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PREDICTIVE ESTIMATION OF FINITE POPULATION MEAN USING COEFFICIENT OF KURTOSIS AND MEDIAN OF AN AUXILIARY VARIABLE UNDER SIMPLE RANDOM SAMPLING SCHEME

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ABSTRACT

The present article deals with the predictive estimation of finite population mean using coefficient of kurtosis and median of the auxiliary variable under simple random sampling scheme. Motivated by Milton et al. [2017], we have proposed an improved ratio type predictive estimator of finite population mean. Bias and mean squared error (MSE) of the proposed estimator are also obtained up to first order of approximation. Theoretical efficiency comparison of proposed estimator with Bahl and Tuteja [1991] estimator and Singh et al. estimator [2014] has also been carried out. Optimum conditions, under which the proposed estimator performs better than the competing estimators are also derived. To amply corroborate the theoretical findings, an empirical study has also been carried out. The percent relative efficiencies of the proposed estimator over existing estimators have also been obtained. The suitability of the proposed estimator can be established and appreciated as it has lesser mean squared error, when compared to other widely used estimators.

Key words: *Predictive estimation, Kurtosis, Median, Bias, and Mean squared error.*

1. INTRODUCTION

It is notable that the use of auxiliary information is very common practice in sampling theory to improve the efficiency of estimators. The theory of basic sample survey as available in standard text books on sampling deals with the case which comprises linear estimators such as mean, variance, total and proportion using simple random sampling without replacement. It continues to supply new and improved procedures for estimation of population means assuming independence of observations. Ratio, difference and regression estimators utilize an auxiliary variable for more efficient estimation of the parameter in question. Such estimators take advantage of the correlation between the auxiliary variable (x) and the study variable y. In a similar manner, then, it seems reasonable that under suitable conditions efficient estimation of the mean of the estimator of the finite population total or mean of the characteristic y is also possible using such estimation techniques. With the increasing growth in the number and diverse uses of sample surveys worldwide, it is often desired to analyse and interpret the resulting voluminous data by swifter methods. A basic requirement of a good survey is that a measure of precision is provided for each estimate computed from survey data collected on the basis of the survey design. An important question is how to choose an appropriate mean estimator.

Many authors have come up with more precise estimators by employing prior knowledge of certain population parameter(s). [Searls [1964] for example attempted use of the coefficient of variation of study variable but prove inadequate for in practice, this parameter is unknown. Motivated by [Searls [1964] work, [Sen[1978]], [Sisodia and Dwivedi [1981]] and [Upadhyaya and Singh [1884]] used the known coefficient of variation but now that of the auxiliary variable for estimating population mean of study variable. Reasoning along the same path [Hirano *et al.* [1973]] used the prior value of coefficient of kurtosis of an auxiliary variable in estimating the population mean of the study variable y.

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Kurtosis in most cases is not reported or used in many research articles, in spite of the fact that fundamentally speaking every statistical package provides a measure of kurtosis. This may be attributed to the likelihood that kurtosis is not well understood or its importance in various aspects of statistical analysis has not been explored fully. Median being the middlemost value in a distribution (when the values are arranged in ascending or descending order) has the advantage of being less affected by the outliers and skewed data, thus is preferred to the mean especially when the distribution is not symmetrical. We can therefore utilize the median and the coefficient of kurtosis of the auxiliary variable to derive a more precise ratio type population mean.

In the predictive approach a model is specified for the population values and is used to predict the non-sampled values. Prediction theory for sample surveys (or model-based theory) can be considered as a general framework for statistical inferences on the character of finite population. Well known estimators of population parameters encountered in the classical theory, as expansion, ratio, regression, another estimators can be predictors in the general prediction theory under some special model. Several authors have applied the predictive approach either to form new predictive estimators or to examine the existing estimators from the predictive view point. Srivastava [1983] has shown that if the usual product estimator is used as a predictor for the mean of the unobserved units of the population, the resulting estimator of the mean of the whole population is different from the customary (usual) product estimator. Agrawal and Roy [1999] and Nayak and Sahoo [2012] provided some predictive estimators for finite population variance. Sahoo and Panda [1999] developed the regression type estimator for two stage sampling procedure. Sahoo and Sahoo [2001] and Sahoo *et al.* [2009] introduced a class of estimators for the finite population mean availing information on two auxiliary variables in two stage sampling. Saini [2013] proposed a class of predictive estimators for two stage design consisting especially of two estimators namely ratio and regression. Singh *et al.* [2014] proposed exponential ratio and product type estimator of population mean based on Bahl and Tuteja [1991] estimator under predictive modeling approach. Yadav and Mishra [2015] also proposed improved predictive estimator of finite population mean using linear combination of Singh *et al.* [2014] estimators. Milton *et al.* [2017] have proposed a ratio type estimator of population variance using coefficient of kurtosis and median of the auxiliary variable under simple random sampling scheme. In the present article we have proposed a new modified ratio type predictive estimator of population mean by using Milton *et al.* [2017] estimator as a predictor of the study variable.

2. NOTATIONS AND METHODOLOGY

Consider a finite population $U = \{U_1, U_2, U_3 \ldots U_N\}$ of N distinct and identifiable units. Let Y be our study and X be corresponding auxiliary variable. Suppose we take a random sample of size n from this bivariate population (Y, X) that is (y_i, x_i), for $i = 1,2,3, \ldots n$ using simple random sampling without replacement (SRSWOR) scheme. Let $\overline{Y}$ and $\overline{X}$ be the population means of the study and auxiliary variable and $\bar{y}$ and $\bar{x}$ be their respective sample means.

The present study is an attempt to improve the efficiency in the estimation of

$$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i \tag{1}$$

Using the coefficient of kurtosis and median of the auxiliary variable x.

We define the following notations which we will use throughout the article.

For the population observations we have,

$$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i\ ,\quad \overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

We also define the following from the sample observations

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \text{ mean of sample observations } y_i\ ,$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \text{ mean of sample observations } x_i$$

In general, we define the following parameters

$$\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \overline{Y})^r\,(x_i - \bar{x})^s$$

$$C_y = \frac{S_y^2}{\overline{Y}^2} = \frac{\mu_{20}}{\overline{Y}^2} = \text{Coefficient of variation for the study variable y.}$$

$$C_x = \frac{S_x^2}{\overline{X}^2} = \frac{\mu_{02}}{\bar{x}^2} = \text{Coefficient of variation for the auxiliary variable x.}$$

$\rho_{xy} = \frac{S_{xy}}{S_x S_y}$ = coefficient of correlation between x and y.

$\kappa_x = \frac{\mu_{04}}{\mu_{02}^2}$ = coefficient of kurtosis for the auxiliary variable x.

M_x = Population median of the auxiliary variable.

3. PREDICTIVE ESTIMATION OF FINITE POPULATION MEAN

Let Y_i $(i = 1, 2, \ldots, N)$ be the real value taken by the variable under study from the finite population of U of size N. Here, the population parameter to be estimated is the population mean on the basis of observed values of y in an ordered sample of the finite population U of size N. Let S denote the collection of all possible samples from the finite population U. Let w(s) denote the effective sample size, for any given $s \in S$ and $\bar{s}$ denote the collection of all those units of U which are not in S.

We now denote:

$$\bar{y}_s = \frac{1}{w(s)} \sum_{i \in s} y_i$$

$$\bar{y}_{\bar{s}} = \frac{1}{N - w(s)} \sum_{i \in \bar{s}} y_i$$

We have,

$$\bar{Y} = \frac{w(s)}{N} \bar{y}_s + \frac{N - w(s)}{N} \bar{y}_{\bar{s}}$$

Basu [1971], asserted that in the representation of $\bar{Y}$ above the sample mean $\bar{y}$s being based on the observed y values on units in the sample s is known, therefore the statistician should attempt a prediction of the mean $\bar{y}_{\bar{s}}$ of the unobserved units of the population on the basis of observed units in s.

For $s \in S$ under simple random sampling without replacement (SRSWOR) with sample size $w(s) = n$ and $\bar{y}_s = \bar{y}$, the population mean $\bar{Y}$ is given by

$$\bar{Y} = \frac{n}{N} \bar{y}_s + \frac{(N-n)}{N} \bar{y}_{\bar{s}} \qquad (2)$$

In view of equation (2) above, an appropriate estimator of population mean $\bar{Y}$ is obtained as

$$t = \frac{n}{N} \bar{y} + \frac{(N-n)}{N} T$$

where T is taken as the predictor of $\bar{y}_{\bar{s}}$.

Let x_i $(i = 1,2, \ldots, N)$ denote the i^{th} observation of the auxiliary variable x and $X_i (i = 1,2, \ldots, N)$ be the values of x on the i^{th} unit of the population U. Auxiliary variable x is correlated with the variable under study y.

Let

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

and

$$\bar{x} = \frac{1}{N} \sum_{i \in s} x_i$$

$$t = \frac{n}{N} \bar{y} + \frac{(N-n)}{N} T \qquad (3)$$

4. REVIEW OF EXISTING ESTIMATORS

(i) Bahl and Tuteja [1991] used the positively correlated auxiliary variable and proposed the following exponential ratio type estimator of population mean of the study variable using known population mean of the auxiliary variable as,

$$t_o = \bar{y} \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$$

The bias and the mean squared error of the above estimator up to the first order of approximation is given by,

$$B(t_0) = \frac{1-f}{8n} \bar{Y} [3C_x^2 - 4C_{yx}]$$

$$MSE(t_0) = \lambda \bar{Y}^2 [C_y^2 + \frac{C_x^2}{4} - C_{yx}] \qquad (4)$$

(ii) Singh *et al.*[2014], proposed the following ratio and product type exponential estimator of population estimator of population mean $\overline{Y}$ using Bahl and Tuteja [1991], ratio and product types exponential estimator of population mean as the predictive estimator of $Y_{\bar{s}}$ respectively as

$$t = t_{Re} = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{\overline{X}_{\bar{s}} - \bar{x}}{\overline{X}_{\bar{s}} + \bar{x}}\right)\right] = \frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{N(\overline{X} - \bar{x})}{N(\overline{X} - \bar{x}) - 2n\bar{x}}\right)$$

$$t = t_{pe} = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{\bar{x} - \overline{X}_{\bar{s}}}{\bar{x} + \overline{X}_{\bar{s}}}\right)\right] = \frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\exp\left(\frac{N(\bar{x} - \overline{X})}{N\overline{X} + (N-2n)\bar{x}}\right)$$

$$\text{Bias}(t_{Re}) = \frac{\phi}{8}\overline{Y}C_x^2[3 - 4(C + f)], \text{ where } \phi = \frac{1}{1-f}$$

$$\text{Bias}(t_{pe}) = \frac{\phi}{8}\overline{Y}C_x^2\left[4C - \frac{1}{(1-f)}\right]$$

$$\text{MSE}(t_{Re}) = \phi\overline{Y}^2\left[C_y^2 + \frac{C_x^2}{4}(1 - 4c)\right] \tag{5}$$

$$\text{MSE}(t_{pe}) = \phi\overline{Y}^2\left[C_y^2 + \frac{C_x^2}{4}(1 + 4c)\right] \tag{6}$$

5. PROPOSED PREDICTIVE ESTIMATOR OF POPULATION MEAN

Motivated by work of Milton [2017] in the improvement of the performance of the population mean of the study variable using known population parameters of an auxiliary variable. We propose the following modified ratio type population mean estimator using a known value of population coefficient of kurtosis κ_x and median M_x of an auxiliary variable x.

$$\bar{y}_{dk} = \bar{y}\left[\frac{\overline{X}\,\kappa_x + M_x^2}{\bar{x}\,\kappa_x + M_x^2}\right] \tag{7}$$

We will use $\bar{y}_{dk}$ as a predictor for population mean. Hence using $T = \bar{y}_{dk}$ in equation (3).

Now the proposed predictive estimator will be given as

$$t = \frac{n}{N}\bar{y} + \frac{(N-n)}{N}\bar{y}_{dk}$$
$$= f\bar{y} + (1-f)\,\bar{y}\,t_1 \tag{8}$$

Where, $t_1 = \bar{y}\left[\frac{\overline{X}\,\kappa_x + M_x^2}{\bar{x}\,\kappa_x + M_x^2}\right]$

To calculate the Bias and MSE of t, we are using the following approximations,

$$\bar{y} = \overline{Y}(1 + e_0),\ \bar{x} = \overline{X}(1 + e_1),\ E(e_0) = E(e_1) = 0, E(e_0^2) = \lambda C_y^2, E(e_1^2) = \lambda C_x^2,\ E(e_0e_1) = \lambda C_{yx}$$

where $\lambda = \frac{1-f}{n}$

$$= \bar{y}\left[\frac{\overline{X}_{\bar{s}}\,\kappa_x + M_x^2}{\bar{x}\,\kappa_x + M_x^2}\right]$$

$$= \overline{Y}(1 + e_0)\left[\frac{\left(\frac{N\overline{X} - n\bar{x}}{N-n}\right)\kappa_x + M_x^2}{\overline{X}(1 + e_1)\,\kappa_x + M_x^2}\right]$$

$$= \overline{Y}(1 + e_0)\left[\frac{\left(\frac{N\overline{X} - n\overline{X}(1 + e_1)}{N-n}\right)\kappa_x + M_x^2}{\overline{X}(1 + e_1)\,\kappa_x + M_x^2}\right]$$

$$= \overline{Y}(1 + e_0)\left[\frac{(\overline{X}\,\kappa_x + M_x^2) - (\frac{n}{N-n})\overline{X}e_1\kappa_x}{(\overline{X}\,\kappa_x + M_x^2) + \overline{X}\kappa_x e_1}\right]$$

$$= \overline{Y}(1 + e_0)\left[\frac{1 - \left(\frac{f}{1-f}\right)\left(\frac{\overline{X}}{\overline{X}\kappa_x + M_x^2}\right)\kappa_x e_1}{(1 + \overline{X}\,\kappa_x\,e_1)}\right]$$

$$= \overline{Y}(1 + e_0)\left[\frac{1 - f_1\theta\kappa_x e_1}{1 + \overline{X}\kappa_x e_1}\right], \quad \text{where } f_1 = \frac{f}{1-f} \text{ and } \theta = \frac{\overline{X}}{\overline{X}\,\kappa_x + M_x^2}$$

$$= \overline{Y}(1 + e_0)(1 - f_1\theta\kappa_x e_1)(1 + \overline{X}\kappa_x e_1)^{-1}$$

$$= \overline{Y}(1 + e_0)[1 - f_1\theta\kappa_x e_1][1 - \overline{X}\kappa_x e_1 + \overline{X}^2\kappa_x^2 e_1^2]$$

$$t_1 = \overline{Y}(1 + e_0)[1 - \overline{X}\kappa_x e_1 + \overline{X}^2 e_1^2\kappa_x^2 - f_1\theta\kappa_x e_1 + f_1\theta\kappa_x^2 e_1^2] \tag{9}$$

Substituting value of t_1 in (8),

$$t = f\bar{Y}(1+e_0) + (1-f)\ \bar{Y}(1+e_0)[1 - \bar{X}\kappa_x e_1 + \bar{X}^2 e_1^2\kappa_x^2 - f_1\theta\kappa_x e_1 + f_1\theta\kappa_x^2 e_1^2]$$

$$t - \bar{Y} = \bar{Y}[\bar{X}\kappa_x e_1 + \bar{X}^2 e_1^2\kappa_x^2 - f_1\theta\kappa_x e_1 + f_1\theta k_x^2 e_1^2 + f\bar{X}\kappa_x e_1 - f\bar{X}e_1^2\kappa_x^2 + ff_1\theta\kappa_x e_1 - ff_1\theta\kappa_x^2 e_1^2 + e_0 - \bar{X}\kappa_x e_0 e_1 - f_1\theta\kappa_x e_0 e_1 + f\bar{X}\kappa_x e_0 e_1 + ff_1\theta\kappa_x e_0 e_1] \quad (10)$$

Taking expectation on both sides

$$E(t - \bar{Y}) = \bar{Y}[\bar{X}\kappa_x E(e_1) + \bar{X}^2\kappa_x^2 E(e_1^2) - f_1\theta\kappa_x E(e_1) + f_1\theta k_x^2 E(e_1^2) + f\bar{X}\kappa_x E(e_1) - f\bar{X}\kappa_x^2 E(e_1^2) + ff_1\theta\kappa_x E(e_1) - ff_1\theta\kappa_x^2 E(e_1^2) + E(e_0) - \bar{X}\kappa_x E(e_0 e_1) - f_1\theta\kappa_x E(e_0 e_1) + f\bar{X}\kappa_x E(e_0 e_1) + ff_1\theta\kappa_x E(e_0 e_1)]$$

using the values of $E(e_0), E(e_1), E(e_1^2), E(e_0 e_1)$, we get the bias (t)

$$\text{Bias}(t) = \lambda\bar{Y}[\bar{X}\kappa_x^2 C_x^2 - f_1\theta C_x^2\kappa_x^2 - f\bar{X}\kappa_x^2 C_x^2 - ff_1\theta\kappa_x^2 C_x^2 - \bar{X}\kappa_x C_{yx} - f_1\theta\kappa_x C_{yx} + f\bar{\bar{X}}\kappa_x C_{yx} + ff_1\theta\kappa_x C_{yx}]$$

Squaring (10) both sides, we get

$$(t - \bar{Y})^2 = \bar{Y}^2[\bar{X}\kappa_x e_1 + \bar{X}^2 e_1^2\kappa_x^2 - f_1\theta\kappa_x e_1 + f_1\theta k_x^2 e_1^2 + f\bar{X}\kappa_x e_1 - f\bar{X}e_1^2\kappa_x^2 + ff_1\theta\kappa_x e_1 - ff_1\theta\kappa_x^2 e_1^2 + e_0 - \bar{X}\kappa_x e_0 e_1 - f_1\theta\kappa_x e_0 e_1 + f\bar{X}\kappa_x e_0 e_1 + ff_1\theta\kappa_x e_0 e_1]^2$$

Using first order approximation, we have

$$= \bar{Y}^2[e_0 + \bar{X}\kappa_x e_1 - f_1\theta\kappa_x e_1 + f\bar{X}\kappa_x e_1 + ff_1\theta\kappa_x e_1]^2$$

$$= \bar{Y}^2[e_0^2 + (\bar{X}\kappa_x e_1 - f_1\theta\kappa_x e_1)^2 + (f\bar{X}\kappa_x e_1 + ff_1\theta\kappa_x e_1)^2 + 2e_0(\bar{X}\kappa_x e_1 - f_1\theta\kappa_x e_1) + 2(\bar{X}\kappa_x e_1 - f_1\theta\kappa_x e_1)(f\bar{X}\kappa_x e_1 + ff_1\theta\kappa_x e_1) + 2e_0(f\bar{X}\kappa_x e_1 + ff_1\theta\kappa_x e_1)]$$

On solving we get

$$= \bar{Y}^2[e_0^2 + \bar{X}^2\kappa_x^2 e_1^2 + f_1^2\theta^2\kappa_x^2 - 2\bar{X}\kappa_x^2\theta e_1^2 + f^2\bar{X}^2\kappa_x^2 e_1^1 + f^2 f_1^2\theta^2\kappa_x^2 e_1^2 + 2f^2 f_1\bar{X}k_x^2 e_1^2 + 2e_0\bar{X}\kappa_x e_1 - 2f_1\theta\kappa_x e_0 e_1 + 2\bar{X}^2 f\kappa_x^2 e_1^2 + 2ff_1\theta\bar{X}\kappa_x^2 e_1^2 - 2f_1 f\theta\kappa_x^2 e_1^2 - 2ff_1^2\theta^2\kappa_x^2 e_1^2 + 2f\bar{X}\kappa_x e_0 e_1 + 2ff_1\theta\kappa_x e_0 e_1]$$

Taking expectation on both sides, we get MSE of proposed estimator t

$$E(t - \bar{Y})^2 = \lambda\bar{Y}^2[E(e_0^2) + \bar{X}^2\kappa_x^2 E(e_1^2) + f_1^2\theta^2\kappa_x^2 - 2\bar{X}\kappa_x^2\theta E(e_1^2) + f^2\bar{X}^2\kappa_x^2 E(e_1^2) + f^2 f_1^2\theta^2\kappa_x^2 E(e_1^2) + 2f^2 f_1\bar{X}k_x^2 E(e_1^2) + 2\bar{X}\kappa_x E(e_0 e_1) - 2f_1\theta\kappa_x E(e_0 e_1) + 2\bar{X}^2 f\kappa_x^2 E(e_1^2) + 2ff_1\theta\bar{X}\kappa_x^2 E(e_1^2) - 2f_1 f\theta\kappa_x^2 E(e_1^2) - 2ff_1^2\theta^2\kappa_x^2 E(e_1^2) + 2f\bar{X}\kappa_x E(e_0 e_1) + 2ff_1\theta\kappa_x E(e_0 e_1)]$$

Substituting the values of $E(e_0^2), E(e_1^2)$ and $E(e_0 e_1)$

$$\text{MSE}(t) = \lambda\bar{Y}^2\left[C_y^2 + \bar{X}^2\kappa_x^2 C_x^2(1+f^2) + f_1^2\theta^2\kappa_x^2 C_x^2 - 2\bar{X}\kappa_x^2\theta C_x^2 + f^2 f_1^2\theta^2\kappa_x^2 C_x^2 + 2f^2 f_1\bar{X}\kappa_x^2 C_x^2 + 2\bar{X}\kappa_x C_{yx} - 2f_1\theta\kappa_x C_{yx} + 2\bar{\bar{X}}^2 f\kappa_x^2 C_x^2 + 2ff_1\theta\bar{X}\kappa_x^2 C_x^2 - 2f_1 f\theta\kappa_x^2 C_x^2 - 2ff_1^2\theta^2\kappa_x^2 C_x^2 + 2f\bar{X}\kappa_x C_{yx} + 2ff_1\theta\kappa_x C_{yx}\right]$$

$$\text{MSE}(t) = \lambda\bar{Y}^2\left[C_y^2 + \{\bar{X}^2\kappa_x^2(1+f^2) + f_1^2\theta^2\kappa_x^2 - 2\bar{X}\kappa_x^2\theta + f^2 f_1^2\theta^2\kappa_x^2 + 2f^2 f_1\bar{X}\kappa_x^2 + 2ff_1\theta\bar{X}\kappa_x^2 + 2\bar{\bar{X}}^2 f\kappa_x^2 - 2f_1 f\theta\kappa_x^2 - 2ff_1^2\theta^2\kappa_x^2\}C_x^2 + \{2\bar{X}\kappa_x - 2f_1\theta\kappa_x + 2f\bar{X}\kappa_x + 2ff_1\theta\kappa_x\}C_{yx}\right] \quad (11)$$

6. THEORETICAL EFFICIENCY COMPARISON

In this section we have made a theoretical comparison of proposed estimator with Bahl and Tuteja [1991] estimator and the Singh *et al.* [2014] estimator and derive the optimal conditions under which proposed estimator performs better than these competing estimators.

i. Proposed estimator t will perform better than the Bahl and Tuteja [1991] estimator t_0, iff MSE (t) $< MSE(t_0)$, from (11) and (4),

$$\begin{bmatrix} C_y^2 + \{\bar{X}^2\kappa_x^2(1+f^2) + f_1^2\theta^2\kappa_x^2 - 2\bar{X}\kappa_x^2\theta + f^2 f_1^2\theta^2\kappa_x^2 + 2f^2 f_1\bar{X}\kappa_x^2 + 2ff_1\theta\bar{X}\kappa_x^2 + 2\bar{\bar{X}}^2 f\kappa_x^2 \\ -2f_1 f\theta\kappa_x^2 - 2ff_1^2\theta^2\kappa_x^2\}C_x^2 + \{2\bar{X}\kappa_x - 2f_1\theta\kappa_x + 2f\bar{X}\kappa_x + 2ff_1\theta\kappa_x\}C_{yx} \end{bmatrix} < \left[C_y^2 + \frac{C_x^2}{4} - C_{yx}\right]$$

ii. Proposed estimator t will perform better than the Singh *et al.*[2014] estimators, iff

(a) MSE (t) $< MSE(t_{Re})$, from (11) and (5),

$$\left[C_y^2 + \{\bar{X}^2\kappa_x^2(1+f^2) + f_1^2\theta^2\kappa_x^2 - 2\bar{X}\kappa_x^2\theta + f^2 f_1^2\theta^2\kappa_x^2 + 2f^2 f_1\bar{X}\kappa_x^2 + 2ff_1\theta\bar{X}\kappa_x^2 + 2\bar{\bar{X}}^2 f\kappa_x^2 - 2f_1 f\theta\kappa_x^2 - 2ff_1^2\theta^2\kappa_x^2\}C_x^2 + \{2\bar{X}\kappa_x - 2f_1\theta\kappa_x + 2f\bar{X}\kappa_x + 2ff_1\theta\kappa_x\}C_{yx}\right] < \phi\left[C_y^2 + \frac{C_x^2}{4}(1-4c)\right]$$

(b) MSE (t) $< MSE(t_{pe})$, from (11) and (6),

$$\left[C_y^2 + \{\bar{X}^2\kappa_x^2(1+f^2) + f_1^2\theta^2\kappa_x^2 - 2\bar{X}\kappa_x^2\theta + f^2 f_1^2\theta^2\kappa_x^2 + 2f^2 f_1\bar{X}\kappa_x^2 + 2ff_1\theta\bar{X}\kappa_x^2 + 2\bar{\bar{X}}^2 f\kappa_x^2 - 2f_1 f\theta\kappa_x^2 - 2ff_1^2\theta^2\kappa_x^2\}C_x^2 + \{2\bar{X}\kappa_x - 2f_1\theta\kappa_x + 2f\bar{X}\kappa_x + 2ff_1\theta\kappa_x\}C_{yx}\right] < \phi\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4}(1+4c)\right]$$

7. NUMERICAL ILLUSTRATION

To see the performance of the proposed estimator and to verify the conditions under which the proposed estimator performs better than mentioned estimators of the population mean in literature, the numerical example taken from Murthy [1967], pg. 228, in which

X= Fixed capital
Y= Output of 80 factories

Table-1: Values of parameters and constants computed from given population

Parameters/Constants	Values
N	80
n	20
F	0.25
$\bar{X}$	11.26
$\bar{Y}$	51.82
κ_x	2.86
ρ_{xy}	0.94
C_y	0.35
C_x	0.75
M_x	10.30
θ	0.081
f_1	0.33
λ	0.037
Φ	1.33
C	0.44
C_{yx}	0.25

Table-2: Mean squared Errors of existing and proposed estimators

S. No	Estimators	MSE
1	Bahl and Tuteja (1991) Estimator	$MSE(t_0)$ = 2,70,383.86
2	Singh et al (2014) Estimator	(i) $MSE(t_{Re}) = 9590029.95$ (ii) $MSE(t_{pe}) = 9591799.89$
3	Proposed Estimator (t)	MSE(t) = 66, 982.32

Table-3: PRE of the proposed estimator with respect to existing estimators

S. No	Estimators	PRE
1	t_0	403.66
2	t_{Re}	14317.25
3	t_{pe}	14319.89

8. CONCLUSIONS, DISCUSSIONS AND RECOMMENDATIONS

In the present article a modified ratio type predictive estimator has been proposed by using coefficient of kurtosis and median of auxiliary variable. Use of coefficient of kurtosis is significant in estimation process since it provides information about the nature of data. Median is also powerful statistical tool since it is unaffected by outliers present in the data. Table-1 describes the values of parameters and constants used for numerical study. From the table-2, it is seen that our proposed estimator has lesser MSE than that of competing estimators. Table-3 gives clear indication that proposed estimator is more efficient with respect to other existing estimators. Therefore proposed estimator is recommended to survey practitioners for estimation of population mean under predictive modelling approach especially in the situations where outliers are present in the data.

REFERENCES

1. Agrawal, M. C., Roy, D. C. (1999). Efficient estimators of population variance with regression-type and ratio-type predictor-inputs. Metron, 1999, 57(3), pp. 4–13
2. Basu, D. (1971). An Essay on the Logical Foundation of Survey Sampling, Part I. Foundations of Statistical Inference, eds. Godambe,V. P., Sprott, D. A. Toronto: Holt, Rinehart and Winston, pp. 203–242.
3. Bahl, S. And Tuteja, R.K. (1991). Ratio and product type exponential estimator, Information and Optimization Sciences, XII (I), 159-163.

4. Cochran, W. G. (1994). Sampling Techniques. Third Edition, Wiley Eastern limted.
5. Cochran, W. G. (1940). The Estimation of the Yields of the Cereal Experiments by Sampling for the Ratio of Grain to Total Produce. The Journal of Agric. Science, 30, 262-275.
6. Hirano, K., Pandey, B.N., Singh, J. (1973). On the Utilization of a Known Coefficient of Kurtosis in the Estimation Procedure of Variance. Annals of the Institute of Statistical Mathematics, 25, 51-55.
7. Kadilar, G.O. (2016). A New Exponential Type Estimator for the Population Mean in Simple Random Sampling, Journal of Modern Applied Statistical Methods, 15, 2, 207-214.
8. Kadilar, C., Cingi, H. (2004). Ratio estimators in simple random sampling, Applied Mathematics and Computation, 151, 893–902.
9. Kadilar, C., Cingi, H. (2006a). An improvement in estimating the population mean by using the correlation co-efficient, Hacettepe Journal of Mathematics and Statistics, 35 (1), 103–109.
10. Kadilar, C., Cingi, H. (2006b). Improvement in estimating the population mean in simple random sampling, Applied Mathematics Letters, 19, 75–79.
11. Kadilar, C., Cingi, H. (2009). Advances in sampling theory - Ratio method of estimation, Bentham Science Publishers.
12. Khan, M. (2016). A ratio chain-type exponential estimator for finite population mean using double sampling, Springer Plus, DOI 10.1186/s40064-016-1717-4.
13. Koyuncu, N., Kadilar, C. (2009). Efficient estimators for the population mean, Hacettepe Journal of Mathematics and Statistics, 38(2), 217–225.
14. Milton, T.K., Odhiambo, R.O. Orwa, G.O. (2017). Estimation of Population Variance Using the Coefficient of Kurtosis and Median of an Auxiliary Variable under Simple Random Sampling. Open Journal of Statistics, 7, 944-955.
15. Misra, S., Singh, R. K, Shukla, A.K. (2013) Partial Predictive Estimation of Finite Population Variance Using Product Type Estimator" International Journal of Statistics and Analysis (IJSA) Number 2 (2013) pp. 211-219.
16. Murthy, M. N. (1967). Sampling theory and methods. Calcutta Statistical Publishing House, India.
17. Mukhopadhyay, P. (2005). Theory and methods of survey sampling, PHI Learning, 2nd edition, New Delhi.
18. Nayak, R., Sahoo, L (2012). Some alternative predictive estimators of population variance. Revista Colombiana de Estadistica, 2012, 35(3), pp. 509–521.
19. Sahoo, L. N., Das, B. C., Sahoo, J. A (2009) Class of Predictive Estimators in Two stage sampling. Journal of the Indian Society of Agricultural Statistics, 63(2), pp. 175–180.
20. Sahoo, L. N., Panda P. (1999). A predictive regression-type estimator in two-stage sampling. Journal of the Indian Society of Agricultural Statistics, 52(3), pp. 303–308.
21. Sahoo, L. N., Sahoo, R. K. (2001). Predictive estimation of finite population mean in two phase sampling using two auxiliary variables. Journal of the Indian Society of Agricultural Statistics, 54(2), pp. 258–264.
22. Saini, M (2013). A class of predictive estimators in two-stage sampling when auxiliary character is estimated at SSU level. International Journal of Pure and Applied Mathematics, 85(2), pp. 285–295.
23. Searls, D.T. (1964). The Utilization of a Known Coefficient of Variation in the Estimation Procedure. Journal of the American Statistical Association, 59, 1225-1226.
24. Sen, A.R. (1978). Estimation of the Population Mean When the Coefficient of variation is Known. Communications in Statistics—Theory and Methods, 7, 657-672.
25. Sisodia, B., Dwivedi, V. (1981) Modified Ratio Estimator Using Coefficient of Variation of Auxiliary Variable. Journal-Indian Society of Agricultural Statistics, 33, 13-18.
26. Upadhyaya, L.N., Singh, H.P. (1984) On the Estimation of the Population Mean With Known Coefficient of Variation. Biometrical Journal , 26, 915-922.
27. Singh, G.N. (2003). On the improvement of product method of estimation in sample surveys, Journal of Indian Society of Agricultural Statistics, 56 (3), 267–265.
28. Singh, D., Chaudhary, F. S. (1986). Theory and analysis of sample survey designs, New Age International Publisher, New Delhi.
29. Singh, H. P., Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population means, Statistics in Transition, 6 (4), 555–560.
30. Singh, H. P., Tailor, R. (2005). Estimation of finite population mean with known co-efficient of variation of an auxiliary variable, Statistica, anno LXV, 3, 301–313.
31. Singh, H. P., Solanki, R. S., Singh, A. K. (2014). Predictive Estimation of Finite Population Mean Using Exponential Estimators. Statistika: Statistics and Economy Journal, 94 (1), pp. 41–53.
32. Srivastava, S. K. (1983). Predictive estimation of finite population mean using product estimator. Metrika, 30, pp. 93–99.
33. Yadav, S. K., Mishra, S. S (2015). Developing Improved Predictive Estimator for Finite Population Mean Using Auxiliary Information, Statistika, 95(1).

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