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FUZZY GENERALIZED CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we investigate fuzzy generalized closed sets in fuzzy topological spaces. Here we also investigate a new class of fuzzy sets called fuzzy strongly g -closed sets and g** -closed sets in fuzzy topological spaces is introduced and its various characterizations are studied. Examples are presented showing that some generalizations cannot be obtained. The concept of generalized closed sets plays a significant role in topology. There are many research papers which deals with different types of generalized closed sets. Moreover, we study some more properties of this type of closed spaces.*

Keyword: *Fuzzy generalized closed sets • Fuzzy strongly g ∗- closed sets • Fuzzy g ∗∗-closed sets.*

I. INTRODUCTION

In 1965, Zadeh introduced the concept of fuzzy sets. Many researchers have been worked in this area and related areas which have applications in different field based on this concept. The concept of generalized closed sets plays a significant role in topology. There are many research papers which deals with different types of generalized closed sets. Levine introduced the generalized closed sets in topological spaces. Chang in introduced the concept of fuzzy topological spaces.

BASIC CONCEPTS

A family τ of fuzzy sets of X is called a fuzzy topology on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complements are called fuzzy closed sets.

Definitions:
Fuzzy α-open if $A \leq int[cl(int A)]$ and a fuzzy α-closed if $cl(int[cl(A)]) \leq A$.
Fuzzy semi pre-open if $A \leq cl(int[cl(A)])$ and a fuzzy semi pre-closed if $int[cl(int(A))] \leq A$.
Fuzzy θ-open if $A=int_\theta(A)$ and a fuzzy θ-closed if $A=cl_\theta(A)$ where $cl_\theta(A) = \wedge\{cl(\mu):A \leq \mu, \mu \in \tau\}$.
Fuzzy generalized closed if $cl(A) \leq G$, whenever $A \leq G$ and G is fuzzy open set in X.
Fuzzy generalized semi closed if $scl(A) \leq G$, whenever $A \leq G$ and G is fuzzy open set in X.
Fuzzy semi-pre-generalized closed in $spcl(A) \leq G$, whenever $A \leq G$ and G is f s-open in X.

Fuzzy Strongly g*-closed sets

Let (X, τ) be a fuzzy topological space. A fuzzy set B of (X, τ) is called fuzzy strongly g*-closed if $cl(int(B)) \leq G$, whenever $B \leq G$ and G is fg-open in X.

Theorem 1: Every fuzzy closed set in fuzzy strongly g*-closed set in a fuzzy topological space (X, τ).

Proof: Let A be fuzzy closed set in X. Let H be a fg-open set in X such that $A \leq H$. Since A is fuzzy closed, $cl(A)=A$.

Therefore $cl(A) \leq H$. Now $cl(int(A)) \leq cl(A) \leq H$. Hence A is fuzzy strongly g*-closed set in X.

The converse of the above theorem need not be true in general. That can be seen by the following example.

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Example: Let X ={a, b, c}. Fuzzy sets A and B are defined by A(a) = 0.7, A(b) = 0.3, A(c) = 0.5, B(a) = 0.2, B(b) = 0.1, B(c) = 0.3. Let τ ={0, A, 1}. Then B is a fuzzy strongly g* -closed set but it is not a fuzzy closed set in (X, τ).

Theorem 2: Every fuzzy g-closed set is fuzzy strongly g* -closed sets in X.

Proof: Obvious.

Converse of the above theorem need not be true. It can be seen by the following example.

Example: Let X={a, b} and the fuzzy sets A and B be defined by A(a) = 0.3, A(b) = 0.3, B(a) = 0.5, B(b) = 0.4. Let τ = {0, B, 1} then A is fuzzy strongly g*-closed but it is not fg-closed.

Theorem 3: Every fuzzy g*- closed set is a fuzzy strongly g*- closed sets in X.

Proof: Suppose that A is fg*-closed set in X. Let H be a fg-open set in X such that A ≤ H. Then cl(A) ≤ H, since A is fg*-closed. Cl(intA) ≤ cl(A) ≤ H. Hence A is fuzzy strongly g*-closed set in X.

However the converse of the above theorem need not be true as seen from the following example.

Example: Let X={a, b}, τ = {0, A, B, D, 1} and the fuzzy sets A, B, D, H are defined as follows A(a) = 0.2, A(b) = 0.4, B(a) = 0.6, B(b) = 0.7, D(a) = 0.4, D(b) = 0.6, H(a) = 0.4, H(b) = 0.5. Then H is fuzzy strongly g*-closed sets but it is not fg*-closed set in (X, τ).

Theorem 4: A fuzzy set A of fuzzy topological space (X, τ) is fuzzy strongly g*-closed if and only if A $\bar{q}$ B then cl(int(A)) $\bar{q}$ B for every fg-closed set B of X.

Theorem 5: If A is a fuzzy strongly g*-closed set in (x, τ) and A≤ B ≤ cl(int(A)) and B is fuzzy strongly g*- closed set in X.

Definition: *A fuzzy set A of (X, τ) is called fuzzy strongly g*-open set in X if and only if 1-A is fuzzy strongly g*-closed in X. In other words, A fuzzy strongly g*-open if and only if H ≤ cl(int(A)), whenever H ≤ A of H is fg-closed in X.*

Theorem 6: Let A be fuzzy g*-open in X and int(cl(A)) ≤ B ≤ A then B is fuzzy stongly g*-open in X.

Theorem 7: If a fuzzy set A of a fuzzy topological space X is both fuzzy open and fuzzy strongly g*-closed then it is fuzzy closed.

Proof: Suppose that a fuzzy set A of X is both fuzzy open and fuzzy strongly g*-closed. Now A ≥ cl(int(A)) ≥ cl(A). That is A ≥ cl(A). Since A ≤ cl(A), we get A=cl(A). Hence A is fuzzy closed in X.

Theorem 8: If a fuzzy set A of a fuzzy topological space X is both fuzzy strongly g*-closed and fuzzy semi open then it is fg*-closed.

Proof: Suppose a fuzzy set A of X is both fuzzy strongly g*-closed and fuzzy semi open in X. Let H be a fg-open set such that A ≤ H. Since A is fuzzy strongly g*-closed, therefore cl(int(A)) ≤ H.

Also since A is f s-open, A ≤ cl(int(A)). We have cl(A) ≤ cl(int(A)) ≤ H. Hence A is fg*-closed in X.

Theorem 9: Every f Ɵ -closed set is a fuzzy strongly g* -closed set.

The following example shows that the converse of the above theorem is not true in general.

Example: Let X={a, b}, τ={0, A, 1} and the fuzzy sets A and B are defined as follows A(a) = 0.3, A(b) = 0.7, B(a) = 0.6, B(b) = 0.5, Then B is fuzzy strongly g* -closed set but it is not f θ-closed set.

FUZZY g -CLOSED SETS**

A subset A of a topological space (X, τ) is called fuzzy g** -closed set if cl(A) ≤ H, whenever A ≤ H and H is fuzzy g*-open in (X, τ). The complement of a fuzzy g**-closed set is fuzzy g** -open set.

Theorem 10: Every fuzzy closed set is fuzzy g** -closed set, but the converse may not be true in general.

Theorem 11: Every fuzzy g* -closed set is fuzzy g** -closed set, but the converse may not be true in general.

Proof: Let A be fuzzy g*- closed set.

By the definition of fuzzy g*-closed set, if cl(A) $\leq$ H whenever A$\leq$H and H is fuzzy g -open. Let A $\leq$ H where H is fuzzy g* -open. Since H is fuzzy g* -open then H is fuzzy g-open. Thus cl(A)$\leq$H, H is fuzzy g* -open, A is fuzzy g** -closed set.

Theorem 12: If A is fuzzy strongly g* -closed and A is fuzzy open then A is fuzzy g**-closed set.

Proof: Let A$\leq$H and H is fuzzy g* -open. Since A is fuzzy strongly g*-closed set cl(int(A)) $\leq$ H, whenever A $\leq$ H and H is fuzzy g-open in X. Since A is open, int(A)=A, also is fuzzy g*-open then is fuzzy g-open. Thus cl(A) $\leq$ H, H is fuzzy g* -open, A is fuzzy g**-closed.

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