Volume 9, No. 5, May - 2018 (Special Issue)
International Journal of Mathematical Archive-9(5), 2018, 45-50
Available online through www.ijma.info ISSN 2229 – 5046

(α, β) – CUT OF INTUITIONISTIC ANTI L-FUZZY M-SUBGROUPS

P. PANDIAMMAL*[1], C. SUBRAMANI[2] AND NIVETHA MARTIN[3]

[*1, 2]Department of Mathematics,
G T N Arts College (Autonomous), Dindigul, Tamil Nadu, India.

Department of Mathematics,
Arul Anandar College (Autonomous), Karumathur, Madurai, Tamil Nadu, India.

E-mail: pandiammal1981@yahoo.com[1], subramani_27maths@yahoo.com
and nivetha.martin710@gmail.com[3]

ABSTRACT

In this paper some interesting properties of (α, β) – cut of Intuitionistic Anti L-fuzzy M-subgroups of a M-group are discussed.

Keywords: *Intuitionistic fuzzy set (IFS), Intuitionistic fuzzy subgroup (IFSG), Intuitionistic fuzzy Normal subgroup (IFNSG), (α, β) – cut, Intuitionistic Anti L-fuzzy M-subgroup, Intuitionistic Anti L-fuzzy Normal M-subgroup.*

1. INTRODUCTION

After the introduction of the concept of fuzzy set by Zadeh [6] several researches were conducted on the generalization of the notion of fuzzy set. The idea of Intuitionistic fuzzy set was given by Krassimiri T. Atanassov [1]. In this paper we study Intuitionistic anti L-fuzzy M-subgroup with the help of some properties of their (α, β) – cut sets.

2. PRELIMINARIES

2.1 Definition: Let $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >: x \in X\}$ be any two IFS's of X, then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$
(iii) $A \cap B = \{< x, (\mu_A \cap \mu_B)(x), (\nu_A \cap \nu_B)(x) >: x \in X\}$,
where $(\mu_A \cap \mu_B)(x) = \text{Min}\{\mu_A(x), \mu_B(x)\} = \mu_A(x) \wedge \mu_B(x)$ & $(\nu_A \cap \nu_B)(x) = \text{Max}\{\nu_A(x), \nu_B(x)\} = \nu_A(x) \vee \nu_B(x)$
(iv) $A \cup B = \{< x, (\mu_A \cup \mu_B)(x), (\nu_A \cup \nu_B)(x) > : x \in X\}$,
where $(\mu_A \cup \mu_B)(x) = \text{Max}\{\mu_A(x), \mu_B(x)\} = \mu_A(x) \vee \mu_B(x)$ & $(\nu_A \cup \nu_B)(x) = \text{Min}\{\nu_A(x), \nu_B(x)\} = \nu_A(x) \wedge \nu_B(x)$

2.2 Definition: An IFS $A = \{< x, \mu_A(x), \nu_A(x) >: x \in G\}$ of a group G is said to be **Intuitionistic Fuzzy Subgroup** of G (IFSG) of G if

(i) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$
(ii) $\mu_A(x^{-1}) = \mu_A(x)$
(iii) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$
(iv) $\nu_A(x^{-1}) = \nu_A(x)$, for all $x, y \in G$

2.3 Definition: An IFSG $A = \{< x, \mu_A(x), \nu_A(x) >: x \in G\}$ of a group G said to be **Intuitionistic Fuzzy Normal Subgroup** of G (IFNSG) of G if

(i) $\mu_A(xy) = \mu_A(yx)$
(ii) $\nu_A(xy) = \nu_A(yx)$, for all $x, y \in G$

CONFERENCE PAPER
National Conference March 1st - 2018, On "Recent Advances in Pure and Applied Mathematics", Organized by Department of Mathematics, Arul Anandar College (Autonomous), Madurai. Tamilnadu, India.

Remark: It is easy to verify that an IFSG A of a group G is **normal** iff
(i) $\mu_A(g^{-1} x g) = \mu_A(x)$ and
(ii) $\nu_A(g^{-1} x g) = \nu_A(x)$, for all $x \in A$ and $g \in G$

Proof: Trivial Proof.

2.4 Definition: Let G be an M-group and μ be an intuitionistic anti fuzzy group of G.
If $\mu_A(mx) \leq \mu_A(x)$ and $\nu_A(mx) \geq \nu_A(x)$ for all x in G and m in M then μ is said to be an intuitionistic anti fuzzy subgroup with operator of G. We use the phrase μ is an **intuitionistic anti L-fuzzy M-subgroup** of G.

2.5 Example: Let H be M-subgroup of an M-group G and let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set in G defined by

$$\mu_A(x) = \begin{cases} 0.3; & x \varepsilon H \\ 0.5; & \text{otherwise} \end{cases}$$

$$\nu_A(x) = \begin{cases} 0.6; & x \varepsilon H \\ 0.3; & \text{otherwise} \end{cases}$$

for all x in G. Then it is easy to verify that $A = (\mu_A, \nu_A)$ is an anti fuzzy M- subgroup of G.

2.6 Definition: Let A and B be any two Intuitionistic Anti L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be **Conjugate Intuitionistic Anti L-fuzzy M-subgroups** of G if for some g in G, $\mu_A(x) = \mu_B(g^{-1}xg)$ & $\nu_A(x) = \nu_B(g^{-1}xg)$, for every x in G.

2.7 Proposition: If $\mu = (\delta\mu, \lambda\mu)$ is **an intuitionistic anti fuzzy M-subgroup of an M- group** G, then for any x, y∈ G and m ∈ M.
(i) $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$,
(ii) $\mu_A(mx^{-1}) \leq \mu_A(x)$ and
(iii) $\nu_A(mxy) \geq \nu_A(x) \wedge \nu_A(y)$,
(iv) $\nu_A(mx^{-1}) \leq \nu_A(x)$, for all x & y in G.

2.8 Theorem: A is an intuitionistic anti L-fuzzy M-subgroup of a M-group (G, ·) iff $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

2.9 Definition: Let A be an Intuitionistic Anti L-fuzzy subset of X. For α and β in L, the (α, β)-level subset of A is the set $A_{(\alpha, \beta)} = \{x \in X : \mu_A(x) \leq \alpha \text{ and } \nu_A(x) \geq \beta\}$. This is called an **Intuitionistic Anti L-fuzzy level subset** of A.

2.10 Definition: Let A be an intuitionistic L-fuzzy M-subgroup of a M-group G. The level M-subgroup $A_{(\alpha, \beta)}$, for α and β in L such that $\alpha \geq \mu_A(e)$ and $\beta \leq \nu_A(e)$ is called an **intuitionistic anti L-fuzzy level M-subgroup** of A.

2.11 Definition: Let (G, ·) be a M-group. An intuitionistic L-fuzzy M-subgroup A of G is said to be an intuitionistic Anti L-fuzzy normal M-subgroup (IALFNMSG) of G if the following conditions are satisfied:
(i) $\mu_A(xy) = \mu_A(yx)$,
(ii) $\nu_A(xy) = \nu_A(yx)$, for all x and y in G.

2.12 Definition: (α, β) **– Cut of Intuitionistic fuzzy set**
Let A be Intuitionistic fuzzy set of a universe set X. Then (α, β)-cut of A is a crisp subset $C_{\alpha, \beta}(A)$ of the IFS A is given by $C_{\alpha, \beta}(A) = \{x: x \in X / \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$, where $(\alpha, \beta) \in [0, 1]$ with $\alpha + \beta \leq 1$.

3. (α, β) – Cut of Intuitionistic Anti fuzzy set (IAFS) and their Properties

3.1 Definition: (α, β) **– Cut of Intuitionistic Anti fuzzy set**
Let A be Intuitionistic Anti fuzzy set of a universe set X. Then (α, β)-cut of A is a crisp subset $C_{\alpha, \beta}(A)$ of the IAFS A is given by $C_{\alpha, \beta}(A) = \{x : x \in X / \mu_A(x) \leq \alpha, \nu_A(x) \geq \beta\}$, where $(\alpha, \beta) \in [0, 1]$ with $\alpha + \beta \leq 1$.

3.2 Proposition: If A and B be two IAFS's of a universe set X, then following holds
(i) $C_{\alpha, \beta}(A) \subseteq C_{\delta, \theta}(A)$ if $\alpha \geq \delta$ and $\beta \leq \theta$
(ii) $C_{1-\beta, \beta}(A) \subseteq C_{\alpha, \beta}(A) \subseteq C_{\alpha, 1-\alpha}(A)$

CONFERENCE PAPER
National Conference March 1st - 2018, On "Recent Advances in Pure and Applied Mathematics", Organized by Department of Mathematics, Arul Anandar College (Autonomous), Madurai. Tamilnadu, India.

(iii) $A \subseteq B$ implies $C_{\alpha,\beta}(A)\ C_{\alpha,\beta}(B)$
(iv) $C_{\alpha,\beta}(A\cap B) = C_{\alpha,\beta}(A)\cap C_{\alpha,\beta}(B)$
(v) $C_{\alpha,\beta}(A\cup B) \supseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$ equality hold if $\alpha+\beta=1$
(vi) $C_{\alpha,\beta}(\cap A_i) = \cap C_{\alpha,\beta}(A_i)$
(vii) $C_{0,1}(A) = X$.

Proof:

(i) Let $x \in C_{\alpha,\beta}(A) \Rightarrow \mu_A(x) \le \alpha$ and $\nu_A(x) \ge \beta$
Since $\delta \le \alpha$ and $\theta \ge \beta$ implies that $\mu_A(x) \le \alpha \le \delta$ and $\nu_A(x) \ge \beta \ge \theta$
$\Rightarrow \mu_A(x) \le \delta$ and $\nu_A(x) \ge \theta$ and so $x \in C_{\delta,\theta}(A)$. Hence $C_{\alpha,\beta}(A) \subseteq C_{\delta,\theta}(A)$

(ii) Since $\alpha+\beta \le 1$ implies that $1-\beta \ge \alpha$ and $\beta \le \beta$
Therefore by part (i) we get $C_{1-\beta,\beta}(A) \subseteq C_{\alpha,\beta}(A)$ (1)
Again $\alpha+\beta \le 1$ implies that $\alpha \ge \alpha$ and $\beta \le 1-\alpha$
Therefore by part (i) we get $C_{\alpha,\beta}(A) \subseteq C_{\alpha,1-\alpha}(A)$ (2)
From (1) and (2) we get $C_{1-\beta,\beta}(A) \subseteq C_{\alpha,\beta}(A) \subseteq C_{\alpha,1-\alpha}(A)$

(iii) Let $x \in C_{\alpha,\beta}(A) \Rightarrow \mu_A(x) \le \alpha$ and $\nu_A(x) \ge \beta$
As $B \supseteq A$ implies $\mu_B(x) \le \mu_A(x) \le \alpha$ and $\nu_B(x) \ge \nu_A(x) \ge \beta$
$\Rightarrow \mu_B(x) \le \alpha$ and $\nu_B(x) \ge \beta$ and
so $x \in C_{\alpha,\beta}(B)$ Hence $C_{\alpha,\beta}(A) \subseteq C_{\alpha,\beta}(B)$

(iv) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$
Therefore by part (i) $C_{\alpha,\beta}(A\cap B) \subseteq C_{\alpha,\beta}(A)$ and $C_{\alpha,\beta}(A\cap B) \subseteq C_{\alpha,\beta}(B)$
$\Rightarrow C_{\alpha,\beta}(A\cap B) \subseteq C_{\alpha,\beta}(A) \cap C_{\alpha,\beta}(B)$ (3)
Also, let $x \in C_{\alpha,\beta}(A) \cap C_{\alpha,\beta}(B) \Rightarrow x \in C_{\alpha,\beta}(A)$ and $x \in C_{\alpha,\beta}(B)$
$\Rightarrow \mu_A(x) \le \alpha$ and $\nu_A(x) \ge \beta$ and $\mu_B(x) \le \alpha$ and $\nu_B(x) \ge \beta$
$\Rightarrow \mu_A(x) \le \alpha$ and $\mu_B(x) \le \alpha$ and $\nu_A(x) \ge \beta$ and $\nu_B(x) \ge \beta$
$\Rightarrow \mu_A(x) \wedge \mu_B(x) \le \alpha$ and $\nu_A(x) \vee \nu_B(x) \ge \beta$
$\Rightarrow (\mu_A \cap \mu_B)(x) \le \alpha$ and $(\nu_A \cap \nu_B)(x) \ge \beta$
$\Rightarrow x \in C_{\alpha,\beta}(A\cap B)$
Thus $C_{\alpha,\beta}(A) \cap C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A\cap B)$ (4)
From (3) and (4), we get $C_{\alpha,\beta}(A\cap B) = C_{\alpha,\beta}(A) \cap C_{\alpha,\beta}(B)$

(v) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$
Therefore by part (i) $C_{\alpha,\beta}(A) \subseteq C_{\alpha,\beta}(A\cup B)$ and $C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A\cup B)$
$\Rightarrow C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A\cup B)$ Now equality hold if $\alpha+\beta = 1$.
We show that $C_{\alpha,\beta}(A\cup B) \subseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$
Let $x \in C_{\alpha,\beta}(A\cup B) \Rightarrow (\mu_A \cup \mu_B)(x) \le \alpha$ and $(\nu_A \cup \nu_B)(x) \ge \beta$
$\Rightarrow \mu_A(x) \vee \mu_B(x) \le \alpha$ and $\nu_A(x) \wedge \nu_B(x) \ge \beta$
If $\mu_A(x) \le \alpha$, then $\nu_A(x) \ge 1-\mu_A(x) \ge 1-\alpha = \beta$
Implies that $x \in C_{\alpha,\beta}(A) \subseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$ (5)
Similarly if $\mu_B(x) \le \alpha$, then $\nu_B(x) \ge 1-\mu_B(x) \ge 1-\alpha = \beta$
Implies that $x \in C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$
Thus we see that $x \in C_{\alpha,\beta}(A\cup B) \Rightarrow x \in C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$,
$C_{\alpha,\beta}(A\cup B) \subseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$ (6)
From (5) and (6), we get $C_{\alpha,\beta}(A\cup B) = C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$

(vi) Let $x \in C_{\alpha,\beta}(\cap A_i) \Rightarrow (\cap \mu_{A_i})(x) \le \alpha$ and $(\cap \nu_{A_i})(x) \ge \beta$
$\wedge \mu_{A_i}(x) \le \alpha$ and $\vee \nu_{A_i}(x) \ge \beta \Rightarrow x \in C_{\alpha,\beta}(A_i)$, for all i
$\Rightarrow x \in \cap C_{\alpha,\beta}(A_i)$ & hence $C_{\alpha,\beta}(\cap A_i) \subseteq \cap C_{\alpha,\beta}(A_i)$

(vii) Follows from definition

3.3 Theorem: If A is Intuitionistic anti L-fuzzy M-subgroup of a M-group G. Then $C_{\alpha,\beta}(A)$ is a M-subgroup of M-group G, where $\mu_A(e) \le \alpha$, $\nu_A(e) \ge \beta$ and e is the identity element of G.

CONFERENCE PAPER
National Conference March 1st - 2018, On "Recent Advances in Pure and Applied Mathematics", Organized by Department of Mathematics, Arul Anandar College (Autonomous), Madurai. Tamilnadu, India.

Proof: Clearly $C_{\alpha,\beta}(A) \neq \phi$ as $e \in C_{\alpha,\beta}(A)$. Let $x, y \in C_{\alpha,\beta}(A)$ be any two elements. Then
$\mu_A(x) \leq \alpha$, $\nu_A(x) \geq \beta$ and $\mu_A(y) \leq \alpha$, $\nu_A(y) \geq \beta$
$\mu_A(x) \wedge \mu_A(y) \leq \alpha$ and $\nu_A(x) \vee \nu_A(y) \geq \beta$

As A is intuitionistic anti L-fuzzy M-subgroup of G
Therefore $\mu_A(mxy^{-1}) \leq \mu_A(x) \wedge \mu_A(y) \leq \alpha$ and $\nu_A(mxy^{-1}) \geq \nu_A(x) \vee \nu_A(y) \geq \beta$
Thus $xy^{-1} \in C_{\alpha,\beta}(A)$. Hence $C_{\alpha,\beta}(A)$ is a M-subgroup of G.

3.4 Theorem: If A be intuitionistic anti L-fuzzy normal M-subgroup of M-group G. Then $C_{\alpha,\beta}(A)$ is normal M-subgroup of M-group G, where $\mu_A(e) \leq \alpha$, $\nu_A(e) \geq \beta$ and e is the identity element of G.

Proof: Let $x \in C_{\alpha,\beta}(A)$ and $g \in G$ be any element. Then $\mu_A(x) \leq \alpha$, $\nu_A(x) \geq \beta$. Also A be Intuitionistic anti L-fuzzy normal M-subgroup of M-group G Therefore, $\mu_A(g^{-1}x\,g) = \mu_A(x)$ and $\nu_A(g^{-1}x\,g) = \nu_A(x)$ for all $x \in A$ and $g \in G$
Therefore $\mu_A(g^{-1}\,x\,g) = \mu_A(x) \leq \alpha$ and $\nu_A(g^{-1}\,x\,g) = \nu_A(x) \geq \beta$ implies that $\mu_A(g^{-1}\,x\,g) \leq \alpha$ and $\nu_A(g^{-1}\,x\,g) \geq \beta$ and so $g^{-1}\,x\,g \in C_{\alpha,\beta}(A)$

Hence $C_{\alpha,\beta}(A)$ is normal M-subgroup of G.

3.5 Theorem: If A is Intuitionistic fuzzy subset of a group G. Then A is intuitionistic anti L-fuzzy M-subgroup of G if and only if $C_{\alpha,\beta}(A)$ is a M-subgroup of M-group G for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$, where $\mu_A(e) \leq \alpha$, $\nu_A(e) \geq \beta$ and e is the identity element of G.

Proof: Firstly let A be intuitionistic anti L-fuzzy M-subgroup of M-group G. Then the result follows by Theorem (3.3)

Conversely, let A is Intuitionistic fuzzy subset of a group G such that $C_{\alpha,\beta}(A)$ is a M-subgroup of M- group G for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$.

To show that A be intuitionistic anti L-fuzzy M-subgroup of M-group G. For this we show that
(i) $\mu_A(mxy) \leq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(mxy) \geq \nu_A(x) \vee \nu_A(y)$ for all $x, y \in G$
(ii) $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$

For (i) Let $x, y \in G$ and let $\alpha = \mu_A(x) \wedge \mu_A(y)$ and $\beta = \nu_A(x) \vee \nu_A(y)$. Then
$\mu_A(x) \leq \alpha$, $\mu_A(y) \leq \alpha$ and $\nu_A(x) \geq \beta$, $\nu_A(y) \geq \beta$
i.e. $\mu_A(x) \leq \alpha$, $\nu_A(x) \geq \beta$ and $\mu_A(y) \leq \alpha$, $\nu_A(y) \geq \beta$
i.e. $x \in C_{\alpha,\beta}(A)$ and $y \in C_{\alpha,\beta}(A)$ and so $xy \in C_{\alpha,\beta}(A)$ [As $C_{\alpha,\beta}$ is a group]
Therefore $\mu_A(mxy) \leq \alpha = \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(mxy) \geq \beta = \nu_A(x) \vee \nu_A(y)$
i.e. $\mu_A(mxy) \leq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(mxy) \geq \nu_A(x) \vee \nu_A(y)$

For (ii) Let $x \in G$ be any element. Let $\mu_A(x) = \alpha$ and $\nu_A(x) = \beta$. Then
$\mu_A(x) \leq \alpha$ and $\nu_A(x) \geq \beta$ is true i.e. $x \in C_{\alpha,\beta}(A)$
As $x \in C_{\alpha,\beta}(A)$ is a M- subgroup of G
Therefore we have $x^{-1} \in C_{\alpha,\beta}(A) \Rightarrow \mu_A(x^{-1}) \leq \alpha$ and $\nu_A(x^{-1}) \geq \beta$
Thus $\mu_A(x^{-1}) \leq \alpha = \mu_A(x)$ and $\nu_A(x^{-1}) \geq \beta = \nu_A(x)$
Thus $\mu_A(x) = \mu_A((x^{-1})^{-1}) \leq \mu_A(x^{-1}) \leq \mu_A(x)$ implies that $\mu_A(x^{-1}) = \mu_A(x)$
And $\nu_A(x) = \nu_A((x^{-1})^{-1}) \geq \nu_A(x^{-1}) \geq \nu_A(x)$ implies that $\nu_A(x^{-1}) = \nu_A(x)$

Hence A is Intuitionistic Anti L-fuzzy M-subgroup of M-group G.

3.6 Theorem: If A and B be two IALFMSG's of a M-group G, then $A \cap B$ is IALFMSG of M-group G .

Proof: By Theorem (3.5), $A \cap B$ is IALFMSG of M-group G if and only if $C_{\alpha,\beta}(A \cap B)$ is a M-subgroup of G. but as $C_{\alpha,\beta}(A \cap B) = C_{\alpha,\beta}(A) \cap C_{\alpha,\beta}(B)$ and both $C_{\alpha,\beta}(A)$ and $C_{\alpha,\beta}(B)$ are M-subgroups of G and intersection of two M-subgroups of a M-group is a subgroup of G implies that $C_{\alpha,\beta}(A \cap B)$ is a M-subgroup of G and hence $A \cap B$ is IALFMSG of M-group G .

CONFERENCE PAPER
National Conference March 1st - 2018, On "Recent Advances in Pure and Applied Mathematics", Organized by Department of Mathematics, Arul Anandar College (Autonomous), Madurai. Tamilnadu, India.

3.7 Remark: Union of two IALFMSG's of a group G need not be IALFMSG of M-group G

3.8 Example: Consider the Klein four group.

$G = \{e, a, b, ab\}$, where $a^2 = e = b^2$ & $ab = ba$
For $0 \le i \le 5$, let $t_i, s_i \in [0, 1]$ such that $1 = t_0 > t_1 > \ldots > t_5$ and $0 < s_0 < s_1 < \ldots < s_5$

Define Intuitionistic fuzzy subset A and B as follows:
$A = \{< x, \mu_A(x), \nu_A(x) >: x \in G\}$ and $B = \{< x, \mu_B(x), \nu_B(x) >: x \in G\}$, where
$\mu_A(e) = t_1$, $\mu_A(a) = t_3$, $\mu_A(b) = \mu_A(ab) = t_4$, $\nu_A(b) = \nu_A(ab) = s_4$, $\nu_A(a) = s_3$, $\nu_A(e) = s_1$
$\mu_B(e) = t_0$, $\mu_B(a) = t_5$, $\mu_B(b) = t_2$, $\mu_B(ab) = t_5$, $\nu_B(b) = \nu_B(ab) = s_5$, $\nu_B(a) = s_2$, $\nu_B(e) = s_0$

Clearly A and B are IALFMSG of the M-group G.
(i) Now $A \cup B = \{< x, (\mu_A \cup \mu_B)(x), (\nu_A \cup \nu_B)(x) > : x \in G\}$,
Where $(\mu_A \cup \mu_B)(x) = \text{Max}\{\mu_A(x), \mu_B(x)\} = \mu_A(x) \vee \mu_B(x)$ and
$(\nu_A \cup \nu_B)(x) = \text{Min}\{\nu_A(x), \nu_B(x)\} = \nu_A(x) \wedge \nu_B(x)$
Here $(\mu_A \cup \mu_B)(e) = t_0$, $(\mu_A \cup \mu_B)(a) = t_3$, $(\mu_A \cup \mu_B)(b) = t_2$, $(\mu_A \cup \mu_B)(ab) = t_4$
$(\nu_A \cup \nu_B)(e) = s_0$, $(\nu_A \cup \nu_B)(a) = s_2$, $(\nu_A \cup \nu_B)(b) = s_4$, $(\nu_A \cup \nu_B)(ab) = s_4$
$C_{t_3, s_4}(A) = \{x: x \in G$ such that $\mu_A(x) \le t_3, \nu_A(x) \ge s_4\} = \{a, e\}$
$C_{t_3, s_4}(B) = \{x : x \in G$ such that $\mu_B(x) \le t_3, \nu_B(x) \ge s_4\} = \{e\}$
$C_{t_3, s_4}(A \cup B) = \{x : x \in G$ such that $\mu_A \cup_B(x) \le t_3, \nu_A \cup \nu_B(x) \ge s_4\}$
$= \{x: x \in G$ such that $\mu_A(x) \vee \mu_B(x) \le t_3, \nu_A(x) \wedge \nu_B(x) \ge s_4\}$
$= \{e, a, b\}$

Since $\{e, a, b\}$ is not a M-subgroup of G i.e. $C_{t_3, s_4}(A \cup B)$ is not a M-subgroup of G and hence $A \cup B$ is not IALFMSG of M-group G.

3.9 Definition: Intuitionistic anti L-fuzzy left and right cosets

Let G be a M-group and A be IALFMSG of M-group G. Let $x \in G$ be a fixed element.
Then the set $xA = \{(g, \mu_{xA}(g), \nu_{xA}(g)) : g \in G\}$ where $\mu_{xA}(g) = \mu_A(x^{-1}g)$ and $\nu_{xA}(g) = \nu_A(x^{-1}g)$ for all $g \in G$ is called intuitionistic anti L-fuzzy left coset of G determined by A and x.
similarly, the set $Ax = \{(g, \mu_{Ax}(g), \nu_{Ax}(g)) : g \in G\}$ where $\mu_{Ax}(g) = \mu_A(gx^{-1})$ and $\nu_{Ax}(g) = \nu_A(gx^{-1})$ for all $g \in G$ is called the intuitionistic anti L-fuzzy right coset of G determined by A and x.

3.10 Remark: It is clear that if A is intuitionistic anti L-fuzzy normal M-subgroup of G, then the intuitionistic anti L-fuzzy left coset and intuitionistic anti L-fuzzy right coset of A on G coincide and in this case, we call intuitionistic anti L-fuzzy coset instead of intuitionistic anti L-fuzzy left or intuitionistic anti L-fuzzy right coset.

3.11 Theorem: Let A be intuitionistic anti L-fuzzy M-subgroup of a M-group G and x be any fixed element of G. Then
(i) $x. C_{\alpha, \beta}(A) = C_{\alpha, \beta}(xA)$
(ii) $C_{\alpha, \beta}(A).x = C_{\alpha, \beta}(Ax)$, for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \le 1$.

Proof:
(i) Now $C_{\alpha, \beta}(xA) = \{ g \in G : \mu_{xA}(g) \le \alpha$ & $\nu_{xA}(g) \ge \beta \}$ with $\alpha + \beta \le 1$
Also $x . C_{\alpha, \beta}(A) = x .\{y \in G: \mu_A(y) \le \alpha$ and $\nu_A(y) \ge \beta\}$
$= \{x y \in G: \mu_A(y) \ge \alpha$ and $\nu_A(y) \ge \beta\}$
Put $xy = g$ so that $y = x^{-1}g$
Therefore $x.C_{\alpha, \beta}(A) = \{g \in G: \mu_A(x^{-1}g) \le \alpha$ and $\nu_A(x^{-1}g) \ge \beta\}$
$= \{g \in G: \mu_{xA}(g) \le \alpha$ and $\nu_{xA}(g) \ge \beta\}$
Thus $x.C_{\alpha, \beta}(A) = C_{\alpha, \beta}(xA)$ for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \le 1$

(ii) Again $C_{\alpha, \beta}(Ax) = \{g \in G: \mu_{Ax}(g) \le \alpha$ and $\nu_{Ax}(g) \ge \beta\}$ with $\alpha + \beta \le 1$
Also $C_{\alpha, \beta}(A).x = \{y \in G: \mu_A(y) \le \alpha$ and $\nu_X(y) \ge \beta\}. x$
$= \{yx \in G: \mu_A(y) \le \alpha$ and $\nu_X(y) \ge \beta\}$ Put $yx = g$ so that $y = gx^{-1}$

CONFERENCE PAPER
National Conference March 1st - 2018, On "Recent Advances in Pure and Applied Mathematics", Organized by Department of Mathematics, Arul Anandar College (Autonomous), Madurai. Tamilnadu, India.

$C_{\alpha,\beta}(A).x = \{g \in G: \mu_A(gx^{-1}) \leq \alpha \text{ and } \nu_X(gx^{-1}) \geq \beta\}$
$= \{g \in G: \mu_{Ax}(g) \leq \alpha \text{ and } \nu_{Ax}(g) \geq \beta\}$
Hence $C_{\alpha,\beta}(A).x = C_{\alpha,\beta}(Ax)$, for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$.

3.12 Theorem: Let A be Intuitionitic Anti L-fuzzy M-subgroup of M-group G. Let x, y be elements of G such that $\mu_A(x) \wedge \mu_A(y) = \alpha$ and $\nu_A(x) \vee \nu_A(y) = \beta$.
Then
(i) $xA = yA \Leftrightarrow x^{-1}y \in C_{\alpha,\beta}(A)$
(ii) $Ax = Ay \Leftrightarrow xy^{-1} \in C_{\alpha,\beta}(A)$

Proof:
(i) Now $xA = yA \Leftrightarrow C_{\alpha,\beta}(xA) = C_{\alpha,\beta}(yA)$
$\Leftrightarrow x.C_{\alpha,\beta}(A) = y.C_{\alpha,\beta}(A)$ [by Theorem (3.11)(i)]
$\Leftrightarrow x^{-1}y \in C_{\alpha,\beta}(A)$ [As $C_{\alpha,\beta}(A)$ is a subgroup of G] (ii) Again $Ax = Ay \Leftrightarrow C_{\alpha,\beta}(Ax) = C_{\alpha,\beta}(Ay)$
$\Leftrightarrow C_{\alpha,\beta}(A).x = C_{\alpha,\beta}(A).y$ [by Theorem (3.11)(ii)]
$\Leftrightarrow xy^{-1} \in C_{\alpha,\beta}(A)$ [As $C_{\alpha,\beta}(A)$ is a subgroup of G]

REFERENCES

1. K.T Atanassov, " Intuitionistic fuzzy sets", Fuzzy Sets and Systems 20(1986), no.1, 87-96
2. D.K Basnet and N.K Sarma, " A note on Intuitionistic Fuzzy Equivalence Relation", International Mathematical Forum, 5, 2010, no. 67, 3301-3307
3. Kul Hur and Su Youn Jang, "The lattice of Intuitionistic fuzzy congruences", International Mathematical Forum, 1, 2006, no. 5, 211-236
4. N. Palaniappan, S. Naganathan and K. Arjunan, "A study on Intuitionistic L- Fuzzy Subgroups", Applied Mathematical Sciences, vol. 3, 2009, no. 53 , 2619-2624
5. Ranjit Biswas "Vague Groups", International Journal of Computational Cognition, Vol. 4, no. 2, June 2006
6. L.A Zadeh, "Fuzzy sets", Information and Control 8, (1965), 338-353
7. P. K. Sharma, "(α, β) – Cut of Intuitionistic Fuzzy Groups", International Mathematical Forum, Vol. 6, 2011, no. 53, 2605 - 2614

Source of support: Proceedings of National Conference March 1st - 2018, On "Recent Advances in Pure and Applied Mathematics (RAPAM - 2018)", Organized by Department of Mathematics, Arul Anandar College (Autonomous), Madurai. Tamilnadu, India.