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REGULAR TOTAL STRONG (WEAK) DOMINATION IN BIPOLAR FUZZY GRAPH

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ABSTRACT

In this paper, we introduced regular total strong (weak) domination in Bipolar Fuzzy Graph and its various classifications. Size, Order and Degree of regular total strong (weak) domination in Bipolar Fuzzy Graph is derived with some examples. Some basic parametric conditions are introduced with suitable examples. The properties of total strong (weak) domination number and regular total strong (weak) domination number in Bipolar Fuzzy Graph are discussed.

Keywords: *Bipolar Fuzzy Graph, Dominating set in BFG, strong (weak) dominating set in BFG, total strong (weak) dominating set in BFG, regular total strong (weak) dominating set in BFG.*

1. INTRODUCTION

The concept of fuzzy graph was proposed by Kaufmann, from the fuzzy relations introduced by Zadeh.Zhang introduced the concept of bipolar fuzzy sets. A bipolar fuzzy set has a pair of positive and negative membership values range is [-1, 1]. In a Bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership value (0, 1] of an element indicates that the element somewhat satisfies the property, the membership value [-1, 0) of an element indicates the element somewhat satisfies the implicit counter property. In 1975, Rosenfeld introduced another elaborated concept, including fuzzy vertex and fuzzy edges and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness and etc. In the year 1998, the concept of domination in fuzzy graphs was investigated by A. Somasundaram, S.Somasundaram. [4] In the year, 2004 A.Somasundaram investigated the concepts of domination in fuzzy graph - II. In the year 2003, A. Nagoor Gani and M. Basheer Ahamed [2] investigated Order and Size in fuzzy graph. In 2010, In 2011, Muhammad Akram [6] introduced Bipolar fuzzy graphs. In the year 2012, Muhammad Akram [7] was proposed regular bipolar fuzzy graphs. In 2012, P.J.Jayalakshmi *et.al* [5] introduced total strong (weak) domination in fuzzy graph.

2. BASIC DEFINITIONS

In this section, Some basic definitions are discussed.

2.1 Definition: Let X be a non-empty set. A **bipolar fuzzy set** B in X is an object having the form $B=\{(x,\mu_B^P(x),\mu_N^P(x))/x\in X\}$ where $\mu_B^P : X \to [0,1]$ *and* $\mu_B^N : X \to [-1,0]$ are mappings.

We use the positive membership degree $\mu^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set B, and the negative membership degree $\mu^N(x)$ to denote the satisfaction degree

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of an element x to some implicit counter-property corresponding to a bipolar fuzzy set B. If $\mu^P(x) \neq 0$ and $\mu^N(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for B. It is possible for an element x to be such $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$, when the membership function of the property overlaps that of its counter property overlaps that of its counter property over some portion of x. For the sake of simplicity, the symbol B=(μ^P,μ^N) is used for the bipolar fuzzy set $B = \{(x, \mu_B^P(x), \mu_B^N(x)) : x \in X\}$

2.2 Definition: By a **bipolar fuzzy graph**, we mean a pair G = (A, B) where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar relation on V such that $\mu_B^P(\{x,y\}) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(\{x,y\}) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all {x, y} $\in$ E. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively. Note that B is symmetric bipolar fuzzy relation on A. We use the notation xy for an element of E. Thus, G = (A, B) is a bipolar graph of G* = (V, E) if $\mu_B^P(xy) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all {xy} $\in$ E.

2.3 Definition: Let G = (A, B) be a bipolar fuzzy graph where $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ be two bipolar fuzzy sets on a non-empty finite set V and E $\subseteq$ V×V respectively. The **positive degree of a vertex** is $d(\mu_A^P(x)) = \sum_{xy \in E} \mu_B^P(xy)$. Similarly, the **negative degree of a vertex** is $d(\mu_A^N(x)) = \sum_{xy \in E} \mu_B^N(xy)$. The **degree of a vertex μ** is d(μ) = ($d^P(\mu)$, $d^N(\mu)$).

2.4 Definition: Let G = (A,B) be a bipolar fuzzy graph. The **order of a bipolar fuzzy graph ,** denoted O(G), is defined as O(G) = (O^P(G), O^N(G)) , where $O^P(G) = \sum_{x \in V} \mu_A{}^P(x), O^N(G) = \sum_{x \in V} \mu_A{}^N(x)$. Similarly, the size of bipolar fuzzy graph, denoted by S(G), is defined as S(G) = (S^P(G), S^N(G)), where The **size of a bipolar fuzzy graph** is $S^P(G) = \sum_{xy \in V} \mu_A{}^P(xy), S^N(G) = \sum_{xy \in V} \mu_A{}^N(xy)$. Here, the vertex cardinality of is

$$p = |V| \sum_{y \in V} \frac{1 + \mu_A{}^P(y) + \mu_A{}^N(y)}{2}$$, and Edge cardinality of

$$q = |E| = \sum_{xy \in V} \frac{1 + \mu_A{}^P(xy) + \mu_A{}^N(xy)}{2}$$

.

2.5 Definition: Let G = (A, B) be a bipolar fuzzy graph. Let x,y $\in$ V. We say that x dominates y in G if $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))$ for all xy $\in$ E. A subset D of V is called a **dominating set** in G if for every y$\notin$D, there exists x$\in$D such that x dominates y.

2.6 Definition: A bipolar fuzzy graph G = (A, B) is called **strong bipolar fuzzy graph**, if $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))$ for all xy $\in$ E.

2.7. Definition: Let G be a bipolar fuzzy graph. The **closed neighbourhood degree of a vertex x in G** is defined by $\deg[x] = (\deg_A[x], \deg_B[x])$ where $\deg_A[x] = \sum_{y \in N(x)} \left[\mu_A{}^P(y) + \mu_A{}^P(x)\right]$ and $\deg_B[x] = \sum_{y \in N(x)} \left[\mu_A{}^N(y) + \mu_A{}^N(x)\right]$.

2.8 Definition: Let G be a bipolar fuzzy graph. Let x and y be any two vertices. Then **x totally strong dominates y (y totally weak dominates x)** if

i. $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))$ for all xy $\in$ E.
ii. $d_N(x) \geq d_N(y)$ for all x $\in$ T, y $\in$ V - T and
iii. every vertex in G dominates x.

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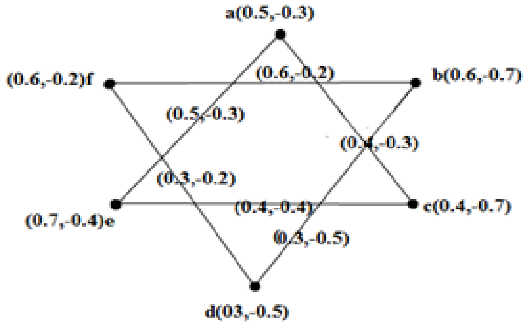
2.9 Definition: Let G = (A, B) be a bipolar Fuzzy Graph. T_b is said to be **total strong (weak) dominating bipolar set of G** if $d_N(x) \geq d_N(y)$ for all $x \in T_b$, $y \in V - T_b$ and x dominates y.

2.10 Definition: A total strong (weak) dominating bipolar fuzzy set T_b is called **minimal total strong (weak) dominating bipolar fuzzy set** of G, if for every vertex $x \in T_b$, T_b - {x}is not a total strong (weak) dominating bipolar fuzzy set of G.

2.11 Definition: The minimum fuzzy cardinality among all minimal total strong (weak) dominating bipolar fuzzy set T_b is called **total strong (weak) dominating bipolar fuzzy set of G** and its total strong (weak) domination bipolar fuzzy number is denoted by $\gamma_{T_b}(G)$.

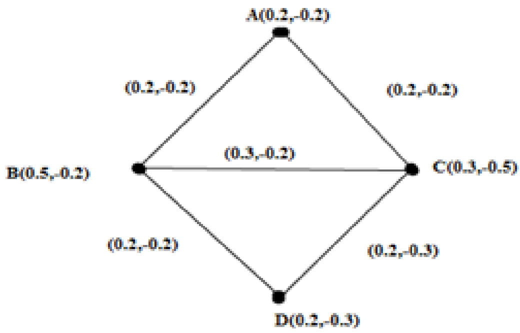
2.12 Example (Strong bipolar fuzzy graph): Let G be a strong bipolar fuzzy graph.



G

Order of G O(G)= P= (3.1,-2.8)
Size of G S(G) = q= (2.8,-1.9)

2.13 Example (Total strong (weak) dominating bipolar fuzzy graph): The Following graph has the
Dominating set, T= {A, C} and V-T= {B, D}
Dominating number, $\gamma_T(G) = 0.9$
Order of G = (1.2, -1.2)
Size of G = (1.1,-1.7)



Total Strong bipolar fuzzy graph

3. MAIN RESULTS

3.1 Definition: Let R_{T_b} be a total strong (weak) dominating set in bipolar fuzzy graph is said to be **Regular total strong(weak) dominating set in bipolar fuzzy graph** if all the vertices have the same degree.

3.2 Definition: A regular total strong (weak) dominating bipolar fuzzy set R_{T_b} is called **minimal regular total strong (weak) dominating bipolar fuzzy set** of G, if $y \in R_{T_b}$, $R_{T_b} - \{y\}$ is not a regular total strong (weak) dominating bipolar fuzzy set of G.

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3.3 Definition: The minimum fuzzy cardinality among all minimal regular total strong (weak) dominating bipolar fuzzy set is called **regular total strong (weak) dominating bipolar fuzzy number** and its regular total strong (weak) dominating bipolar fuzzy number and it is denoted by $\gamma_{R_{T_b}}(G)$.

3.4 Theorem: If G is a Regular Total strong(weak) dominating bipolar fuzzy graph then,

i. $\gamma_{R_{T_b}} \geq O^{p}(G) \geq S^{P}(G)$. ii. $O^{N}(G) \leq S^{N}(G) \leq \gamma_{R_{T_b}}$

Proof:

(i) Let R_{T_b} be a Regular total strong (weak) domination in bipolar fuzzy graph in G. Let $\gamma_{R_{T_b}}(G)$ be the Order of positive regular total strong (weak) domination in bipolar fuzzy graph in G. Then, the scalar cardinality of $V - R_{T_b}$ is less than or equal to the scalar cardinality of X x X.

Obviously, $O^{P}(G) \leq \gamma_{R_{T_b}}(G)$

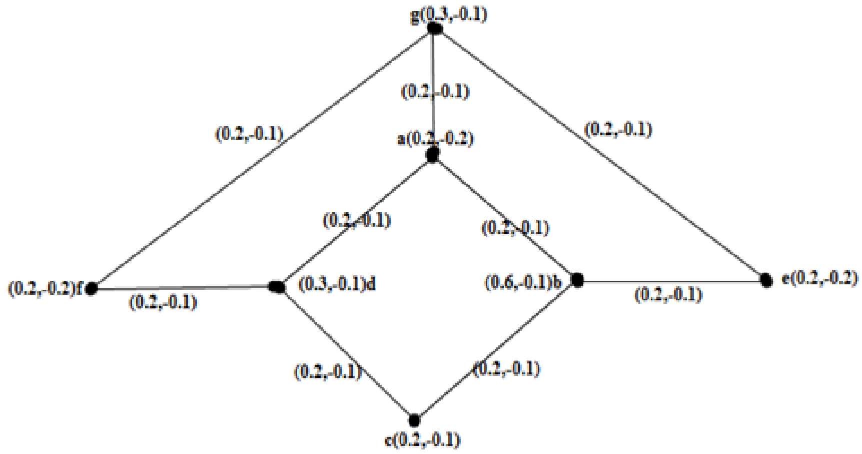
That is $\gamma_{NS_{T_b}}(G) \geq O^{P}(G)$ (1)

Now, Let A be the vertex with size of positive $S^{P}(G)$. Let $\gamma_{R_{T_b}}(G)$ be the Size of positive regular total strong (weak) domination in bipolar fuzzy graph in G.Clearly, $R_{T_b} - \{v\}$ is a regular total strong (weak) domination in bipolar fuzzy graph and hence $O^{P}(G) \geq S^{P}(G)$ (2)

From (1) & (2), we get $\gamma_{NS_{T_b}}(G) \geq O^{P}(G) \geq S^{P}(G)$.

(ii) as same as above said.

3.5 Example: Let G be regular total strong (weak) domination bipolar fuzzy graph.



G

Dominating Set, $\gamma_{R_{T_b}} = \{a, e, g, f\}$; v-$\gamma_{R_{T_b}} = \{b, c, d\}$

Dominating number, $\gamma_T(G) = 2.1$

Order of a graph, O(G) = (2, -1)

Size of a graph, S(G) = (1.8,-1)

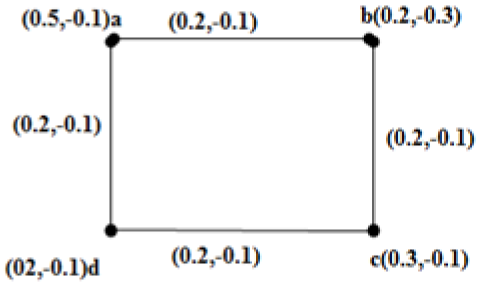
Here $\gamma_{R_{T_b}} \geq O^{p}(G) \geq S^{P}(G)$ and $O^{N}(G) \leq S^{N}(G) \leq \gamma_{R_{T_b}}$

Hence the theorem.

3.6 Proposition: Total Strong (Weak) domination bipolar fuzzy graph need not be Regular Total Strong (weak) domination in bipolar fuzzy graph.

3.7 Proposition: Every Regular Total Strong (Weak) dominating bipolar fuzzy graphs has the following inequalities (a) $O^{P}(G) \geq S^{P}(G)$ (b) $O^{N}(G) \leq S^{N}(G)$

3.8 Example: Let G be a regular total strong (weak) domination bipolar fuzzy graph.



Regular Total strong(weak) dominating BFG -G

$d_N(a) = (0.4, -0.4)$, $d_N(b) = (0.8,-0.2)$, $d_N(c) = (0.4,-0.4)$, $d_N(d) = (0.8,-0.2)$
$O(G) = (1.2, -0.6)$ and $S(G)= (0.8,-0.4)$

By usual calculations we get $O^P(G)=1.2$, $S^P(G) =-0.6 \Longrightarrow O^{P}(G) \geq S^{P}(G)$

Similarly, $O^N(G)=-0.6$, $S^N(G)=-0.4$, $\Longrightarrow O^{N}(G) \leq S^{N}(G)$

4. CONCLUSIONS

In this paper bipolar fuzzy graph, total strong bipolar fuzzy graph are discussed and introduced Regular total strong bipolar fuzzy graph. We can extend our research work to irregular bipolar fuzzy graph and some various bipolar fuzzy graph.

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