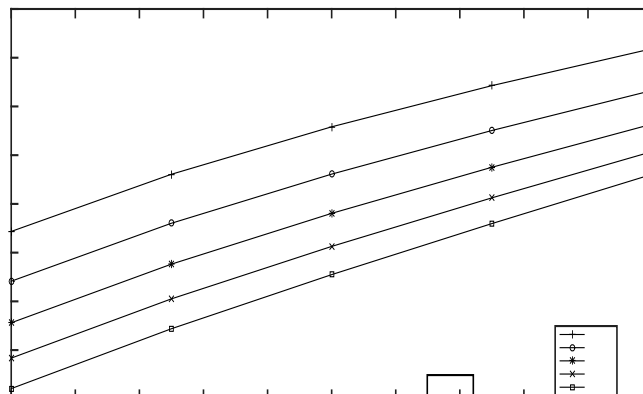
**Figure-9:** Influence of frequency of excitation on skin friction

10. The consolidated effect of frequency of excitation and Grashoff number with respect to porosity parameter has been depicted in figure 10. It is noticed that, as the Grashoff number increases, the skin friction on the boundary surface decreases considerably. In addition to the above as the frequency of excitation increases, The profiles for skin friction are noticed to be increasing and are linear in their behavior.



**Figure-10:** Influence of Grashoff Number on skin friction

11. Fig 11 illustrates that the combined effects of frequency of excitation with respect to the porosity of the boundary surface for a constant Grashoff number. It is observed. For a fixed porosity parameter, as the frequency of excitation increases, the skin friction also increases. The profile for the skin friction seems to be more or less linear in its nature.



**Figure-11:** Influence of porosity on skin friction



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