International Journal of Mathematical Archive-9(5), 2018, 216-220
Available online through www.ijma.info ISSN 2229 – 5046

BIANCHI TYPE I SOLUTION IN $f(\mathrm{T})$ GRAVITY

CHANDRA REKHA MAHANTA[1] AND NITIN SARMA*[2]

[1]Department of Mathematics, Gauhati University, Guwahati-14, India.

[2]Department of Mathematics, A. D. P. College, Nagaon-02, India.

(Received On: 28-02-18; Revised & Accepted On: 02-05-18)

ABSTRACT

We study locally rotationally symmetric Bianchi type-I space-time model in f (T) gravity, where T is the torsion scalar. The field equations are solved by using some physical assumptions. Some important cosmological physical quantities such as expansion scalar θ, shear scalar σ^2, average Hubble parameter are evaluated and studied the physical behaviour of the dark energy dominated model. These parameters are infinite at the initial epoch and tend to zero at $t \to \infty$ showing thereby that there is a point type singularity in the model. It is observed that the equation of state parameter ω is negative under certain condition which gives the negative deceleration parameter. We find our results to be in agreement with the current observations.

Keywords: *Bianchi type I, f (T) gravity, Torsion.*

1. INTRODUCTION

The observational data of distant type Ia Supernovae [1, 2] suggest that our universe entered a phase of accelerated expansion in the recent past. A number of cosmological observations such as Wilkinson Microwave Anisotropy Probe (WMAP), Cosmic Microwave Background Anisotropy, Large Scale Structure etc. support this cosmic acceleration which is supposed to be due to some hitherto unknown physical entity, dubbed dark energy (DE), with negative pressure. The simplest candidate for dark energy is the cosmological constant Λ which fits the observation well, but is plagued with the fine-tuning and cosmic coincidence problem [3, 4, 5]. For these reasons the cosmological constant with dynamical character is preferred over the constant one.

According to the latest observations [6], nearly 68.3% of our universe consists of the dark energy and nearly 26.8% consists of another dark component of the universe, called Dark Matter. So, the dark energy becomes the greatest puzzle in Modern Cosmology. In order to investigate the true nature of dark energy, many dynamical dark energy models have been proposed in the literature such as quintessence [7], k-essence [8,9], Chaplygin gas [10], tachyon [11], phantom [12], quintom[13], holographic dark energy [14], agegraphic dark energy [15] etc. But, it is seen that many DE models get into trouble when tested by some old red-shift objects [16, 17] posing a fundamental theoretical challenge to theories of gravitation. One possibility in explaining the recent observations is by assuming that at large scales the gravity model of the General Theory of Relativity breaks down. To describe the accelerated expansion of the universe, another approach is adopted in the literature by modifying the standard theories of gravity, viz. the General Theory of Relativity or the Teleparallel Equivalent of General Relativity. Among the various modifications of the General Theory of Relativity, Brans-Dicke (BD) theory[18], $f(R)$ gravity [19,20,21,22,23], $f(R,T)$ gravity [24] where R is the Ricci scalar and T is the trace of energy momentum tensor, Gauss-Bonnett gravity or $f(G)$ gravity [25,26,27,28,29], $f(R,G)$ gravity [30], $f(G,T)$ gravity[31] etc. are discussed in the literature. Most of the works in the modified theory of gravity starts from the usual curvature based formulation. However, there is also a reason to start from the Teleparallel Equivalent of General Relativity to build a gravitational modification. The simplest such modification is the $f(\mathrm{T})$ theory of gravity where T is the torsion scalar [32,33]. In $f(\mathrm{T})$ theory of gravity, the Lagrangian is taken to be a non-linear function of Teleparallel Equivalent of General Relativity Lagrangian.

Recently many researchers have studied the $f(\mathrm{T})$ theory of gravity [34,35,36,37,38,39,40,41,42,43] in different aspects. Motivated by these we study the anisotropic homogeneous model in $f(\mathrm{T})$ theory of gravity.

Corresponding Author: Nitin Sarma*[2]
[2]Department of Mathematics, A. D. P. College, Nagaon-02, India.

This paper is organized as follows: In Sec.2 we present some basics of $f(\mathrm{T})$ theory of gravity. We present the field equations for a homogeneous and anisotropic LRS Bianchi type-I metric in Sec.3. The Sec.4 is devoted to the solutions of the field equations. We conclude the paper with a brief discussion in Sec.5.

2. SOME BASICS OF f(T) THEORY OF GRAVITY

The $f(\mathrm{T})$ theory of gravity [44] is defined in the Weitzenbock's space time in which the line element is described by,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \eta_{ij}\theta^i\theta^j \tag{1}$$

where $g_{\mu\nu}$ are the metric tensors and $\eta_{ij} = diag\,(+1,-1,-1,-1)$.

$$dx^\mu = e_i{}^\mu\,\theta^i,\ \ \theta^i = e^i{}_\mu\,dx^\mu \tag{2}$$

and

$$e_i^{\ \mu}e^i{}_\nu = \delta_\nu^\mu\ ,\ e_i{}^\mu e^j{}_\mu = \delta_i^j. \tag{3}$$

The square root of the metric determinant is given by $\sqrt{-g} = \det[e^i{}_\mu] = e$ and the matrix $e^a{}_\mu$ are called tetrads and represent the dynamic fields of the theory.

By using these fields, Weitzenbock's connection can be defined as

$$\Gamma^\alpha_{\mu\nu} = e_i^\alpha\partial_\nu e^i{}_\mu = -e^i{}_\mu\,\partial_\nu e_i{}^\alpha \tag{4}$$

The main geometrical objects of the space time are constructed from this connection and the components of the torsion tensor are defined by the anti-symmetric part of this connection

$$\mathrm{T}^\alpha{}_{\mu\nu} = \Gamma^\alpha_{\nu\mu} - \Gamma^\alpha_{\mu\nu} = e_i{}^\alpha\left(\partial_\mu e^i{}_\nu - \partial_\nu e^i{}_\mu\right) \tag{5}$$

The components of the contorsion tensor are defined as

$$K^{\mu\nu}{}_\alpha = -\frac{1}{2}\left(\mathrm{T}^{\mu\nu}{}_\alpha - \mathrm{T}^{\nu\mu}{}_\alpha - \mathrm{T}_\alpha{}^{\mu\nu}\right) \tag{6}$$

In order to make the definition of the scalar equivalent to the curvature scalar of General Theory of Relativity more clear, a new tensor $S_\alpha{}^{\mu\nu}$ constructed from the components of the torsion and contorsion tensors is defined as

$$S_\alpha{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_\alpha + \delta^\mu_\alpha\mathrm{T}^{\beta\nu}{}_\beta - \delta^\nu_\alpha\mathrm{T}^{\beta\mu}{}_\beta\right) \tag{7}$$

The torsion scalar is then defined by the following contraction

$$\mathrm{T} = \mathrm{T}^\alpha{}_{\mu\nu}S_\alpha{}^{\mu\nu} \tag{8}$$

The action of the theory is defined by generalizing the Teleparallel Theory of Gravity [34] as

$$S = \int e[f(\mathrm{T}) + L_{matter}]\,d^4x \tag{9}$$

where $f(\mathrm{T})$ is an algebraic function of the torsion scalar T.

Making the functional variation of the action (9) with respect to the tetrads, the field equations are derived [38, 39, 45] in the following form

$$S_\mu^{\nu\rho}\partial_\rho\mathrm{T}f_{\mathrm{TT}} + \left[e^{-1}e^i_\mu\partial_\rho\left(ee^\alpha_i S_\alpha^{\nu\rho}\right) + \mathrm{T}^\alpha_{\lambda\mu}S_\alpha^{\nu\lambda}\right]f_\mathrm{T} + \frac{1}{4}\delta^\nu_\mu f = 4\pi T^\nu_\mu \tag{10}$$

where T^ν_μ is the energy momentum tensor, $f_\mathrm{T} = \frac{df}{d\mathrm{T}}$ and $f_{\mathrm{TT}} = \frac{d^2f}{d\mathrm{T}^2}$.

The contribution of the interaction with the matter fields is given by the energy momentum tensor which, in this case, is defined as

$$T^\nu_\mu = diag\,\left(1,-\omega_x,-\omega_y,-\omega_z\right)\rho \tag{11}$$

where the $\omega_i(i = x,y,z)$ are the parameters of equations of state related to the pressures $p_x,\ p_y, p_z$ respectively.

3. METRIC AND FIELD EQUATIONS

The line element for a flat, homogeneous and anisotropic LRS Bianchi type-I space time is

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)[dy^2 + dz^2] \tag{12}$$

Using Eqs. (2), (3) and (12), we then obtain the tetrad components as follows

$$e^i{}_\mu = diag\,(1,A,B,B)$$
$$e_\mu{}^i = diag\,(1,\ A^{-1},B^{-1},B^{-1}) \tag{13}$$

The components of the torsion tensor (5) for the tetrads (13) are

$$\mathrm{T}^1{}_{01} = \frac{\dot{A}}{A}\,,\ \mathrm{T}^2{}_{02} = \mathrm{T}^3{}_{03} = \frac{\dot{B}}{B} \tag{14}$$

and the components of the corresponding contorsion tensor are

$$K^{01}{}_1 = \frac{\dot{A}}{A},\ K^{02}{}_2 = \frac{\dot{B}}{B},\ K^{03}{}_3 = \frac{\dot{B}}{B} \tag{15}$$

where a dot denotes derivative with respect to cosmic time *t*.

The components of the tensor $S_\alpha{}^{\mu\nu}$ in (7) are

$$S_1{}^{10} = \frac{\dot{B}}{B},\quad S_2{}^{20} = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right),\quad S_3{}^{30} = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{16}$$

Using the components (14) and (16), the torsion scalar (8) is given by

$$\mathrm{T} = -2\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2}\right) \tag{17}$$

The field equations corresponding to the metric (12) are obtained by

$$f + 4f_{\mathrm{T}}\left[\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right] = 16\pi\rho \tag{18}$$

$$f + 4f_{\mathrm{T}}\left[\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right] + 4\frac{\dot{B}}{B}\dot{\mathrm{T}}f_{\mathrm{TT}} = -16\pi p_x \tag{19}$$

$$f + 2f_{\mathrm{T}}\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right] + 2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{\mathrm{T}}f_{\mathrm{TT}} = -16\pi p_y \tag{20}$$

$$p_y = p_z \tag{21}$$

The physical quantities which are important in cosmological observations are the average scale factor a, the average Hubble's parameter H, the scalar expansionθ, deceleration parameter q, shear scalar σ^2 and the average anisotropy parameter A_m. For the metric (12), these are given by

$$a = (AB^2)^{\frac{1}{3}} \tag{22}$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{23}$$

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{24}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{25}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right]^2 \tag{26}$$

$$A_m = \frac{2}{9H^2}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{27}$$

4. COSMOLOGICAL SOLUTION

To solve the field equations (18) to (20), we need some additional constraints. One of the commonly used additional constraints is that the expansion scalar θ is proportional to shear scalar σ which gives [46,47,48,49,50,51]

$$A = B^m \tag{28}$$

Using this condition, Eqs. (18) to (20) give

$$f + 4f_{\mathrm{T}}(1+2m)\frac{\dot{B}^2}{B^2} = 16\pi\rho \tag{29}$$

$$f + 4f_{\mathrm{T}}\left[\frac{\ddot{B}}{B} + (1+m)\frac{\dot{B}^2}{B^2}\right] + 4\frac{\dot{B}}{B}\dot{\mathrm{T}}f_{\mathrm{TT}} = -16\pi p \tag{30}$$

$$f + 2f_{\mathrm{T}}\left[(1+m)^2\frac{\dot{B}^2}{B^2} + (1+m)\frac{\ddot{B}}{B}\right] + 2(1+m)\frac{\dot{B}}{B}\dot{\mathrm{T}}f_{\mathrm{TT}} = -16\pi p \tag{31}$$

where we have assumed for simplicity that $p_x = p_y = p_z = p$ and EOS $p = \omega\rho$.

Now,

$$f_{\mathrm{T}} = \frac{df}{d\mathrm{T}} = \frac{\dot{f}}{\dot{\mathrm{T}}} \text{ and } f_{\mathrm{TT}} = \frac{\dot{\mathrm{T}}\ddot{f} - \dot{f}\ddot{\mathrm{T}}}{\dot{\mathrm{T}}^3} \tag{32}$$

Subtracting (31) from (30) and using (32), we get

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2}(1+m) + \frac{\dot{B}}{B}\frac{\dot{F}}{F} = 0 \tag{33}$$

where $F = f_{\mathrm{T}}$.

Here $F = f_{\mathrm{T}} = \frac{\dot{f}}{\dot{\mathrm{T}}}$ and the scale factor B both are functions of the cosmic time t, so it is reasonable to take a relation between F and B as

$$F = kB^{1+m} \tag{34}$$

where k is the constant of proportionality.

Using this in Eq. (33), it follows that

$$\frac{\ddot{B}}{B}+(2+2m)\frac{\dot{B}^2}{B^2}=0 \tag{35}$$

Put $\dot{B}=h(B)$ in this equation, we get

$$\frac{dh^2}{dB}+\frac{4+4m}{B}h^2=0 \tag{36}$$

which leads to the solution

$$h^2=\frac{c}{B^{4+4m}} \tag{37}$$

where c is constant of integration.

Solving (37), we get

$$B(t)=\{(3+2m)(c_1t+c_2)\}^{\frac{1}{3+2m}} \tag{38}$$

Using Eq.(38), Eqs. (17), (28), (29), (31), (34) give

$$A(t)=\{(3+2m)(c_1t+c_2)\}^{\frac{m}{3+2m}} \tag{39}$$

$$\mathrm{T}=-2\frac{(1+2m)}{(3+2m)^2}{c_1}^2(c_1t+c_2)^{-2} \tag{40}$$

$$f=-4k\frac{(1+2m)}{5+3m}(3+2m)^{-\frac{2+m}{3+2m}}\ {c_1}^2(c_1t+c_2)^{-\frac{5+3m}{3+2m}} \tag{41}$$

$$\rho=\frac{1}{4\pi}k\frac{(1+2m)(2+m)}{(5+3m)}{c_1}^2\{(3+2m)(c_1t+c_2)\}^{-\frac{5+3m}{3+2m}} \tag{42}$$

$$p=\frac{1}{4\pi}k\frac{1+2m}{5+3m}(3+2m)^{-\frac{2+m}{3+2m}}\ {c_1}^2(c_1t+c_2)^{-\frac{5+3m}{3+2m}} \tag{43}$$

The expressions for the Hubble parameter H, scalar of expansion θ, shear scalar σ and the average anisotropy parameter A_m are obtained as

$$\theta=3H=c_1\frac{2+m}{3+2m}(c_1t+c_2)^{-1} \tag{44}$$

$$\sigma^2=\frac{1}{3}\left(\frac{m-1}{3+2m}\right)^2{c_1}^2(c_1t+c_2)^{-2} \tag{45}$$

$$A_m=\frac{2(m-1)^2}{(m+2)^2} \tag{46}$$

The value of the deceleration parameter is found to be

$$q=\frac{6m+9}{m+2}-1 \tag{47}$$

which is constant. The sign of q indicates whether the model inflates or not. The current observations of SNe Ia and CMBR favors accelerating model. Our model is in agreement with the current observation for <-1.4 .

Using (42) and (43), the EOS parameter ω is obtained as

$$\omega=\frac{2m+3}{m+2} \tag{48}$$

This shows that the EOS parameter is dependent on m and $\omega<0$ for $-2<m<-1.5$. It supports the accelerated expanding universe. From Eqs. (44) to (46), it is seen that all the kinematical parameter H, θ and σ diverge at the initial singularity. There is also a Point Type singularity [52] at $t=-\frac{c_2}{c_1}$ in the model. The mean anisotropic parameter is constant and it depends on m. Since $\frac{\sigma^2}{\theta^2}=constant$, the model does not approach isotropy throughout the whole evolution of the universe.

5. CONCLUSION

In this paper, we study Bianchi type-I space-time in *f* (T) gravity theory where T denotes the torsion scalar. The model is derived by using some assumptions. The first assumption is that the expansion scalar θ is proportional to shear scalar σ. It gives $A=B^m$, where A, B are the metric co-efficient and m is a real constant. Second assumption is that, $p_x=p_y=p_z=p$ and an EoS $p=\omega\rho$. A power law relation between F and the scale factor B is also used to find the solution. Some important cosmological physical parameters such as expansion scalar θ, shear scalar σ^2, average Hubble parameter H are evaluated. These parameters are infinite at the initial epoch when $t=-\frac{c_2}{c_1}$ and tend to zero at $t\to\infty$. It means that there is a point type singularity at $t=-\frac{c_2}{c_1}$ in the model. It is observed that the EOS parameter ω is negative when m lies between $(-2,-1.5)$ and these values of m give the negative deceleration parameter. It is supported by current observations namely SNe Ia and CMBR.

REFERENCES

1. Riess, A. G., et al.: Astron. J. 116, 1009 (1998).
2. Perlmutter, S., et al.: Nature (London) 391, 51 (1998).
3. Weinberg, S.: Rev. Mod. Phys. 61, 1 (1989).
4. Overduin, J. M., Coperstock, F. I.: Phys. Rev. D 58, 043506 (1998).
5. Sahni, V., Starobinsky, A.: Int. J. Mod. Phys. D 9, 373 (2000).
6. Ade, P. A. R., et al.: A & A 571, A1 (2014).
7. Barreiro, T., et al.: Phys. Rev. D 61, 127301 (2000).
8. Chiba, T., et al.: Phys. Rev. D 62, 023511 (2000).
9. Armendoriz-Picon. C., et al.: Phys. Rev. Lett. 85, 4438 (2000).
10. Bento, M. C., et al.: Phys. Rev. D 66, 043507 (2002).
11. Padmanabhan, T.: Phys. Rev. D 66, 021301 (2002).
12. Caldwell, R. R., et al.: Phys. Rev. Lett. 91, 071301 (2003).
13. Guo, et al.: Phys. Lett. B 608, 177 (2005).
14. Granda, et al.: Phys. Lett. B 669, 275 (2008).
15. Wei, H., Cai, R. G.: Phys. Lett. B 660, 113 (2008).
16. Friaca, A., et al.: Mon. Not. Roy. Astron. Soc. 362, 1295 (2005).
17. Wang, S., et al.: Phys. Rev. D82, (2010).
18. Brans, C. and Dicke, R. H.: Phys. Rev. 124, 925 (1961).
19. Carroll, S. M., et al.: Phys. Rev. D70, 043528 (2004).
20. Copeland, E. J., et al.: Int. J. Mod. Phys. D15, 1753 (2006).
21. Nojiri, S., Odintsov, S.: Phys. Rev. D 74, 086005 (2006).
22. Nojiri, S., Odintsov, S.: Phys. Rev. D 77,026007 (2008).
23. Nojiri, S., Odintsov, S.: Phys. Rept. 505, 59 (2011).
24. Harko, T., et al.,: Phys. Rev. D84, 024020 (2011).
25. Nojiri, S., Odintsov, S.: Phys. Lett. B 631, 1 (2005).
26. Nojiri, S., et al.,: Gen. Relativ. Gravit. 42, 1997 (2010).
27. Bamba, K., et al.: Eur. Phys. J. C67, 295 (2010).
28. Banijamali, A., Fazlpour, B., Setare, M.: Astrophys. Space Sci. 338, 327 (2012).
29. Myrzakulov, R., Saez-Gomez, D., Tureanu, A.: Gen. Relt. Grav. 4, 1671(2011).
30. Bamba, K., Geng, C., Lee, C.: arXiv: 1008. 4036 v [astro-ph. Co] (2010).
31. Sharif, M. and Ikram, A.: Eur. Phys. J. C 76, 640 (2016).
32. Ferraro, R., Fiorini, F.: Phys. Rev. D 75, 084031 (2007).
33. Ferraro, R., Fiorini, F.: Phys. Rev. D78, 124019 (2008).
34. Linder, E. V.: Phys. Rev. D 81, 127301 (2010).
35. Wang, T.: Phys. Rev. D84, 024042 (2011).
36. Ferraro, R., Fiorini, F.: Phys. Lett. B75, 702 (2011).
37. Haghani, Z., et al., Phys. Rev. D88, 044024 (2013).
38. Li, B., Sotiriou, T. P., Barrow, J. D.: Phys. Rev. D 83, 064035 (2011).
39. Li, M., Miao, R. X., Miao, Y. G.: JHEP 1107, 108 (2011).
40. Wu, P., Yu, H.: Phys. Lett. B 692, 176 (2010).
41. Wu, P., Yu, H.: Phys. Lett. B 693, 415 (2010).
42. Sharif, M., Amir, M. J.: Mod. Phys. Lett. A 22, 425 (2007).
43. Bengochea, G. R.: Phys. Lett. B 695, 405 (2011).
44. Rodrigues, M. E., et al.,: Astrophys. Space. Sci. 357, 129 (2015).
45. Daouda, M. H., et al.: Eur. Phys. J. C71, 1817 (2011).
46. Karami, K., Abdolmaleki, A.: JCAP 04007 (2012).
47. Bali, R., Kumawat, P.: Phys. Lett. B 665, 332 (2008).
48. Amirhashchi, H.: Phys. Lett. B 697, 429 (2011)..
49. Sharif, M., Zubair, M.: Astrophys. Space Sci. 330, 399 (2010).
50. Sharif, M., Jawad, A.: Astrophys. Space Sci. 331, 257 (2011).
51. Pradhan, A., Singh, A. K., Amirhashchi, H.: Int. J. Theor. Phys. 51, 3769 (2012).
52. Mac Callum, M. A. H.: Commun. Math. Phys. 20, 57 (1971).

Source of support: Nil, Conflict of interest: None Declared.