

SOME SUPRA MAPS VIA SUPRA \tilde{g} - CLOSED SETS

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ABSTRACT

In this paper, We introduce new class of maps called supra \tilde{g} -continuous maps, supra \tilde{g} - closed maps and supra \tilde{g} - irresolute maps. Subsequently, we investigate several properties of these classes of maps.

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1. INTRODUCTION

In 2008, Devi *et al.* [1] introduced and studied a class of sets called supra α -open and a class of maps called α -continuous maps between topological spaces, respectively. In 2010, Ravi *et al.* [10] have introduced and studied a class of sets called supra g -closed and a class of maps called supra g -continuous and supra g -closed respectively. Quite Recently G. Ramkumar *et al.* [8] have introduced and studied a class of sets called supra \tilde{g} -closed. In line with the research, In this paper, We introduce new class of maps called supra \tilde{g} -continuous maps, supra \tilde{g} - closed maps and supra \tilde{g} - irresolute maps. Subsequently, we investigate several properties of these classes of maps.

2. PRELIMINARIES

Throughout this paper (X, τ) , (Y, σ) and (Z, ν) (or simply, X , Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1 [5, 11]: Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ where $P(X)$ is the power set of X is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions.

The pair (X, μ) is called a supra topological space.

The elements of μ are said to be supra open in (X, μ) .

Complements of supra open sets are called supra closed sets.

Definition 2.2 [4]: A map $f: X \rightarrow Y$ is said to be

- (i) continuous if the inverse image of each open set of Y is an open set in X .
- (ii) closed if the image of each closed set of X is a closed set in Y .
- (iii) g -closed if the image of each closed set of X is a g -closed set in Y .

Definition 2.3 [1]: Let A be a subset of (X, μ) . Then

- (i) The supra closure of a set A is, denoted by $cl^\mu(A)$, defined as $cl^\mu(A) = \bigcap \{B : B \text{ is a supra closed and } A \subseteq B\}$;
- (ii) The supra interior of a set A is, denoted by $int^\mu(A)$, defined as $int^\mu(A) = \bigcup \{G : G \text{ is a supra open and } A \supseteq G\}$.

Definition 2.4 [5]: Let (X, τ) be a topological space and μ be a supra topology on X . We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.

Definition 2.5: Let (X, μ) be a supra topological space. A subset A of X is called

- (i) supra semi-open set [1] if $A \subseteq \text{cl}^\mu(\text{int}^\mu(A))$;
- (ii) supra α -open set [1, 12] if $A \subseteq \text{int}^\mu(\text{cl}^\mu(\text{int}^\mu(A)))$;
- (iii) supra β -open set [9] if $A \subseteq \text{cl}^\mu(\text{int}^\mu(\text{cl}^\mu(A)))$;
- (iv) supra pre-open set [12] if $A \subseteq \text{int}^\mu(\text{cl}^\mu(A))$.

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.6: Let (X, μ) be a supra topological space. A subset A of X is called

- i) supra g -closed [10] if $\text{cl}^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X
- ii) supra ω -closed (= supra \hat{g} -closed) [7] if $\text{cl}^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in X .
- iii) supra $*g$ closed [6] if $\text{cl}^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra ω -open in X .
- iv) supra $^{\#}gs$ closed [7] if $\text{scl}^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra $*g$ -open in X .
- v) supra \tilde{g} - closed [8] if $\text{cl}^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra $^{\#}gs$ -open in X .

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.7: Let (X, τ) and (Y, σ) be two topological spaces with $\tau \subseteq \mu$. A map $f : (X, \mu) \rightarrow (Y, \sigma)$ is called

- a) supra continuous [1] if the inverse image of each open set of Y is a supra open set in X .
- b) g -continuous [10] if the inverse image of each closed set of Y is a supra g -closed set in X .
- c) supra ω -continuous [7] if the inverse image of each closed set of Y is a supra ω -closed set in X .
- d) supra $^{\#}gs$ -continuous [7] if the inverse image of each closed set of Y is a supra $^{\#}gs$ -closed set in X .

3. SUPRA \tilde{g} - MAPS

Definition 3.1: Let (X, μ) be a supra topological space. $A \subseteq X$ then

- i) Supra \tilde{g} -closure of A is denoted by $\text{cl}_{\tilde{g}}^\mu(A)$, defined by the intersection of all supra \tilde{g} -closed sets containing A
(i.e) $\text{cl}_{\tilde{g}}^\mu(A) = \bigcap \{F, A \subseteq F \text{ and } F \text{ is supra } \tilde{g} \text{-closed}\}$
- ii) Supra \tilde{g} - interior of A is denoted by $\text{int}_{\tilde{g}}^\mu(A)$, defined by the union of all supra \tilde{g} -open sets contained in A .
(i.e) $\text{int}_{\tilde{g}}^\mu(A) = \bigcup \{F, A \subseteq F \text{ and } F \text{ is supra } \tilde{g} \text{-open}\}$

Remark 3.2: For the subsets A, B of a supra topological space (X, μ) , the following statements hold.

- i) $\text{cl}_{\tilde{g}}^\mu(A)$ is the smallest supra \tilde{g} -closed set containing A .
- ii) A is supra \tilde{g} -closed if and only if $\text{cl}_{\tilde{g}}^\mu(A) = A$.
- iii) If $A \subseteq B$ then $\text{cl}_{\tilde{g}}^\mu(A) \subseteq \text{cl}_{\tilde{g}}^\mu(B)$.
- iv) $\text{cl}_{\tilde{g}}^\mu(A) \cup \text{cl}_{\tilde{g}}^\mu(B) \subseteq \text{cl}_{\tilde{g}}^\mu(A \cup B)$.
- v) $X \setminus \text{int}_{\tilde{g}}^\mu(A) = \text{cl}_{\tilde{g}}^\mu(A^c)$.
- vi) $\text{int}_{\tilde{g}}^\mu(\text{int}_{\tilde{g}}^\mu(A)) = \text{int}_{\tilde{g}}^\mu(A)$.
- vii) $X \setminus \text{cl}_{\tilde{g}}^\mu(A) = \text{int}_{\tilde{g}}^\mu(A^c)$.
- viii) If $A \subseteq B$ then $\text{int}_{\tilde{g}}^\mu(A) \subseteq \text{int}_{\tilde{g}}^\mu(B)$
- ix) $\text{int}_{\tilde{g}}^\mu(A) \cup \text{int}_{\tilde{g}}^\mu(B) \subseteq \text{int}_{\tilde{g}}^\mu(A \cup B)$
- x) $\text{int}_{\tilde{g}}^\mu(A) \cap \text{int}_{\tilde{g}}^\mu(B) \subseteq \text{int}_{\tilde{g}}^\mu(A \cap B)$

Lemma 3.3: Let (X, τ) be topological space $\tau \subseteq \mu$. For any $A \subseteq X$, $\text{int}^\mu(A) \subseteq \text{int}_{\tilde{g}}^\mu(A) \subseteq A$.

Proof: Since every supra open set is supra \tilde{g} -open.

Definition 3.4: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is said to be

- i) Supra \tilde{g} -continuous if $f^{-1}(V)$ is supra \tilde{g} -closed in X for every closed set V of Y .
- ii) Supra \tilde{g} -irresolute if $f^{-1}(V)$ is supra \tilde{g} -closed in X for every supra \tilde{g} -closed set V of Y .
- iii) Supra \tilde{g} -closed [resp. supra \tilde{g} -open] if $f(V)$ is supra \tilde{g} -closed [resp. supra \tilde{g} -open] in Y for every closed set [resp. open] V of X .
- iv) Supra \tilde{g}^* -continuous if $f^{-1}(V)$ is supra \tilde{g} -closed in X for every supra closed set V of Y .
- v) Supra \tilde{g}^* -closed if $f(V)$ is supra \tilde{g} -closed in Y for every supra closed set V of X .

Theorem 3.5: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is supra \tilde{g}^* -closed if and only if $\text{cl}_{\tilde{g}}^\mu(f(A)) \subseteq f(\text{cl}^\mu(A))$ for every subset A of X .

Proof: Suppose that f is supra \tilde{g}^* -closed and $A \subseteq X$. Then $f(\text{cl}^\mu(A))$ is supra \tilde{g} -closed in Y . We have $f(A) \subseteq f(\text{cl}^\mu(A))$ and by Remark 3.2 $\text{cl}_{\tilde{g}}^\mu(f(A)) \subseteq \text{cl}_{\tilde{g}}^\mu(f(\text{cl}^\mu(A))) = f(\text{cl}^\mu(A))$.

Conversely, Let A be any supra closed in X . By hypothesis and Remark 3.2 we have $A = \text{cl}^\mu(A)$ and so $f(A) = f(\text{cl}^\mu(A)) \subseteq \text{cl}_{\tilde{g}}^\mu(f(A))$. Therefore, $f(A) = \text{cl}_{\tilde{g}}^\mu(f(A))$. Hence $f(A)$ is supra \tilde{g} -closed in Y and hence f is supra \tilde{g} -closed.

Theorem 3.6: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is a supra \tilde{g}^* -closed mapping, then for each subset A of X , $\text{cl}^\lambda(\text{int}^\lambda f(A)) \subseteq f(\text{cl}^\mu(A))$.

Proof: Let f be a supra \tilde{g}^* -closed map and $A \subseteq X$. Since $\text{cl}^\mu(A)$ is a supra closed set in X . We have $f(\text{cl}^\mu(A))$ is supra \tilde{g} -closed and hence supra pre-closed. Therefore $\text{cl}^\lambda(\text{int}^\lambda (f(\text{cl}^\mu(A)))) \subseteq f(\text{cl}^\mu(A))$ (i.e) $\text{cl}^\lambda(\text{int}^\lambda f(A)) \subseteq f(\text{cl}^\mu(A))$.

Theorem 3.7: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \tau) \rightarrow (Y, \lambda)$ is a supra \tilde{g} -closed if and only if for each subset S of Y for each open set U containing $f^{-1}(S)$ there is supra \tilde{g} -open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Suppose that f is a supra \tilde{g} -closed map. Let $S \subseteq Y$ and U be an open set of X such that $f^{-1}(S) \subseteq U$. Thus $V = Y \setminus f(X \setminus U)$ is a supra \tilde{g} -open set containing S such that $f^{-1}(V) \subseteq U$.

Conversely, suppose that F is a closed set of X . Then $f^{-1}(Y \setminus f(F)) \subseteq X \setminus F$ and $X \setminus F$ is open. By hypothesis, there exist a supra \tilde{g} -open set V of Y such that $Y \setminus (f(F)) \subseteq V$ and $f^{-1}(V) \subseteq X \setminus F$. Therefore $F \subseteq X \setminus f^{-1}(V)$. Hence $Y \setminus V \subseteq f(F) \subseteq f(X \setminus f^{-1}(V)) \subseteq Y \setminus V$ which implies $f(F) = Y \setminus V$. Since $Y \setminus V$ is supra \tilde{g} -closed, $f(F)$ is supra \tilde{g} -closed set in Y and thus f is supra \tilde{g} -closed map.

Theorem 3.8: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is a supra $\#$ gs-irresolute and supra \tilde{g}^* -closed and A is a supra \tilde{g} -closed subset of X , Then $f(A)$ is supra \tilde{g} -closed.

Proof: Let U be a supra $\#$ gs-open in Y such that $f(A) \subseteq U$. Since f is supra $\#$ gs-irresolute. $f^{-1}(U)$ is a supra $\#$ gs-open set containing A . Hence $\text{cl}^\mu(A) \subseteq f^{-1}(U)$ as A is supra \tilde{g} -closed in X . Since f is supra \tilde{g}^* -closed. $f(\text{cl}^\mu(A))$ is a supra \tilde{g} -closed set containing in the supra $\#$ gs-open set U , which implies that $\text{cl}^\mu(f(\text{cl}^\mu(A))) \subseteq U$ and hence $\text{cl}^\mu(f(A)) \subseteq U$. Therefore $f(A)$ supra \tilde{g} -closed set.

Remark 3.9: The composition of two supra \tilde{g} -closed maps need not be supra \tilde{g} -closed.

Corollary 3.10: Let (X, τ) , (Y, σ) and (Z, η) be three topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$ and $\nu \subseteq \eta$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ be a supra \tilde{g} -closed map and $g: (Y, \lambda) \rightarrow (Z, \eta)$ be a supra \tilde{g}^* -closed and supra $\#$ gs-irresolute map then their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is supra \tilde{g} -closed.

Proof: Let A be a closed set of X . Since f is supra \tilde{g} -closed, $f(A)$ is supra \tilde{g} -closed set in Y . Since g is both supra $\#$ gs-irresolute and supra \tilde{g} -closed by Theorem 3.8, $g(f(A)) = (g \circ f)(A)$ is supra \tilde{g} -closed in Z and therefore $g \circ f$ is supra \tilde{g} -closed.

Definition 3.11: A Supra topological (X, μ) is said to be

- 1) Supra $T_{\tilde{g}}$ -space if every supra \tilde{g} -closed subset of X is supra closed in X .
- 2) Supra $T_{1/2}$ -space if every supra g -closed subset of X is supra closed in X .
- 3) Supra semi $^* T_{1/2}$ -space if every supra \hat{g} -closed subset of X is supra closed in X .
- 4) Supra $T_{1/2}^*$ -space if every supra g -closed subset of X is closed in X .
- 5) Supra semi $T_{1/2}^{**}$ -space if every supra \hat{g} -closed subset of X is closed in X .

Proposition 3.12: Let (X, τ) , (Y, σ) and (Z, ν) be three topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$ and $\nu \subseteq \eta$. If $f: (X, \mu) \rightarrow (Y, \lambda)$ and $g: (Y, \lambda) \rightarrow (Z, \eta)$ are supra \tilde{g}^* -closed and Y is a supra $T_{\tilde{g}}$ -space, then their composition $g \circ f: (X, \mu) \rightarrow (Z, \eta)$ is supra \tilde{g}^* -closed map.

Proof: Let A be supra closed set of X . Then $f(A)$ is supra \tilde{g} -closed in Y . Since Y is $T_{\tilde{g}}$ -space $f(A)$ is supra closed in Y and g is supra \tilde{g}^* -closed then $g(f(A))$ is supra \tilde{g}^* -closed set in Z that is $(g \circ f)(A)$ is supra \tilde{g} -closed in Z and so $g \circ f$ is supra \tilde{g} -closed.

Definition 3.13: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is called supra g^* -closed if $f(V)$ is supra g -closed in Y for every supra closed V in X .

Proposition 3.14: Let (X, τ) , (Y, σ) and (Z, ν) be three topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$ and $\nu \subseteq \eta$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ be a supra \tilde{g}^* -closed map and $g: (Y, \lambda) \rightarrow (Z, \eta)$ be a supra g^* -closed and Y is supra $T_{\tilde{g}}$ -space then their composition $g \circ f: (X, \mu) \rightarrow (Z, \eta)$ is supra g^* -closed.

Proof: Similar to Proposition 3.12.

Proposition 3.15: Let (X, τ) , (Y, σ) and (Z, ν) be three topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$ and $\nu \subseteq \eta$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ be a closed map and $g: (Y, \lambda) \rightarrow (Z, \eta)$ be a supra \tilde{g} -closed map then their composition $g \circ f: (X, \mu) \rightarrow (Z, \eta)$ is supra \tilde{g} -closed.

Proof: Similar to Proposition 3.12.

Theorem 3.16: Let (X, τ) , (Y, σ) and (Z, ν) be three topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$ and $\nu \subseteq \eta$. Let $f: (X, \mu) \rightarrow (Y, \lambda)$ and $g: (Y, \lambda) \rightarrow (Z, \eta)$ be two mappings such that $g \circ f: (X, \mu) \rightarrow (Z, \eta)$ is supra \tilde{g} -closed mapping then the following statements are true if

- i) f is a supra continuous and surjective, then g is supra \tilde{g}^* -closed.
- ii) g is a supra \tilde{g} -irresolute and injective, then f is supra \tilde{g} -closed.
- iii) f is supra g -continuous, surjective and X is supra $T_{1/2}^*$ -space, then g is supra \tilde{g} -closed.
- iv) f is supra \tilde{g} -continuous, surjective and X is supra semi $T_{1/2}^{**}$ -space, then g is supra \tilde{g} -closed.

Proof:

- (i) Let A be a closed set of Y. Since f is continuous $f^{-1}(A)$ is closed in X and since gof is supra \tilde{g} -closed, $(\text{gof})(f^{-1}(A))$ is supra \tilde{g} -closed in Z. Therefore $g(A)$ is supra \tilde{g} -closed in Z, since f is surjective. Therefore, g is supra \tilde{g} -closed map.
- (ii) Let B be closed in X. Since gof is supra \tilde{g} -closed, $(\text{gof})(B)$ is supra \tilde{g} -closed in Z. $g^{-1}((\text{gof})(B))$ is supra \tilde{g} -closed in Y. i.e, $f(B)$ is supra \tilde{g} -closed in Y, since g is injective.
Thus, f is supra \tilde{g} -closed map.
- (iii) Let A be a closed set of Y. Since f is supra \tilde{g} -continuous, $f^{-1}(A)$ is supra \tilde{g} -closed in X.
Since X is a supra $T_{1/2}^*$ -space $f^{-1}(A)$ is closed in X and so as in (i) g is supra \tilde{g} -closed map.
- (iv) Let A be a closed set of Y. Since f is supra \tilde{g} -continuous, $f^{-1}(A)$ is supra \tilde{g} -closed set in X. Since every supra \tilde{g} -closed set is supra \hat{g} -closed and X is supra semi $T_{1/2}^{**}$ -space. $f^{-1}(A)$ is closed in X and so as in (i) g is supra \tilde{g} -closed map.

Definition 3.17: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu, \sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is said to be

- i) Supra strongly \tilde{g} -continuous if $f^{-1}(V)$ is open in X for every supra \tilde{g} -open V in Y.
- ii) Supra strongly \tilde{g}^* -continuous if $f^{-1}(V)$ is supra open in X for every supra \tilde{g} -open V in Y.

Theorem 3.18: Let $(X, \tau), (Y, \sigma)$ and (Z, ν) be three topological space with $\tau \subseteq \mu, \sigma \subseteq \lambda$ and $\nu \subseteq \eta$. Let $f: (X, \mu) \rightarrow (Y, \lambda)$ and $g: (Y, \lambda) \rightarrow (Z, \eta)$ be two mappings such that their composition $\text{gof}: (X, \mu) \rightarrow (Z, \eta)$ be a supra \tilde{g} -closed mapping. If g is strongly \tilde{g} -continuous and injective, then f is closed.

Proof: Let D be a closed set of X. Since gof is supra \tilde{g} -closed, $(\text{gof})(D)$ is supra \tilde{g} -closed in Z. Since g is supra strongly \tilde{g} -continuous $g^{-1}((\text{gof})(D))$ is closed in Y, $f(D)$ is closed in Y, since g is injective. Therefore f is closed map.

Theorem 3.19: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu, \sigma \subseteq \lambda$. Any bijection map $f: (X, \mu) \rightarrow (Y, \lambda)$ the following statements are equivalent

- (i) $f^{-1}: (Y, \lambda) \rightarrow (X, \mu)$ is supra \tilde{g} -continuous
- (ii) f is supra \tilde{g} -open map and
- (iii) f is supra \tilde{g} -closed map

Proof:

i) \Rightarrow ii): Let U be open set of X. Then by assumption $(f^{-1})^{-1}(U) = f(U)$ is supra \tilde{g} -open in Y and so f is supra \tilde{g} -open.

ii) \Rightarrow iii): Let F be a closed set of X. The F^c is open in X. By assumption $f(F^c)$ is supra \tilde{g} -open in Y. i.e, $f(F^c) = (f(F))^c$ is supra \tilde{g} -open in Y and therefore $f(F)$ is supra \tilde{g} -closed in Y. Hence f is supra \tilde{g} -closed.

iii) \Rightarrow i): Let F be closed set in X. By assumption $f(F)$ is supra \tilde{g} -closed in Y.

But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is supra \tilde{g} -continuous.

Theorem 3.20: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is Supra \tilde{g} -open if and only if for any subset B of Y and for any closed set S containing $f^{-1}(B)$, there exist Supra \tilde{g} -closed set A of Y containing B such that $f^{-1}(A) \subseteq S$.

Proof: Similar to Theorem 3.7

Definition 3.21: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is said to be supra \tilde{g}^{**} -closed map if $f(V)$ is Supra \tilde{g} -closed in Y for every supra \tilde{g} -closed set V in X.

Proposition 3.22: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map $f: (X, \mu) \rightarrow (Y, \lambda)$ is said to be Supra \tilde{g}^{**} -closed if and only if $cl_g^{\mu}(f(A)) \subseteq f(cl_g^{\lambda}(A))$ for every subset A of X.

Proof: Similar to Theorem 3.5.

Theorem 3.23: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. For any bijection map $f: (X, \mu) \rightarrow (Y, \lambda)$ the following statements are equivalent

- i) $f^{-1}: (Y, \lambda) \rightarrow (X, \mu)$ is supra \tilde{g} -irresolute
- ii) f is supra \tilde{g}^{**} -open map and
- iii) f is supra \tilde{g}^{**} -closed map

Proof: Similar to Theorem 3.19

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