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ALMOST SUPRA*g-CLOSED MAPS AND STRONGLY SUPRA*g-CLOSED MAPS IN SUPRA TOPOLOGICAL SPACES

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ABSTRACT

In 1983, Mashhour et al. introduced the supra topological spaces and studied S-continuous functions and S-continuous functions. In this paper, In this paper, we introduce the concept of supra*g-closed maps and we obtain almost supra*g-closed map, strongly supra*g-closed map and basic properties and their relationships with other forms of supra*g-closed maps In supra topological spaces.*

Keywords: *Supra closed set, Supra*g- closed set, supra *g-closed map, almost supra*g-closed map, strongly supra*g-closed map.*

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1.INTRODUCTION

In 1983, Mashhour *et al.* [1] introduced the supra topological spaces and studied S-continuous functions and S*-continuous functions. In 2011, Ravi *et al.* [3] introduced and investigated several properties of supra generalized closed sets, supra sg-closed sets and gs-closed sets in supra topological spaces .In topological space the arbitrary union condition is enough to have a supra topological space. Here every topological space is a supra topological space but the converse is not always true. Many researchers are Introducing many new notions and investigating the properties and characterizations of such new notions The purpose of this paper is to introduce the concept of supra*g-closed maps and studied its basic properties. Also we defined almost supra*g-closed maps, strongly supra*g-closed maps and investigated their relationship to other functions in supra topological spaces.

2. PRELIMINARIES

Throughout this paper, X, Y and Z denote the supra topological spaces (X, μ), (Y, λ) and (Z, η) respectively, which no separation axioms are assumed. For a subset A of a space X, $cl^{\mu}(A)$ and $int^{\mu}(A)$ denote the closure of A and the interior of A respectively.

Definition 2.1 [1]: A subfamily μ of X is said to be a supra topology on X, if
- (i) $X, \varphi \in \mu$,
- (ii) If $A_i \in \mu$ for all $i \in J$, then $\cup A_i \in \mu$. The pair (X, μ) is called the supra topological space.

The elements of μ are called supra open sets in (X, μ) and the complement of a supra open set is called a supra closed set.

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Definition 2.2[1]: The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is defined as $cl^{\mu}(A) = \cap\{B : B$ is supra closed and $A \subseteq B\}$. The supra interior of a set A is denoted by $int^{\mu}(A)$ and is defined as $int^{\mu}(A) = \cup\{B : B$ is supra open and $A \supseteq B\}$.

Definition 2.3 [1]: Let (X, τ) be a topological space and μ be a supra topology associated with τ, if $\tau \subset \mu$.

Definition 2.4: A subset A of a supra topological space X is called

(i) a supra pre-open set [5] if $A\subseteq int^{\mu} (cl^{\mu} (A))$ and a supra pre-closed set if $cl^{\mu} (int^{\mu}(A))\subseteq A$
(ii) a supra semi-open set [6] if $A\subseteq cl^{\mu} (int^{\mu} (A))$ and a supra semi closed set if $int^{\mu}(cl^{\mu}(A))\subseteq A$
(iii) a supra semi-preopen set [6] if $A \subseteq cl^{\mu} (int^{\mu}(cl^{\mu}(A)))$ and a supra semi-pre closed if $int^{\mu} (cl^{\mu}int^{\mu}(A))\subseteq A$.
(iv) a supra α open set if [5] $A\subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$ and supra α closed set if $c^{\mu}l(int^{\mu}(cl^{\mu}(A)))\subseteq A$
(v) a supra regular-open set [5] if $A = int^{\mu}(cl^{\mu}(A))$ and a supra regular-closed set if $A = cl^{\mu}(int^{\mu}(A))$.
(vi) supra ω-closed set if [4] $cl^{\mu}(A)\subseteq U$ whenever $A\subseteq U$ and U is supra semi-open in (X, μ).

Definition 2.5: Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ.
A function $f : (X, \tau) \to (Y, \sigma)$ is called supra*g- Continuous if $f^{-1}(V)$ is supra*g-closed in (X, τ) for every closed set V of (Y, σ).

Definition 2.6: Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function $f : (X, \tau) \to (Y, \sigma)$ is called supra*g- irresolute if $f^{-1}(V)$ is supra*g-closed in (X, τ) for every supra*g-closed set V of (Y, σ).

3. SUPRA*g-CLOSED MAPS

In this section, we introduce the concept of Supra*g-closed Maps by using supra supra*g-open and supra*g-closed and investigate some of the basic properties for this class of maps

Definition 3.1: A map $f: (X, \tau) \to (Y, \sigma)$ is called supra*g-closed map(resp. supra*g-open) if for every supra closed(resp. supra open) F of X, f(F) is supra*g-closed (resp. supra *g-open) in Y.

Theorem 3.2: Every supra closed map is supra*g-closed map.

Proof: Let $f:(X, \tau) \to (Y, \sigma)$ be supra closed map. Let V be supra closed set in X, Since f is supra closed map then f(V) is supra closed set in Y. We know that every supra closed set is supra*g-closed, then f(V) is supra*g- closed in Y. Therefore f is supra*g-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.3: Let X = Y = {1, 2, 3} and $\tau = \{X, \phi, \{1\},\{2, 3\}\}$, $\sigma = \{Y, \phi, \{1\}\}$.
$f:(X, \tau) \to (Y, \sigma)$ be the function defined by f(1)=2, f(1)=2, f(3)=1. Here f is supra*g-closed map but not supra closed map, since V = {2, 3} is closed in X but f ({2, 3}) = {1, 3} is supra*g-closed set but not supra closed in Y.

Theorem 3.4: A map $f :(X, \tau) \to (Y, \sigma)$ is supra*g-closed iff $f(cl^{\mu}(V))$= supra*g-open $cl^{\mu}(f(V))$

Proof: Suppose f is supra *g-closed map. Let V be supra closed set in (X, τ). Since V is supra closed, $cl\mu(V)=V$. f(V) is supra*g-closed in (Y, σ). Since f is supra *g-closed map, then $f(cl^{\mu}(V))=f(V)$. Since f(V) is supra *g-closed, we have supra *g-open $cl^{\mu}(f(V))=f(V)$. Hence $f(cl^{\mu}(V))$= supra*g-open $cl^{\mu}(f(V))$

Conversly, suppose $f(cl^{\mu}(V))$= supra *g-open $cl^{\mu}(f(V))$. Let V be supra closed set in (X, τ), then $cl^{\mu}(V)=V$. since f is a mapping, f(V) is in (Y, σ) and we have $f(cl^{\mu}(V))=f(V)$. Since $f(cl^{\mu}(V))$= supra*g-open $cl^{\mu}(f(V))$, we have f(V)= supra *g-open $cl^{\mu}(f(V))$,implies f(V) is supra*g-closed in (Y, σ). Therefore f is supra *g-closed map.

Theorem 3.5: A map $f:(X, \tau) \to (Y, \sigma)$ is supra*g-open iff $f (int^{\mu}(V))$= supra*g-open $int^{\mu}(f(V))$

Proof: Suppose f is supra*g-open map. Let V be supra open set in (X, τ). Since V is supra open, $int^{\mu}(V)=V$, f(V) is supra *g-open in (Y, σ). Since f is supra*g-open map, Therefore $f(int^{\mu}(V))=f(V)$. Since f(V) is supra*g-open, we have supra*g-open $int^{\mu}(f(V))=f(V)$. Hence $f(int^{\mu}(V))$= supra *g-open $int^{\mu}(f(V))$

Conversely, suppose $f(int^{\mu}(V))$= supra*g-open $int^{\mu}(f(V))$. Let V be a supra open set in (X, τ), then $int^{\mu}(V)=V$. Since f is a mapping, f(V) is in (Y, σ) and we have $f(int^{\mu}(V))=f(V)$. Since $f(int^{\mu}(V))$= supra*g-open $int^{\mu}(f(V))$, we have f(V)= supra*g-open $int^{\mu}(f(V))$, implies f(V) is supra *g-open in (Y, σ). Therefore f is supra*g-open map.

Remark 3.6: If f:(X, τ)→ (Y, σ) is supra *g -closed map and g: (Y, σ)→ (Z, ρ) is supra*g -closed map then its composite need not be supra*g-closed map in general and this is shown by the following example.

Example 3.7: Let X=Y=Z={1, 2, 3} and τ = {X, ϕ, {1} {2, 3}} , σ = {Y, ϕ, {1}}. ρ ={Z, ϕ {1},{2},{1,2},{2,3}}. f:(X, τ) → (Y, σ) be the function defined by f(1) = 2, f(2) = 3, f(3) = 1. and g: (Y, σ) → (Z, ρ) be the function defined by g(1) = 2, g(2) = 3, g(3) = 1. Here f and g is supra*g-closed map, but its composition is not supra *g-closed map, since gof{2, 3}= {1, 2} is not supra *g-closed in (Z, ρ) .

Theorem 3.8: If f: (X, τ) → (Y, σ) is supra closed map and g: (Y, σ) → (Z, ρ) is supra *g-closed map then the composition gof is supra*g-closed map.

Proof: Let V be supra closed set in X. Since f is a supra closed map, f(V) is supra closed set in Y. Since g is supra*g-closed map, g(f(V)) is supra*g-closed in Z. This implies gof is supra*g-closed map.

4. ALMOST SUPRA*g-CLOSED MAPS AND STRONGLY SUPRA*g-CLOSED MAPS

In this section, we introduce the concept of Almost supra*g-closed map and strongly supra*g-closed map .by using supra supra*g- open and supra*g-closed and investigate some of the basic properties for this class of maps

Definition 4.1: A map f:(X, τ) → (Y, σ) is said to be almost supra*g-closed map if for every supra regular closed set F of X, f(F) is supra*g-closed in Y.

Definition 4.2: A map f :(X, τ) → (Y, σ) is said to be strongly supra*g-closed map if for every supra *g closed set F of X, f(F) is supra*g-closed in Y.

Theorem 4.3: Every strongly supra*g-closed map is supra*g-closed map.

Proof: Let V be supra closed set in X. Since every supra closed set is supra*g-closed set, then V is supra*g-closed in X. Since f is strongly supra*g-closed map, f(V) is supra*g-closed set in Y. Therefore f is supra*g-closed map. The converse of the above theorem need not be true. It is shown by the following example.

Example 4.4: Let X=Y={1, 2, 3} and τ = {X, ϕ, {1}}, σ = {Y, ϕ, {2},{1,2},{2,3}}. f:(X, τ) → (Y, σ) be the function defined by f(1)=2, f(2)=3, f(3)=1. Here f is supra*g-closed map but not strongly supra*g-closed map, since V={1,2} is supra*g-closed set in X, but f ({1,2}) = {2,3} is not a supra*g-closed set in Y.

Theorem 4.5: Every supra*g-closed map is almost supra*g-closed map.

Proof: Let V be a supra regular closed set in X. We know that every supra regular closed set is supra closed set. Therefore V is supra closed set in X. Since f is supra*g-closed map, f(V) is supra*g-closed set in Y. Therefore f is almost supra*g-closed map. The converse of the above theorem need not be true. It is shown by the following example.

Example 4.6: Let X=Y={1, 2, 3} and τ = {X, ϕ, {1}, {2}, {1,2},{2, 3}}, σ = {Y, ϕ, {1},{3},{1,3}}. f:(X, τ) → (Y, σ) be a function defined by f(1) =3, f(2) =2, f(3) = 1. Here f is almost supra*g- closed map but it is not supra*g-closed map, since V={1, 3} is supra closed set in X but f ({1, 3}) = {1, 3} is not supra*g-closed set in Y.

Theorem 4.7: Every strongly supra*g-closed map is almost supra*g-closed map.

Proof: Let V be supra regular closed set in X. We know that every supra regular closed set is supra closed set and every supra closed set is supra*g-closed set. Therefore V is supra*g-closed set in X. Since f is strongly supra*g-closed map, f(V) is supra*g-closed set in Y. Therefore f is almost supra*g-closed map. The converse of the above theorem need not be true. It is shown by the following example.

Example 4.8: Let X=Y={1, 2, 3} and τ = {X, ϕ, {1}, {3}, {1,3}} , σ = {Y, ϕ,{1}, {2},{1,2},{2,3}}. f:(X, τ) → (Y, σ) be the function defined by f(1)=2, f(2)=3, f(3)=1. Here f is almost supra *g- closed map but it is not strongly supra*g-closed map, since V={1} is supra*g-closed in X but f ({1}) = { 1} is not supra*g-closed set in Y.

Theorem 4.9: If f :(X, τ)→(Y, σ) is strongly supra*g-closed map and g: (Y, σ)→ (Z, ρ) is strongly supra*g -closed map then its composition gof is strongly supra*g-closed map.

Proof: Let V be supra*g-closed set in X. Since f is strongly supra*g-closed, then f(V) is supra*g-closed in Y. Since g is strongly supra*g-closed, then g(f(V)) is supra*g-closed in Z. Therefore gof is strongly supra*g-closed map.

Theorem 4.10: If f:(X, τ) → (Y, σ) is almost supra*g-closed map and g: (Y, σ)→ (Z, ρ) is strongly supra*g -closed map then its composite gof is almost supra*g-closed map.

Proof: Let V be supra regular closed set in X. Since f is almost supra*g-closed, then f(V) is supra*g-closed set in Y. Since g is strongly supra*g-closed, then g(f(V)) is supra*g-closed in Z. Therefore gof is almost supra*g-closed map.

Theorem 4.11: Let f:(X, τ) → (Y, σ) and g: (Y, σ) → (Z, ρ) be two mappings such that their composition gof:(X, τ) → (Z, ρ) be a supra*g-closed mapping then the following statements are true:

(i) If f is supra continuous and surjective then g is supra*g-closed map
(ii) If g is supra*g-irresolute and injective then f is supra*g-closed map.

Proof:

i) Let V be a supra closed set of (Y, σ). Since f is supra continuous $f^{-1}(V)$ is supra closed set in (X, τ).Since gof is supra*g-closed map, We have (gof)($f^{-1}(V)$) is supra*g-closed in (Z, ρ). Therefore g(V) is supra*g-closed in (Z, ρ), since f is surjective. Hence g is supra*g-closed map.
ii) Let V be supra closed set of (X, τ). Since gof is supra *g-closed, we have gof(V) is supra*g-closed in (Z, ρ) . Since g is injective and supra *g-irresolute g^{-1}(gof(V) is supra*g-closed in (Y, σ).Therefore f(V) is supra*g-closed in (Y, σ). Hence f is supra*g-closed map.

CONCLUSIONS

Many different forms of open functions and closed functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, this paper we introduce almost supra*g-closed map, strongly supra*g-closed map in supra topological spaces and investigate some of the basic properties. This shall be extended in the future Research with some applications

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