International Journal of Mathematical Archive-9(5), 2018, 119-126
Available online through www.ijma.info ISSN 2229 – 5046

MHD FREE CONVECTIVE FLOW OF CASSON FLUID PAST OVER AN OSCILLATING VERTICAL POROUS PLATE

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(Received On: 30-03-18; Revised & Accepted On: 25-04-18)

ABSTRACT

This article studies the unsteady MHD free flow of a Casson fluid past an oscillating vertical plate with constant wall temperature. The fluid is electrically conducting and passing through a porous medium.This phenomenon is modelled in the form of partial differential equations with initial and boundary conditions. Some suitable non-dimensional variables are introduced. The dimensionless partial differential equations of governing equations of the flow field are solved numerically using closed analytical method. The velocity and temperature profiles are discussed through graphically and discussed qualitatively.

***Keywords**: Casson fluid, MHD, Porous medium, Free convection, Heat transfer.*

1. INTRODUCTION

The study of magnetohydrodynamic flow of non-Newtonian fluid in presence a porous medium has involved many researchers. Of course, it is due to the fact that such spectacleare mostly found in the optimization of solidification developments of metals and metal alloys, the geothermal sources study and nuclear fuel debris treatment. Still, non-Newtonian fluids are refined compare to Newtonian fluids. Indeed, the resulting equations of non-Newtonian fluids give highly non-linear differential equations which are usually difficult to solve. These equations add further complexities when MHD flows in aporous space have been taken into account. For an application for the MHD flows of non-Newtonian fluids in a porous medium are encountered in heat-storage beds, textile, irrigation problems, biological systems, process of petroleum, paper and polymer composite industries.

Ali [1] studied closed form solutions for unsteady free convection flow of a second-grade fluid over an oscillating vertical plate. Hussanan *et al.* [2] performed unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating. Kishore *et al.* [3] studied the effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions. Hari *et al.* [4] studied radiation and chemical reaction effects on MHD Casson fluid flow past an oscillating vertical plate embedded in porous medium, engineering science and technology. Nabil *et al.* [5] analysed non-Darcy couette flow through a porous medium of magnetohydrodynamic visco-elastic fluid with heat and mass transfer. Nabil *et al.* [6] discussed Magnetohydrodynamic flow of non-Newtonian visco-elastic fluid through a porous medium near an accelerated plate. Hameed *et al.* [7] analysed unsteady MHD flow of a non-Newtonian fluid on a porous plate. Rajesh [8] studied MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Sengupta [9] studied thermal diffusion effect of free convection mass transfer flow past a uniformly accelerated porous plate with heat sink. Samiulhaq *et al.* [10] discussed an unsteady magnetohydrodynamic free convection flow of a second-grade fluid in a porous medium with ramped wall temperature. Nadeem *et al.* [11] examined MHD three-dimensional Casson fluid flow past a porous linearly stretching sheet. Nadeem *et al.* [12] discussed MHD flow of a Casson fluid over an exponentially shrinking sheet. Shaw *et al.* [13] analysed Casson fluid flow through a stenosed bifurcated artery. Mahanta *et al.* [14] discussed 3D Casson fluid flow past a porous linearly stretching sheet with convective boundary condition. Imran *et al.* [15] examined effects of slip condition and Newtonian heating on MHD flow of Casson fluid over a nonlinearly stretching sheet saturated in a porous medium. Pramanik [16] studied Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. Animasaun *et al.* [17] analysed Casson fluid flow with variable thermo-physical property along exponentially stretching sheet with suction and exponentially decaying internal heat generation using the homotopy analysis method. Bala [18] studied magnetohydrodynamic flow of a Casson fluid over an exponentially inclined

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permeable stretching surface with thermal radiation and chemical reaction. Reddy [19] examined the Soret and the Dufour effects on MHD free convective flow past a vertical porous plate in the presence of heat generation. Exact solutions when the plate performs sine and cosine oscillations with constant wall temperature are obtained by using the Laplace transform technique [20]. Reddy *et al.* [21] discussed Soret and Dufour effects on MHD flow with heat and mass transfer past a permeable stretching sheet in presence of thermal radiation.

2. FORMULATION OF THE PROBLEM

We consider the Casson fluid over an infinite vertical flat plate embedded in a saturated porous medium. The flow being confined to y > 0, where y is the coordinate measured in the normal direction to the plate. The fluid is assumed to be electrically conducting with a uniform magnetic field B of strength B0, applied perpendicular to the plate. The magnetic Reynolds number is assumed to be small enough to neglect the effects of applied magnetic field. Initially, for time t = 0 both the fluid and the plate are at rest with uniform temperature. At time $t = 0^+$ the plate begins to oscillate in its plane $(y = 0)$ according to

$$V = UH(t)\cos(\omega t)i; \text{ or } \quad V = U\sin(\omega t)i; \ t > 0, \tag{1}$$

where the constant U is the amplitude of the plate oscillations, $H(t)$ is the unit step function, i is the unit vector in the vertical flow direction and ω is the frequency of oscillation of the plate. At the same time, the plate temperature is raised to T_W which is thereafter maintained constant.

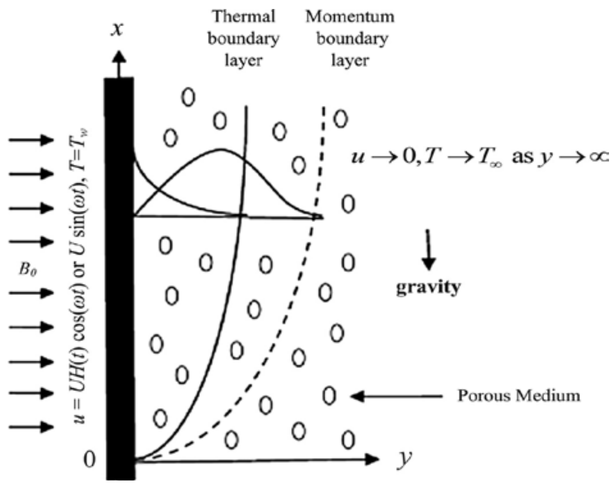

Figure-1: Physical sketch of the problem.

The rheological equation of state for the Cauchy stress tensor of Casson fluid is written as, $\tau = \tau_0 + \mu\gamma^*$,
Or

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{Py}{\sqrt{2\pi}}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \dfrac{Py}{\sqrt{2\pi_c}}\right)e_{ij}, \pi < \pi_c \end{cases},$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} is the $(i,j)^{ih}$ component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non- Newtonian model, μ_B is plastic dynamic viscosity of the non- Newtonian fluid and P_y is yield stress of fluid. Before we derive the governing equations, the following assumptions are made, rigid plate, incompressible flow, unsteady flow, unidirectional flow, one dimensional flow, non-Newtonian flow, free convection, oscillating vertical plate and viscous dissipation term in the energy equation is neglected. Under these conditions we get the following set of partial differential equations

$$\rho\frac{\partial u}{\partial t} = \mu_B\left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u - \frac{\mu\varphi}{k_1}u + \rho g\beta(T - T_\infty), \tag{2}$$

$$\rho c_p\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

together with initial and boundary conditions

$$t<0: u=0, T=T_{\infty} \text{ for all } \; y>0,$$

$$t \geq 0: \begin{cases} u=UH(t)\cos(\omega t) \text{ or } u=U\sin(\omega t), T=T_W & \text{at } y=0, \\ u \to 0, T \to T_{\infty} & \text{as } \; y \to \infty, \end{cases} \tag{4}$$

where $u,t,T,\mu_B,\gamma,\rho,g,\beta,c_p,k,\sigma,\phi,$ an k_1 are the velocity of the fluid in $x-$direction, time, temperature, plastic dynamic viscosity, Casson parameter, the constant density, the gravitational acceleration, volumetric coefficient of thermal expansion, specific heat at constant pressure and thermal conductivity, electric conductivity of the fluid, porosity and permeability of the fluid, respectively.

We introduce the following dimensionless variables

$$u^*=\frac{u}{U}, y^*=\frac{U}{\nu}y, t^*=\frac{U^2}{\nu}t, \theta=\frac{T-T_{\infty}}{T_W-T_{\infty}}, \omega^*=\frac{\omega\nu}{U^2}, \tau^*=\frac{\tau}{\rho u^2}, \tag{5}$$

into Equations. (2) - (4), and we get (* symbols are dropped for simplicity)

$$\frac{\partial u}{\partial t}=\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2}-\left(M^2+\frac{1}{K}\right)u+Gr\theta, \tag{6}$$

$$\Pr\frac{\partial \theta}{\partial t}=\frac{\partial^2 \theta}{\partial y^2}, \tag{7}$$

with associated initial and boundary conditions

$$t<0: \; u=0, \theta=0 \; \text{ for all } y>0,$$

$$t \geq 0: \begin{cases} u=H(t)\cos(\omega t) \text{ or } u=\sin(\omega t), \theta=1 \text{ at } y=0, \\ u \to 0, \theta \to 0 & \text{as } y \to \infty, \end{cases} \tag{8}$$

where

$$\Pr=\frac{\mu c_p}{k}, M^2=\frac{\sigma\upsilon B_0^2}{\rho U^2}, \frac{1}{K}=\frac{\nu\phi^2}{k_1 U^2}, Gr=\frac{\upsilon g\beta(T_w-T_{\infty})}{U^3} \text{ and } \gamma=\frac{\mu B\sqrt{2\pi_c}}{P_y},$$

here Pr is the Prandtl number, M is the magnetic parameter called Hartmann number, K is the dimensionless permeability parameter, Gr is the Grashof number and γ is the Casson parameter.

SOLUTION OF THE PROBLEM

The governing equations along with the boundary conditions cannot be solved numerically. So, the numerical solution remains the only possible solution one could take. To solve the governing equations is basedon the Laplace transform technique by Asma [20]. This was based on the discretization of the governing equations using the central differencing in space. The discretization equations were solved by numerically using closed analytical method. we assume the trial solution for the velocity and temperature as:

$$u(y,t)=u_0(y)e^{i\omega t} \tag{9}$$

$$\theta(y,t)=\theta_0(y)e^{i\omega t} \tag{10}$$

Substituting Equations (9) and (10) in Equations (6), (7) and (8), we obtain:

$$u_0''-k_2^2 u_0=-(Gr\theta_0) \tag{11}$$

$$\theta_0''-\Pr i\omega\theta_0=0 \tag{12}$$

where $k_2^2=\left(\dfrac{M^2+\frac{1}{K}+i\omega}{1+\frac{1}{\gamma}}\right)$, Here the primes denote the differentiation with respect to y.

The corresponding boundary conditions can be written as

$$\begin{aligned} u_0 &= e^{-i\omega t}\sin(\omega t), & \theta_0 &= e^{-i\omega t} & \text{at} \quad & y=0 \\ u_0 &\to 0, & \theta_0 &\to 0, & \text{as} \quad & y \to \infty \end{aligned} \tag{13}$$

The analytical solutions of equations (11) and (12) with satisfying the boundary conditions (13) are given by

$$u(y,t) = (\sin(\omega t) - A1)e^{-k2y} + A1e^{-k1y} \tag{14}$$

$$\theta(y,t) = e^{-k1y} \tag{15}$$

RESULTS AND DISCUSSION

In this section, the obtained exact solutions are studied numerically in order to determine the effects of several involved parameters such as Prandtl number Pr, Grashof number Gr, Casson parameter γ, magnetic parameter M, permeability of porous medium K, angle of inclination ω and time t. To examine the effects of various parameters on different profiles, we assign values to the parameters as

$$i = 2; \omega = \pi / 4; M = 0.5; \gamma = 0.2; Gr = 5; K = 0.2; t = 0.2; .$$

These Numerical values of skin-friction and Nusselt number are computed and presented in tables for different parameters. Physical sketch of the problem is shown in Fig. 1.

Fig. (2) exhibits the velocity profiles for different values of Prandtl number (Pr), when the other parameters are fixed. It is observed that velocity of the fluid decreases with increasing Prandtl number (Pr). It is depicted from Fig. (3) that, the temperature decreases as the Prandtl number Pr increases. It is justified due to the fact that thermal conductivity of the fluid decreases with increasing Prandtl number Pr and hence decreases the thermal boundary layer thickness.

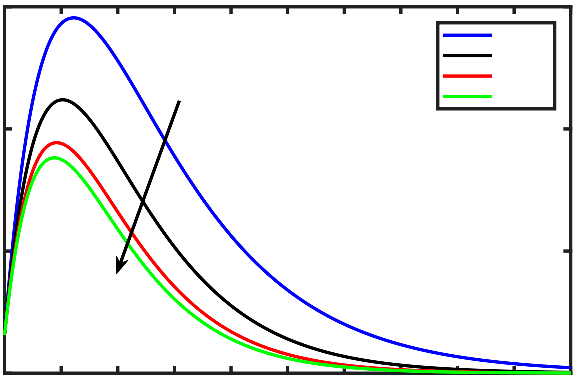

Figure-2: Velocity field for different values of Prandtl number (Pr).

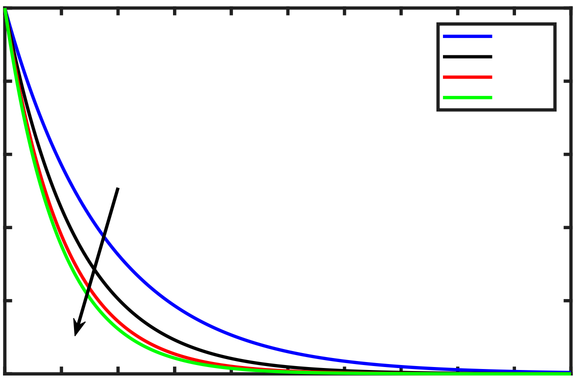

Figure-3: Temperature field for different values of Prandtl number (Pr).

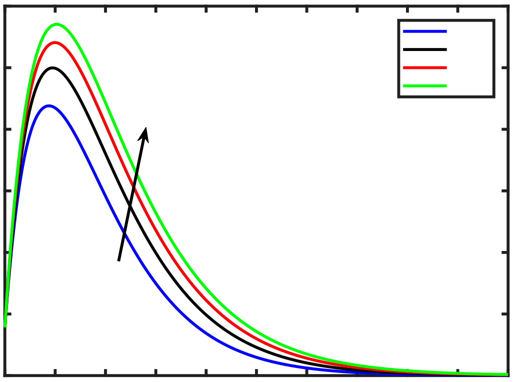

Figure-4: Velocity field for different values of permeability of porous medium (K).

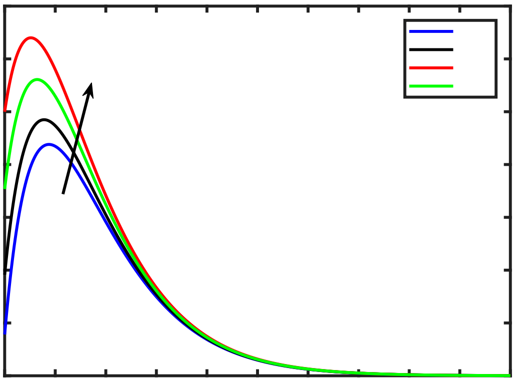

Figure-5: Velocity field for different values of time (t).

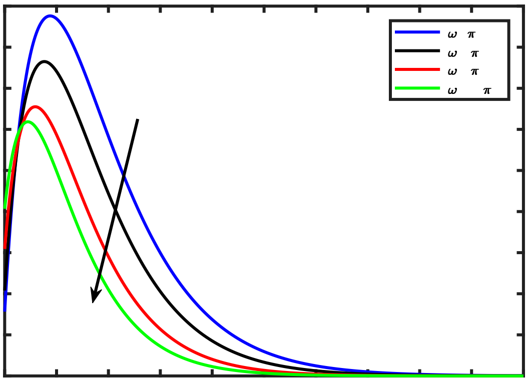

Figure-6: Velocity field for different values of angle of inclination (ω).

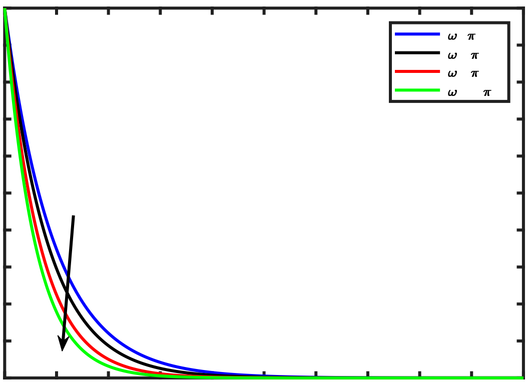

Figure-7: Temperature field for different values of angle of inclination (ω).

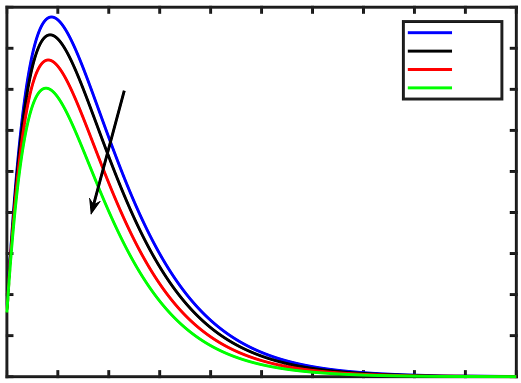

Figure-8: Velocity field for different values of Magneticparameter (M).

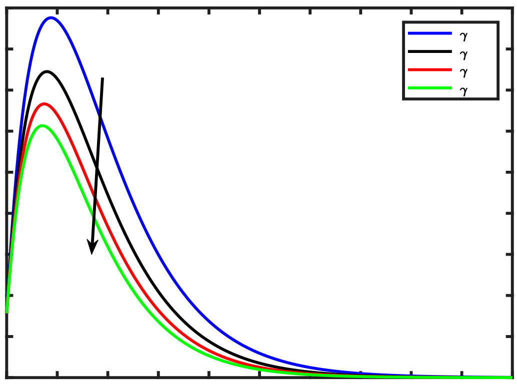

Figure-9: Velocity field for different values of Casson parameter (γ).

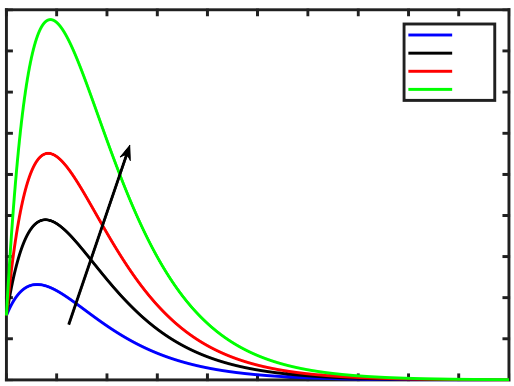

Figure-10: Velocity field for different values of Grashof number (Gr).

In Fig. (4), the profiles of velocity have been plotted for various values of permeability parameter K by keeping other parameters fixed. It is observed that for large values of K, velocity and boundary layer thickness increase which explains the physical situation that as K increases, the resistance of the porous medium is lowered which increases the momentum development of the flow regime, ultimately enhances the velocity field. Fig. (5) the influence of dimensionless time t on the velocity profiles is shown. It is found that the velocity is an increasing function of time t.. The graphical results for the phase different values of angle of inclination (w) shown in Fig. (6) and (7). It is observed that the velocity and temperature decrease as the angle of inclination (w) increases. Fig. (8). illustrate that the influence of magnetic field parameter on the velocity profiles of the Casson fluid. It is evident that an increase in the magnetic field parameter depreciates the velocity profiles. Generally, with an increase in the magnetic field parameter the opposite force to flow direction which is called the Lorentz's force is developed. Due to this reason, a fall in the velocity profiles of the flow is seen. It is interesting to mention here that the heat transfer performance is high in the injection case while compared with the suction case. The influence of Casson fluid parameter γ on velocity profiles is shown in Fig.(9). It is found that velocity decreases with increasing values of γ. It is important to note that an increase in Casson parameter γ makes the velocity boundary layer thickness shorter. It is further observed from this graph that when the Casson parameter γ is large enough i.e. $\gamma \to \infty$, the non-Newtonian behaviours disappear, and the fluid purely increment in velocity boundary layer thickness. Fig. (10) illustrates the profiles of velocity for different values of Gr: It is observed that velocity increases with increasing values of Gr the flow is accelerated due to the enhancement in the buoyancy forces corresponding to the increasing values of Grashof number, i.e., free convection effects.

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow.

Skin friction
Knowing the velocity field, the skin – friction at the plate can be obtained, which in non –dimensional form is given by

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Nusselt number
Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non –dimensional form is given, in terms of the Nusselt number, is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

6. CONCLUSION

In this paper an exact analysis is performed to investigate the unsteady boundary layer flow of a Casson fluid past an oscillating vertical plate with constant wall temperature. The dimensionless governing equations are solved by using the perturbation technique. The results for velocity and temperature are obtained and plotted graphically. The numerical results for skin-friction and Nusselt number are computed in tables. The main conclusions of this study are as follows:

- Velocity decreases with increasing as Prandtl number Pr, Casson parameter γ, magnetic parameter M, angle of inclination ω.
- Velocity increases with increasing as Grashof number Gr, permeability of porous medium K and time t.
- Temperature decreases with increasing Prandtl number Prand angle of inclination ω

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Source of support: Nil, Conflict of interest: None Declared.