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(α, β) – CUT OF INTUITIONISTIC ANTI L-FUZZY M-SUBGROUPS

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ABSTRACT

In this paper some interesting properties of (α, β) – cut of Intuitionistic Anti L-fuzzy M-subgroups of a M-group are discussed.

Keywords: Intuitionistic fuzzy set (IFS), Intuitionistic fuzzy subgroup (IFSG), Intuitionistic fuzzy Normal subgroup (IFNSG), (α, β) – cut, Intuitionistic Anti L-fuzzy M-subgroup, Intuitionistic Anti L-fuzzy Normal M-subgroup.

1. INTRODUCTION

After the introduction of the concept of fuzzy set by Zadeh [6] several researches were conducted on the generalization of the notion of fuzzy set. The idea of Intuitionistic fuzzy set was given by Krassimir T. Atanassov [1]. In this paper we study Intuitionistic anti L-fuzzy M-subgroup with the help of some properties of their (α, β) – cut sets.

2. PRELIMINARIES

2.1 Definition: Let A = \{< x, μA(x), νA(x) >: x ∈ X\} and B = \{< x, μB(x), νB(x) >: x ∈ X\} be any two IFS’s of X, then
   (i) A ≤ B if and only if μA(x) ≤ μB(x) and νA(x) ≥ νB(x) for all x ∈ X
   (ii) A = B if and only if μA(x) = μB(x) and νA(x) = νB(x) for all x ∈ X
   (iii) A ∩ B = \{< x, (μA ∩ μB)(x), (νA ∩ νB)(x) >: x ∈ X\},
       where (μA ∩ μB)(x) = Min{μA(x), μB(x)} = μA(x) ∧ μB(x) & (νA ∩ νB)(x) = Max{νA(x), νB(x)} = νA(x) ∨ νB(x)
   (iv) A ∪ B = \{< x, (μA ∪ μB)(x), (νA ∪ νB)(x) >: x ∈ X\},
       where (μA ∪ μB)(x) = Max{μA(x), μB(x)} = μA(x) ∨ μB(x) & (νA ∪ νB)(x) = Min{νA(x), νB(x)} = νA(x) ∧ νB(x)

2.2 Definition: An IFS A = \{< x, μA(x), νA(x) >: x ∈ G\} of a group G is said to be Intuitionistic Fuzzy Subgroup of G (IFSG) of G if
   (i) μA(xy) ≥ μA(x) ∧ μA(y)
   (ii) μA(x⁻¹) = μA(x)
   (iii) νA(xy) ≤ νA(x) ∧ νA(y)
   (iv) νA(x⁻¹) = νA(x), for all x, y ∈ G

2.3 Definition: An IFSG A = \{< x, μA(x), νA(x) >: x ∈ G\} of a group G is said to be Intuitionistic Fuzzy Normal Subgroup of G (IFNSG) of G if
   (i) μA(xy) = μA(yx)
   (ii) νA(xy) = νA(yx), for all x, y ∈ G
Remark: It is easy to verify that an IFSG $A$ of a group $G$ is normal iff:

(i) $\mu_A(g^{-1}xg) = \mu_A(x)$ and
(ii) $\nu_A(g^{-1}xg) = \nu_A(x)$, for all $x \in G$ and $g \in G$.

Proof: Trivial Proof.

2.4 Definition: Let $G$ be an $M$-group and $\mu$ be an intuitionistic anti fuzzy group of $G$. If $\mu_A(mx) \leq \mu_A(x)$ and $\nu_A(mx) \geq \nu_A(x)$ for all $x \in G$ and $m \in M$ then $\mu$ is said to be an intuitionistic anti fuzzy subgroup with operator of $G$. We use the phrase $\mu$ is an intuitionistic anti $L$-fuzzy $M$-subgroup of $G$.

2.5 Example: Let $H$ be $M$-subgroup of an $M$-group $G$ and let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set in $G$ defined by

$$\mu_A(x) = \begin{cases} 0.3, & x \in H \\ 0.5, & \text{otherwise} \end{cases}$$

$$\nu_A(x) = \begin{cases} 0.6, & x \in H \\ 0.3, & \text{otherwise} \end{cases}$$

for all $x \in G$. Then it is easy to verify that $A = (\mu_A, \nu_A)$ is an anti fuzzy $M$-subgroup of $G$.

2.6 Definition: Let $A$ and $B$ be any two Intuitionistic Anti L-fuzzy $M$-subgroups of a $M$-group $(G, \cdot)$. Then $A$ and $B$ are said to be Conjugate Intuitionistic Anti $L$-fuzzy $M$-subgroups of $G$ if for some $g \in G$, $\mu_A(x) = \mu_B(g^{-1}xg)$ & $\nu_A(x) = \nu_B(g^{-1}xg)$, for every $x \in G$.

2.7 Proposition: If $\mu = (\delta \mu, \lambda \mu)$ is an intuitionistic anti fuzzy $M$-subgroup of an $M$-group $G$, then for any $x, y \in G$ and $m \in M$,

(i) $\mu_A(mx) \leq \mu_A(x)$ & $\mu_A(y)$,
(ii) $\mu_A(mx^{-1}) \leq \mu_A(x)$ and
(iii) $\nu_A(mx) \geq \nu_A(x) \land \nu_A(y)$,
(iv) $\nu_A(mx^{-1}) \geq \nu_A(x)$, for all $x$ & $y$ in $G$.

2.8 Theorem: $A$ is an intuitionistic anti $L$-fuzzy $M$-subgroup of a $M$-group $(G, \cdot)$ iff $\mu_A(mx^{-1}) \leq \mu_A(x)$ & $\mu_A(y)$ and $\nu_A(mx^{-1}) \geq \nu_A(x) \land \nu_A(y)$, for all $x$ and $y$ in $G$.

2.9 Definition: Let $A$ be the Intuitionistic Anti $L$-fuzzy subset of $X$. For $\alpha$ and $\beta$ in $L$, the $(\alpha, \beta)$-level subset of $A$ is the set $A_{(\alpha, \beta)} = \{x \in X : \mu_A(x) \leq \alpha \land \nu_A(x) \geq \beta\}$. This is called an Intuitionistic Anti $L$-fuzzy level subset of $A$.

2.10 Definition: Let $A$ be an intuitionistic L-fuzzy $M$-subgroup of a $M$-group $G$. The level $M$-subgroup $A_{(\alpha, \beta)}$, for $\alpha$ and $\beta$ in $L$ such that $\alpha \geq \mu_A(e)$ and $\beta \leq \nu_A(e)$ is called an intuitionistic anti $L$-fuzzy $M$-subgroup of $A$.

2.11 Definition: Let $(G, \cdot)$ be a $M$-group. An intuitionistic L-fuzzy $M$-subgroup $A$ of $G$ is said to be an intuitionistic Anti $L$-fuzzy normal $M$-subgroup (IALFNMSG) of $G$ if the following conditions are satisfied:

(i) $\mu_A(xy) = \mu_A(yx)$,
(ii) $\nu_A(xy) = \nu_A(yx)$, for all $x$ and $y$ in $G$.

2.12 Definition: $(\alpha, \beta)$ – Cut of Intuitionistic fuzzy set

Let $A$ be Intuitionistic fuzzy set of a universe set $X$. Then $(\alpha, \beta)$-cut of $A$ is a crisp subset $C_{\alpha, \beta}(A)$ of the IF $A$ is given by $C_{\alpha, \beta}(A) = \{x \in X : \mu_A(x) \geq \alpha \land \nu_A(x) \leq \beta\}$, where $(\alpha, \beta) \in [0, 1]$ with $\alpha + \beta \leq 1$.

3. $(\alpha, \beta)$ – Cut of Intuitionistic Anti fuzzy set (IAFS) and their Properties

3.1 Definition: $(\alpha, \beta)$ – Cut of Intuitionistic Anti fuzzy set

Let $A$ be Intuitionistic Anti fuzzy set of a universe set $X$. Then $(\alpha, \beta)$-cut of $A$ is a crisp subset $C_{\alpha, \beta}(A)$ of the IAFS $A$ is given by $C_{\alpha, \beta}(A) = \{x \in X : \mu_A(x) \geq \alpha \land \nu_A(x) \leq \beta\}$, where $(\alpha, \beta) \in [0, 1]$ with $\alpha + \beta \leq 1$.

3.2 Proposition: If $A$ and $B$ be two IAFS’s of a universe set $X$, then following holds

(i) $C_{\alpha, \beta}(A) \subseteq C_{\delta, \theta}(A)$ if $\alpha \geq \delta$ and $\beta \leq \theta$,
(ii) $C_{1-\delta, 1-\theta}(A) \subseteq C_{\alpha, \beta}(A) \subseteq C_{\alpha, \beta}(A)$.
3.3 Theorem: If $A$ is Intuitionistic anti L-fuzzy M-subgroup of a M-group $G$. Then $C_{\alpha, \beta}(A)$ is a M-subgroup of M-group $G$, where $\mu_A(e) \leq \alpha \cdot v_A(e) \geq \beta$ and $e$ is the identity element of $G$. 

\( (\alpha, \beta) - \text{Cut of Intuitionistic Anti L-Fuzzy M-Subgroups / UMA-9(5), May-2018, (Special Issue)} \)
Proof: Clearly $C_{α,β}(A) ≠ φ$ as $e ∈ C_{α,β}(A)$. Let $x, y ∈ C_{α,β}(A)$ be any two elements. Then

$μ_A(x) ≤_α v_A(x) ≥_β$ and $μ_A(y) ≤_α v_A(y) ≥_β$

Thus $xy^{-1} ∈ C_{α,β}(A)$. Hence $C_{α,β}(A)$ is a M-subgroup of G.

3.4 Theorem: If A be intuitionistic anti L-fuzzy normal M-subgroup of M-group G. Then $C_{α,β}(A)$ is normal M-subgroup of M-group G, where $μ_A(e) ≤_α v_A(e) ≥_β$ and e is the identity element of G.

Proof: Let $x ∈ C_{α,β}(A)$ and $g ∈ G$ be any element. Then $μ_A(x) ≤_α v_A(x) ≥_β$. Also A be Intuitionistic anti L-fuzzy normal M-subgroup of M-group G. Therefore, $μ_A(g^{-1} x g) = μ_A(x)$ and $v_A(g^{-1} x g) = v_A(x)$ for all $x ∈ A$ and $g ∈ G$

Therefore $μ_A(g^{-1} x g) = μ_A(x) ≤_α$ and $ν_A(g^{-1} x g) = ν_A(x) ≥_β$ implies that $μ_A(g^{-1} x g) ≤_α$ and $ν_A(g^{-1} x g) ≥_β$ and so $g^{-1} x g ∈ C_{α,β}(A)$

Hence $C_{α,β}(A)$ is normal M-subgroup of G.

3.5 Theorem: If A is Intuitionistic fuzzy subset of a group G. Then A be intuitionistic anti L-fuzzy normal M-subgroup of G if and only if $C_{α,β}(A)$ is a M-subgroup of M-group G for all $α, β ∈ [0, 1]$ with $α + β ≤ 1$, where $μ_A(e) ≤_α v_A(e) ≥_β$ and e is the identity element of G.

Proof: Firstly let A be intuitionistic anti L-fuzzy M-subgroup of M-group G. Then the result follows by Theorem (3.3)

Conversely, let A is Intuitionistic fuzzy subset of a group G such that $C_{α,β}(A)$ is a M-subgroup of M-group G for all $α, β ∈ [0, 1]$ with $α + β ≤ 1$.

To show that A be intuitionistic anti L-fuzzy M-subgroup of M-group G. For this we show that

(i) $μ_A(xy) ≤_α μ_A(x) ∧ μ_A(y)$ and $ν_A(xy) ≥_β ν_A(x) ∨ ν_A(y)$ for all $x, y ∈ G$

(ii) $μ_A(x^{-1}) = μ_A(x)$ and $ν_A(x^{-1}) = ν_A(x)$

For (i) Let $x, y ∈ G$ and let $α = μ_A(x) ∧ μ_A(y)$ and $β = ν_A(x) ∨ ν_A(y)$. Then

$μ_A(x) ≤_α μ_A(y) ≤_α$ and $ν_A(x) ≥_β ν_A(y) ≥_β$

i.e. $μ_A(x) ≤_α v_A(x) ≥_β$ and $μ_A(y) ≤_α v_A(y) ≥_β$

i.e. $x ∈ C_{α,β}(A)$ and $y ∈ C_{α,β}(A)$ and so $xy ∈ C_{α,β}(A)$ [As $C_{α,β}(A)$ is a group]

Therefore $μ_A(xy) ≤_α μ_A(x) ∧ μ_A(y)$ and $ν_A(xy) ≥_β ν_A(x) ∨ ν_A(y)$

i.e. $μ_A(xy) ≤_α μ_A(x) ∧ μ_A(y)$ and $ν_A(xy) ≥_β ν_A(x) ∨ ν_A(y)$

For (ii) Let $x ∈ G$ be any element. Let $μ_A(x) = α$ and $ν_A(x) = β$. Then

$μ_A(x) ≤_α$ and $ν_A(x) ≥_β$ is true i.e. $x ∈ C_{α,β}(A)$

As $x ∈ C_{α,β}(A)$ is a M-subgroup of G

Therefore we have $x^{-1} ∈ C_{α,β}(A) ⇒ μ_A(x^{-1}) ≤_α$ and $ν_A(x^{-1}) ≥_β$

Thus $μ_A(x^{-1}) ≤_α μ_A(x)$ and $ν_A(x^{-1}) ≥_β ν_A(x)$

Thus $μ_A(x) = μ_A((x^{-1})^{-1}) ≤_α μ_A(x^{-1}) ≤_α μ_A(x)$ implies that $μ_A(x^{-1}) = μ_A(x)$

And $ν_A(x) = ν_A((x^{-1})^{-1}) ≥_β ν_A(x^{-1}) ≥_β ν_A(x)$ implies that $ν_A(x^{-1}) = ν_A(x)$

Hence A is Intuitionistic Anti L-fuzzy M-subgroup of M-group G.

3.6 Theorem: If A and B be two IALFMSG’s of a M-group G, then $A ∩ B$ is IALFMSG of M-group G.

Proof: By Theorem (3.5), $A ∩ B$ is IALFMSG of M-group G if and only if $C_{α,β}(A ∩ B)$ is a M-subgroup of G. but as $C_{α, β}(A ∩ B) = C_{α, β}(A) ∩ C_{α, β}(B)$ and both $C_{α, β}(A)$ and $C_{α, β}(B)$ are M-subgroups of G and intersection of two M-subgroups of a M-group is a subgroup of G implies that $C_{α, β}(A ∩ B)$ is a M-subgroup of G and hence $A ∩ B$ is IALFMSG of M-group G.
3.7 Remark: Union of two IALFMSG’s of a group G need not be IALFMSG of M-group G.

3.8 Example: Consider the Klein four group.

G = {e, a, b, ab}, where a = e = b & ab = ba
For 0 ≤ i ≤ 5, let t_i, s_i ∈ [0, 1] such that 1 ≥ t_0 > t_1 > … > t_4 and 0 ≤ s_0 ≤ s_1 ≤ … ≤ s_5

Define Intuitionistic fuzzy subset A and B as follows:

\[ A = \{ (x, \mu_A(x), \nu_A(x)) : x \in G \} \]
\[ B = \{ (x, \mu_B(x), \nu_B(x)) : x \in G \} \]

Where

\[ \mu_A(x) = \mu_A(g) = \mu_A(x^{-1}g) \]
\[ \nu_A(x) = \nu_A(g) = \nu_A(x^{-1}g) \]

Clearly A and B are IALFMSG of the M-group G.

(i) Now \( A \cup B = \{ x : (\mu_A(x), \nu_A(x)) \cup (\nu_B(x)) : x \in G \} \)

Here \( \mu_A(x) = \mu_A(g) \), \( \nu_A(x) = \nu_A(g) \)
\[ \mu_B(x) = \mu_B(g) = \mu_B(x^{-1}g) \]
\[ \nu_B(x) = \nu_B(g) = \nu_B(x^{-1}g) \]
\[ \mu_A(x) \cup \mu_B(x) \]
\[ \nu_A(x) \cup \nu_B(x) \]

3.9 Definition: Intuitionistic anti L-fuzzy left and right cosets

Let G be a M-group and A be IALFMSG of M-group G. Let x ∈ G be a fixed element. Then the set \( A \cdot x = \{ g \in G : \mu_A(g) \leq \alpha \} \) and \( x \cdot A = \{ g \in G : \nu_A(g) \leq \beta \} \) for all \( g \in G \) is called intuitionistic anti L-fuzzy left coset of G determined by A and x.

Similarly, the set \( A \cdot x = \{ g \in G : \mu_A(g) \leq \alpha \} \) and \( x \cdot A = \{ g \in G : \nu_A(g) \leq \beta \} \) for all \( g \in G \) is called the intuitionistic anti L-fuzzy right coset of G determined by A and x.

3.10 Remark: It is clear that if A is intuitionistic anti L-fuzzy normal M-subgroup of G, then the intuitionistic anti L-fuzzy left coset and intuitionistic anti L-fuzzy right coset of A on G coincide and in this case, we call intuitionistic anti L-fuzzy coset instead of intuitionistic anti L-fuzzy left or intuitionistic anti L-fuzzy right coset.

3.11 Theorem: Let A be intuitionistic anti L-fuzzy M-subgroup of a M-group G and x be any fixed element of G. Then

(i) \( x \cdot A, \beta(A) = C_{\alpha, \beta}(A) \)

(ii) \( A, \beta(A) \cdot x = C_{\alpha, \beta}(A) \)

for all \( \alpha, \beta \in [0, 1] \) with \( \alpha + \beta \leq 1 \).

Proof:

(i) Now \( C_{\alpha, \beta}(A) = \{ g \in G : \mu_A(g) \leq \alpha \} \) and \( \nu_A(g) \leq \beta \] with \( \alpha + \beta \leq 1 \)

Also \( x \cdot C_{\alpha, \beta}(A) = \{ y \in G : \mu_A(y) \leq \alpha \} \) and \( \nu_A(y) \leq \beta \]
\[ = \{ x \in G : \mu_A(g) \leq \alpha \} \] and \( \nu_A(g) \leq \beta \]

Put \( xy = g \) so that \( y = x^{-1}g \)
Therefore \( x \cdot C_{\alpha, \beta}(A) = \{ g \in G : \mu_A(x^{-1}g) \leq \alpha \} \)
\[ = \{ g \in G : \mu_A(g) \leq \alpha \} \]

Thus \( x \cdot C_{\alpha, \beta}(A) = C_{\alpha, \beta}(A) \)

for all \( \alpha, \beta \in [0, 1] \) with \( \alpha + \beta \leq 1 \)

(ii) Again \( C_{\alpha, \beta}(A) = A, \beta(A) = \{ g \in G : \mu_A(g) \leq \alpha \} \) and \( \nu_A(g) \leq \beta \] with \( \alpha + \beta \leq 1 \)

Also \( C_{\alpha, \beta}(A) = A, \beta(A) = \{ y \in G : \mu_A(y) \leq \alpha \} \) and \( \nu_A(y) \leq \beta \]
\[ = \{ yx \in G : \mu_A(y) \leq \alpha \} \]

Put \( yx = g \) so that \( y = gx^{-1} \)

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\[ C_{\alpha,\beta}(A).x = \{ g \in G : \mu_A(gx^{-1}) \leq \alpha \text{ and } \nu_X(gx^{-1}) \geq \beta \} \]

\[ = \{ g \in G : \mu_{A}(g) \leq \alpha \text{ and } \nu_{X}(g) \geq \beta \} \]

Hence \( C_{\alpha,\beta}(A).x = C_{\alpha,\beta}(Ax) \), for all \( \alpha, \beta \in [0, 1] \) with \( \alpha + \beta \leq 1 \).

3.12 Theorem: Let \( A \) be Intuitionistic Anti L-fuzzy M-subgroup of M-group \( G \). Let \( x, y \) be elements of \( G \) such that \( \mu_A(x) \wedge \mu_A(y) = \alpha \) and \( \nu_A(x) \vee \nu_A(y) = \beta \).

Then

(i) \( xA = yA \iff x^{-1}y \in C_{\alpha,\beta}(A) \)

(ii) \( Ax = Ay \iff xy^{-1} \in C_{\alpha,\beta}(A) \)

Proof:

(i) Now \( xA = yA \iff C_{\alpha,\beta}(xA) = C_{\alpha,\beta}(yA) \)

\[ \iff x . C_{\alpha,\beta}(A) = y . C_{\alpha,\beta}(A) \]

[by Theorem (3.11)(i)]

\[ \iff x^{-1}y \in C_{\alpha,\beta}(A) \]

[As \( C_{\alpha,\beta}(A) \) is a subgroup of \( G \)]

(ii) Again \( Ax = Ay \iff C_{\alpha,\beta}(Ax) = C_{\alpha,\beta}(Ay) \)

\[ \iff C_{\alpha,\beta}(A) . x = C_{\alpha,\beta}(A) . y \]

[by Theorem (3.11)(ii)]

\[ \iff xy^{-1} \in C_{\alpha,\beta}(A) \]

[As \( C_{\alpha,\beta}(A) \) is a subgroup of \( G \)]

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