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DOMINATION ON ANTI FUZZY GRAPH

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ABSTRACT

In this paper, the concept of dominating set and domination number on anti fuzzy graph is introduced. The bounds on domination number of anti fuzzy graph are obtained. Some properties of adjacency fuzzy matrix are discussed and using strong adjacency matrix, An algorithms are derived for finding minimal dominating set of anti fuzzy graph G_A. These concepts are applied on anti cartesian product of anti fuzzy graphs and obtained the results on them.

Keywords: *Anti fuzzy graph, Fuzzy graph, Dominating set, Domination number, v-nodal anti fuzzy graph, uninodal anti fuzzy graph.*

Mathematical Classification: *05C62, 05E99, 05C07.*

I. INTRODUCTION

A real life problem can be solved by representing the situation as a mathematical model. Graph is one of the best suitable ways of representing the relationship between objects and it is the easiest way to arrive at the solution. In graph, vertices and edges represent the objects and their relationship. In some cases, there exist uncertainties in the description of the objects or in its relationships or in both which has to be designed as fuzzy graph model. In fuzzy graph, the relation attains only the minimum value among the objects. Sometimes, if complexity occurs among the relation, then it may lead to obtain maximum values. Such case, the problem is constructed as 'Anti fuzzy Graph Model'. R.Seethalakshmi and R.B.Gnanajothi [7] introduced the definition of anti fuzzy graph. R.Muthuraj and A. Sasireka [5] defined some types of anti fuzzy graph. R. Muthuraj and A. Sasireka [3, 4] illustrated the concepts of some types of regular, edge regular anti fuzzy graph and some operations on anti fuzzy graphs such as anti union, anti join, anti cartesian product and anti composition. R.Muthuraj and A. Sasireka [2] explained the concept of domination on cartesian product of fuzzy graph. Normally, the concept of domination is used to obtain the minimalism in the model. But in some situation, we need to get maximum value with minimal objects. These types of problem arises the concept of domination in anti fuzzy graph. In this paper, we introduce the concept of domination number on anti fuzzy graphs. Fuzzy matrix is defined on anti fuzzy graph and also define strong adjacency fuzzy matrix on anti fuzzy graph. Using this matrix, two algorithms are illustrated for finding minimal dominating set. The domination concepts applied on anti cartesian product of anti fuzzy graphs also. The results are examined and some theorems are derived for them.

II. PRELIMINARIES

In this section, basic concepts of anti fuzzy graph are discussed. Notations and more formal definitions which are followed as in [3, 4, 5].

Definition 2.1 [3]: A fuzzy graph $G = (\sigma, \mu)$ is said to be an anti fuzzy graph with a pair of functions $\sigma : V\to [0,1]$ and $\mu : V\times V\to[0,1]$, where for all $u, v \in V$, we have $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_A(\sigma, \mu)$.

Note: μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. i.e., undirected anti fuzzy graphs are only considered.

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Notation: Without loss of generality let us simply use the letter G_A to denote an anti fuzzy graph.

Definition 2.2 [3]: The order p and size q of an anti fuzzy graph $G_A = (V, \sigma, \mu)$ are defined to be $p=\sum_{x\in V}\sigma(x)$ and $q=\sum_{xy\in E}\mu(xy)$. It is denoted by O(G) and S(G).

Definition 2.3 [3]: Two vertices u and v in G_A are called adjacent if $(½)[\sigma(u) \vee \sigma(v)] \le \mu(u,v)$.

Definition 2.4 [3]: The anti complement of anti fuzzy graph $G_A(\sigma,\mu)$ is an anti fuzzy graph $\overline{G_A} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u,v) = \mu(u,v) - (\sigma(u) \vee \sigma(v))$ for all u,v in V.

Definition 2.5 [5]: An anti fuzzy graph $G_A= (\sigma,\mu)$ is a strong anti fuzzy graph of $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v)\in\mu^*$ and G_A is a complete anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \mu^*$ and $u,v\in \sigma^*$. Two vertices u and v are said to be neighbors if $\mu(u,v) > 0$.

Definition 2.6 [3]: An edge e = {u, v} of an anti fuzzy graph G_A is called an effective edge if $\mu(u, v) = \sigma(u) \vee \sigma(v)$.

Definition 2.7 [3]: u is a vertex in an anti fuzzy graph G_A then N(u) = {v: (u,v) is an effective edge} is called the neighborhood of u and $N[u] = N(u) \cup \{u\}$ is called closed neighborhood of u.

Definition 2.8 [3]: A path P_A in an anti fuzzy graph is a sequence of distinct vertices $u_0, u_1, u_2, \ldots, u_n$ such that $\mu(u_{i-1},u_i)>0$, $1\le i \le n$. Here $n \ge 0$ is called the length of the path P_A. The consecutive pairs (u_{i-1}, u_i) are called the edges of the path.

Definition 2.9[3]: A cycle in G_A is said to be an anti fuzzy cycle if it contains more than one weakest edge.

Definition 2.10[4]: Let $G_A^* = G_{A_1}^* \times G_{A_2}^* = (V, E')$ be the anti cartesian product of anti fuzzy graphs where $V = V_1 \times V_2$ and $E' = \{(u_1,u_2),(u_1,v_2) / u_1\in V_1 , (u_2,v_2)\in E_2\}\cup\{(u_1,w_2),(v_1,w_2) / w_2\in V_2 , (u_1,v_1)\in E_1\}$. Then the anti cartesian product of two anti fuzzy graphs, $G_A=G_{A_1}\times G_{A_2}$: $(\sigma_1\times\sigma_2, \mu_1\times\mu_2)$ is an anti fuzzy graph and is defined by
$(\sigma_1 \times \sigma_2)(u_1, u_2) = \max \{\sigma_1 (u_1), \sigma_2(u_2)\}$ for all $(u_1, u_2)\in V$
$(\mu_1 \times \mu_2)((u_1, u_2), (u_1, v_2)) = \max\{\sigma_1(u_1), \mu_2(u_2, v_2)\}$ for all $u_1\in V_1$ and $(u_2, v_2)\in E_2$
$(\mu_1 \times \mu_2)((u_1, w_2), (v_1, w_2)) = \max \{\sigma_2 (w_2), \mu_1(u_1, v_1)\}$ for all $w_2\in V_2$ and $(u_1, v_1) \in E_1$,
Then the anti fuzzy graph $G_A = (\sigma_1\times\sigma_2, \mu_1\times\mu_2)$ is said to be the anti cartesian product of $G_{A_1}= (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$.

Definition 2.11 [5]: Every vertex in an anti fuzzy graph G_A has unique fuzzy values then G_A is said to be v-nodal anti fuzzy graph. i.e. $\sigma(u) = c$ for all $u\in V(G_A)$.

Definition 2.12[5]: Every edge in an anti fuzzy graph G_A has unique fuzzy values then G_A is said to be e-nodal anti fuzzy graph. i.e. $\mu(u,v) = c$ for all $uv\in E(G_A)$

Definition 2.13 [5]: Every vertices and edges in an anti fuzzy graph G_A have the unique fuzzy value then G_A is called as uninodal anti fuzzy graph.

Definition 2.14: The strong neighborhood of an edge e_i in an anti fuzzy graph G_A is $N_s(e_i) = \{e_j \in E(G) / e_j$ is an effective edge with $\vee N(e_i)$ in G_A and adjacent to $e_i\}$.

Definition 2.15: An edge e = {u, v} of an anti fuzzy graph G_A is called a strong edge if $\mu^\infty(u, v) = \mu(u, v)$ where $\mu(u, v)$ is an effective edge.

Definition 2.16: An edge e = {u, v} of an anti fuzzy graph G_A is called an weak edge if $\mu(u,v) \ne \sigma(u) \vee \sigma(v)$.

Definition 2.17: Maximum number of edges are effective edge in anti cartesian product of anti fuzzy graphs G_{A_1} and G_{A_2}. Then the type of anti fuzzy graph $G_A(= G_{A_1}\times G_{A_2})$ is called as partially strong anti fuzzy graph.

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III. DOMINATION ON ANTI FUZZY GRAPH

In this section, we define the definition of dominating set and domination number on anti fuzzy graph G_A. These concepts are applied on some types of simple anti fuzzy graph G_A, few elementary bounds on domination number are described, the corresponding theorems and results are presented. To prove the theorems and bounds, we consider that the number of vertices in an anti fuzzy graph as m and number of edges in an anti fuzzy graph as n. Order, $p=\sum_{x\in V}\sigma(x)$ and size, $q=\sum_{xy\in E}\mu(xy)$.

Definition 3.1: A set $D\subseteq V(G_A)$ is said to be a dominating set of an anti fuzzy graph G_A if for every vertex $v\in V(G_A) \setminus D$ there exists u in D such that v is a strong neighborhood of u with $\mu(u,v) = \sigma(u) \vee \sigma(v)$ otherwise it dominates itself.

A dominating set D with minimum number of vertices is called a minimal dominating set if no proper subset of D is a dominating set.

The maximum fuzzy cardinality taken over all minimal dominating set in G_A is called a domination number of anti fuzzy graph G_A and is denoted by $\gamma(G_A)$ or γ_A. ie, $|D|_f = \sum_{v\in D} \sigma(v)$.

Example 3.2:

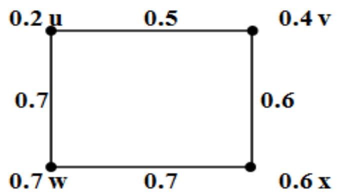

Fig. 1. Anti fuzzy graph G_A

In figure 1, w is the strong neighbour to x and u, x is the strong neighbour to v and w, u is the strong neighbour to w. $\mu(u,v)$ is a weakest edge then u and v are not strong neighbours to each other. That is, u and v cannot dominate each other.
Then the dominating sets are, {w, x}, {w, v}, {x, u}.

The corresponding domination number is 1.3, 1.1 and 0.8.

Therefore, $\gamma(G_A)=1.3$

Note: To construct a dominating set in an anti fuzzy graph G_A, first choose the vertex with maximum fuzzy value with maximum number of neighbourhood vertices. Then easily we get a minimal dominating set.

Theorem 3.3: Every connected anti fuzzy graph G_A contains a minimal dominating set D such that every vertex $v\in D$ and there exists a vertex $u\in V\setminus D$ such that $N(u)\cap D=\{v\}$.

Theorem 3.4: A dominating set D of G_A is a minimal dominating set if and only if for any $u\in D$, and the following conditions are hold.

1. u isolates in D or it dominates itself.
2. There is a node $v\in V\setminus D$ such that $N[v] \cap D = \{u\}$.

Proof: Let G_A be an anti fuzzy graph and D be a dominating set. If G_A contains any isolated vertices (say u) then they dominates itself and $u \in D$. Therefore u isolates in D.

If G_A is a simple connected anti fuzzy graph and D be a minimal dominating set. Let $u\in D$. suppose $D\setminus\{u\}$ is not minimal dominating set of G_A. That is, there is a vertex $v\in(V\setminus D)\cup\{u\}$ is not dominated by $D\setminus\{u\}$. Thus v=u may be isolated in D otherwise it dominate itself unless $v\in V\setminus D$ and v is not dominated by $D\setminus\{u\}$ which implies $N[v] \cap D = \{u\}$.

Converse part similarly we can prove it.

Theorem 3.5: If G_A is an anti fuzzy graph without isolated vertices and D is a minimal dominating set then V \ D is a dominating set of G_A.

Theorem 3.6: Let G_A be an uninodal anti fuzzy graph with the order p, size q and minimum degree δ then $q - p \leq \gamma_A \leq p - \delta$.

Proof: Let G_A be an uninodal anti fuzzy graph with the order p, size q and D be a dominating set with γ vertices and the fuzzy values γ_A.

$\therefore |V - D|_f = p - \gamma_A$.

Then, there exists atmost $\deg(G_A)/2$ edges incident from V \ D to D.

$\Rightarrow p - \gamma_A \leq q$

$\Rightarrow q - p \geq -\gamma_A$

$\Rightarrow q - p \leq \gamma_A$.

Let u be a vertex with minimum degree δ. u must be adjacent to strongly dominate vertices in G_A. Therefore, V\ N(u) is a dominating set . Hence $\gamma_A \leq p - \delta$.

Remark: The above theorem is fail for v-nodal anti fuzzy graph

Theorem 3.7

i. If G_A is star with n vertices then $\gamma(G_A)=\sigma(u)$ where u is a root node.
ii. If G_A is complete anti fuzzy graph with n vertices then $\gamma(G_A)=\sigma(u_1)$ where $u\in V(G_A)$ and $\sigma(u_1)= \max\{ \sigma(u_i)\in V(G_A) \}$.

Proof: If G_A is a star with n vertices, then there exists n-1 pendent vertices and which are adjacent to a single n^{th} vertex (root) with maximum adjacent fuzzy value. Hence the minimal dominating set contains n^{th} vertex only. Therefore, $\gamma(G_A)=\sigma(u)$.

If G_A is complete anti fuzzy graph with n vertices then every vertices in G_A are adjacent to each other. That is, each vertex (say u_i) is adjacent to n-1 vertices with their maximum fuzzy value between their neighbourhood and its. Therefore, $\gamma(G_A)=\sigma(u_i)$ where $u_i\in V(G_A)$ and u_i attains maximum fuzzy value in G_A.

Theorem 3.8: For an uninodal anti fuzzy graph G_A then $\gamma(G_A) \leq p/2$.

Proof: Let us consider that G_A be an uninodal anti fuzzy graph and D be its minimal dominating set. By the definition of uninodal anti fuzzy graph, every vertex in G_A has unique fuzzy value. To construct D, we choose the vertices which have maximum number of neighbourhood vertices in G_A. By theorem 3.3, there exists a vertex $u\in V\backslash D$ and $v\in D$ and every vertex in D dominates atleast one vertex in V\D. Since there is vertex in D is a strong neighbour to a vertex in V\D. also V\D is a dominating set but which is contradict to minimal of a dominating set. Therefore, $\gamma(G_A) \leq p/2$.

Theorem 3.9: Let G_A be an uninodal anti fuzzy graph with $m \geq 2$ then $\gamma_A \leq p -\Delta(G_A)$.

Proof: Let G_A be an uninodal anti fuzzy graph and $u\in V(G_A)$ which has maximum neighbour in G_A. Therefore, $d(u)=\Delta(G_A)$. u should be dominates its neighbour and $u\in D$. Hence $\gamma_A \leq p -\Delta(G_A)$.

Theorem 3.10: Let G_A be a uninodal anti fuzzy graph with q = 2p then $\gamma(G_A) = p/2$.

Proof: Let D be minimal dominating set of anti fuzzy graph G_A. By the definition of uninodal anti fuzzy graph, every vertex in G_A has unique fuzzy value. To construct D, we choose the vertices which have maximum number of neighbourhood vertices. This yields m/2 vertices in D and the vertices having same fuzzy value. Therefore, $\gamma(G_A) = p/2$.

Theorem 3.11: For a connected anti fuzzy graph G_A with $m \geq 2$ then $\gamma(G_A) \geq \frac{p}{1+\Delta(G_A)}$

Proof: Let D be a minimal dominating set with maximum fuzzy value of anti fuzzy graph G_A then $|D|_f. \Delta(G_A) \geq |V\backslash D|_f$

$\Rightarrow |D|_f . \Delta(G_A) \geq \sum_{u\in V} \sigma(u) - |D_f|$

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$\Rightarrow \gamma\,(G_A).\ \Delta(G_A) \geq p - \gamma$
$\Rightarrow \gamma\,(G_A).\ \Delta(G_A) + \gamma\,(G_A) \geq p$
$\Rightarrow \gamma\,(G_A)\ [\ \Delta(G_A) + 1] \geq p$
$\Rightarrow \gamma\,(G_A)\ \geq \frac{p}{1+\Delta(G_A)}$

Theorem 3.12:

1. If $\gamma(G_A) + \gamma\,(\overline{G_A}) \leq 3p/2$ then G_A is a simple connected anti fuzzy graph
2. If $\gamma(G_A) + \gamma(\overline{G_A}) \leq p + \sum \sigma(u_i)$ then G_A contains atleast one isolated vertices(say u_i).
3. If $\gamma(G_A) + \gamma(\overline{G_A}) = p + \sigma(u)$ then G_A is a complete anti fuzzy graph

Proof:

1. Let us consider that G_A and $\overline{G_A}$ may not have any isolated vertices then $\gamma(G_A) \leq p/2$ and $\gamma(\overline{G_A}) \leq p$.
   $\Rightarrow \gamma(G_A) + \gamma(\overline{G_A}) \leq p/2 + p$
   $\Rightarrow \gamma(G_A) + \gamma(\overline{G_A}) \leq 3p/2.$
2. If G_A has atleast one isolated vertices then $\gamma(G_A) \leq p/2$ and $\gamma(\overline{G_A}) \leq \sum \sigma(u_i)$. Since every vertex in $\overline{G_A}$ is adjacent to u_i with their maximum member hood values.
   Therefore, $\gamma(G_A) + \gamma(\overline{G_A}) \leq p + \sum \sigma(u_i)$
3. If G_A is a complete anti fuzzy graph then in $\overline{G_A}$, all vertices are isolated. Therefore, by theorem 3.7(ii),
   $\gamma(G_A) = \sigma(u)$ and $\gamma(\overline{G_A}) = p$.
   Hence $\gamma(G_A) + \gamma(\overline{G_A}) = p + \sigma(u)$

Note: For a connected anti fuzzy graph G_A,

1. $0.1 \leq \gamma(G_A) \leq p$
2. $\gamma(G_A) \neq p/2$

IV. ALGORITHMS TO FIND DOMINATING SET OF ANTI FUZZY GRAPH

In this section, we define the definition of adjacency matrix and strong adjacency matrix on an anti fuzzy graph. Some properties of adjacency fuzzy matrix is discussed. Using strong adjacency matrix, an algorithms are derived for finding minimal dominating set of anti fuzzy graph G_A.

Definition 4.1[9]: An anti fuzzy graph $G_A=(\sigma,\mu)$ with the fuzzy relation μ to be reflexive and symmetric is completely determined by the adjacency fuzzy matrix and it is denoted by M_μ.

Where, $(M_\mu)_{i,j} = \begin{cases} \mu(v_i, v_j), & \text{for } i \neq j \\ \sigma(v_i), & \text{for } i = j \end{cases}$

If σ^* contains n elements then M_μ is a square matrix of order n.

Definition 4.2: Let G_A be a simple connected anti fuzzy graph. Then the strong adjacency matrix $(M_\mu)'$ is defined as,

$(M_\mu)' = \begin{cases} \mu(v_i, v_j), & \text{for } i \neq j \text{ and } v_i \text{ is a strong neighbourhoos to } v_j \\ \sigma(v_i), & \text{for } i = j \end{cases}$

Properties 4.3:

i. M_μ is a symmetric matrix whose all entries in main diagonal are $\sigma(v_i)$
ii. There is no row exists with 0 fuzzy value entries.
iii. The value exists in main diagonal only. Then the corresponding vertex is isolated vertex in G_A.
iv. In (M_μ), a i^{th} row contains r number of entries then the corresponding vertex is a neighbour to atleast r vertices in G_A.
v. $(M_\mu)'$ is asymmetric matrix whose all entries in main diagonal are $\sigma(v_i)$
vi. Sum of the main diagonal elements gives p.
vii. In $(M_\mu)'$, Sum of each row($R(u_i)$) is equal to sum of each column($C(u_i)$). *i.e.*, $\sum R(u_i) = \sum C(u_i)$
viii. In $(M_\mu)'$, a i^{th} row contains r number of entries then the corresponding vertex is a strong neighbour to r vertices in G_A.

Algorithm 4.4: Let us define the algorithm for finding minimal dominating set in an anti fuzzy graph G_A with m vertices. Two types of algorithm are defined for finding minimal dominating set. They are row maxima method and column maxima method.

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Algorithm 1 - Row Maxima Method

Step-1: For G_A, to construct the $(M_\mu)'$ and p = sum of main diagonal elements.
Step-2: Let D = Φ
Step-3: Find $\sum R(u_i)$ and $\sum C(u_i)$
Step-4: In $(M_\mu)'$, choose the maximum value in $R(u_i)$. If tie occurs then choose the vertex which have maximum $\sigma(u_i)$ in main diagonal otherwise break it arbitrarily.
Step-5: Select the corresponding vertex in the selected row as a dominated vertex. And write the vertex in the set D.
Step-6: Cut the selected row with horizontal line. By using vertical lines, cover all the entries in column wise which corresponding to the selected row. The resulting reduced matrix is called as $(M_\mu)'_1$.
Step-7: If the reduced matrix $(M_\mu)'_1$ has any rows go to step 3. Otherwise STOP.
Step-8: Finally get the dominating set D of G_A and it will be minimal.

Algorithm 2 - Column Maxima Method

Step-1: For G_A, to construct the $(M_\mu)'$ and p=sum of main diagonal elements.
Step-2: Let D = Φ
Step-3: Find $\sum R(u_i)$ and $\sum C(u_i)$
Step-4: In $(M_\mu)'$, choose the maximum value in $C(u_i)$. If tie occurs then choose the vertex which have maximum $\sigma(u_i)$ in main diagonal otherwise break it arbitrarily.
Step-5: Select the corresponding vertex in the selected column as a dominated vertex. And write the vertex in the set D.
Step-6: Cut the selected column with horizontal line. By using vertical lines, cover all the entries in row wise which corresponding to the selected column. The resulting reduced matrix is called as $(M_\mu)'_1$.
Step-7: If the reduced matrix $(M_\mu)'_1$ has any columns go to step 3. Otherwise STOP.
Step-8: Finally get the dominating set D of G_A and it will be minimal.

Example 4.5:

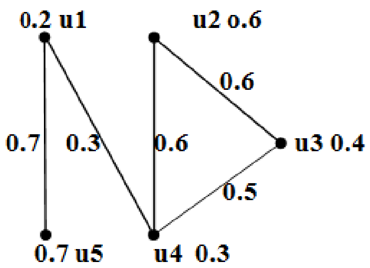

Fig. 2. Anti fuzzy graph G_A

First use the row maxima algorithm to find the minimal dominating set of anti fuzzy graph G_A.

Step-1: for the given anti fuzzy graph G_A, the strong adjacency matrix $(M_\mu)'$ is given as

$$(M_\mu)' = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{c} \begin{array}{ccccc} u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\ \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0.7 \\ 0 & 0.6 & 0.6 & 0.6 & 0 \\ 0 & 0.4 & 0.6 & 0 & 0 \\ 0 & 0.6 & 0 & 0.3 & 0 \\ 0.7 & 0 & 0 & 0 & 0.7 \end{bmatrix} \end{array}$$

Step-2: Let the dominating set, D=Φ

Step-3:

$$(M_\mu)' = \begin{array}{c|ccccc|c} & u_1 & u_2 & u_3 & u_4 & u_5 & \Sigma R(u_i) \\ \hline u_1 & 0.2 & 0 & 0 & 0 & 0.7 & 0.9 \\ u_2 & 0 & 0.6 & 0.6 & 0.6 & 0 & 1.8 \rightarrow \\ u_3 & 0 & 0.4 & 0.6 & 0 & 0 & 1.0 \\ u_4 & 0 & 0.6 & 0 & 0.3 & 0 & 0.9 \\ u_5 & 0.7 & 0 & 0 & 0 & 0.7 & 1.4 \\ \hline \Sigma C(u_i) & 0.9 & 1.6 & 1.2 & 0.9 & 1.4 & \end{array}$$ [6]

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Step-4: In $(M_\mu)'$, $R(u_2)$ has maximum value. Then select u_2. And $D=\{u_2\}$

Step-5: Delete the selected row u_2 and the corresponding column u_2,u_3 and u_4 in $(M_\mu)'$. The resulting reduced matrix say $(M_\mu)'_1$.

Step-6: $(M_\mu)'_1 = \begin{array}{c} \\ u_1 \\ u_3 \\ u_4 \\ u_5 \end{array}\begin{array}{c} \begin{array}{cc} u_1 & u_5 \end{array} \\ \begin{bmatrix} 0.2 & 0.7 \\ 0 & 0 \\ 0 & 0 \\ 0.7 & 0.7 \end{bmatrix}\end{array} = \begin{array}{c} \\ u_1 \\ u_5 \end{array}\begin{array}{c} \begin{array}{cc} u_1 & u_5 \end{array} \\ \begin{bmatrix} 0.2 & 0.7 \\ 0.7 & 0.7 \end{bmatrix}\end{array}$

$M_\mu)'_1$ has 4 rows but u_3 rd and u_4 th row contains only zero. We ignore such rows.

Step-7: $(M_\mu)'_1$ has two rows. Then go to step 3.

Step-3a:

$$(M_\mu)'_1 = \begin{array}{c|cc|c} & u_1 & u_5 & \Sigma R(u_i) \\ u_1 & 0.2 & 0.7 & 0.9 \\ u_5 & 0.7 & 0.7 & 1.4 \rightarrow \\ \Sigma C(u_i) & 0.9 & 1.4 & [2.3] \end{array}$$

Step-4a: In $(M_\mu)'_1$, $R(u_5)$ has maximum value. Then select u_5. And $D=\{ u_2, u_5\}$

Step-5a: Delete the selected row u_5 and the corresponding column u_1 and u_5 in $(M_\mu)'_1$. Now $(M_\mu)'_1$ has no rows. STOP.

The resulting set $D = \{u_2, u_5\}$ is a dominating set of the given anti fuzzy graph G_A and which is also minimal. The domination number is $\gamma=1.3$

Similarly, we can apply column maxima method to find minimal dominating set for a given anti fuzzy graph. We get minimal dominating set as $D =\{u_2, u_5\}$.

Note: In some cases, when applying row maxima algorithm and column maxima algorithm to a given anti fuzzy graph, we get different minimal dominating set.

V. DOMINATION ON ANTI CARTESIAN PRODUCT OF ANTI FUZZY GRAPHS

In this section, initially we apply anti cartesian product on same types of anti fuzzy graphs such as cycle, path and complete anti fuzzy graph. Next, we apply domination number to the resulting anti fuzzy graphs and obtain the bounds on them. In this paper, to derive the theorems consider that the number of vertices in G_{A_1} (say m_1) should be greater than or equal to the number of vertices in G_{A_2} (say m_2).

Theorem 5.1[4]: Let G_A be an anti cartesian product of anti fuzzy graphs G_{A_1} and G_{A_2} where $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$ then $G_A = (\sigma_1\times\sigma_2, \mu_1\times\mu_2)$ is an anti fuzzy graph.

Note: In G_A, the vertices of anti dominating set incident with atleast one effective edge of G_A and every vertex in G_A is a strong neighbourhood to atleast one vertex in G_A.

Theorem 5.1[4]:

G_{A_1}:

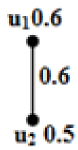

Fig 3. Anti Fuzzy path on 2 vertices

G_{A_2}:

Fig 4. Anti Fuzzy path on 4 vertices

$G_{A_1} \times G_{A_2}$:

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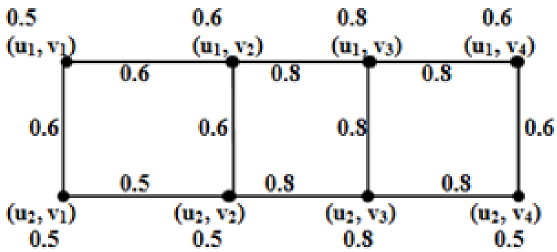

Fig 5. Anti Cartesian Product of Anti Fuzzy path on 2 and 4 vertices

In Figure 5, the dominating sets are $D_1=\{(u_1,v_1), (u_2,v_3), (u_1,v_3)\}$, $\gamma_A = 0.5+0.8+0.8 = 2.1$
$D_2 = \{(u_2,v_1), (u_1,v_2), (u_2,v_4)\}$, $\gamma_A = 0.5+0.6+0.5 = 1.6$
$D_3 = \{(u_2,v_1), (u_1,v_3), (u_2,v_4)\}$, $\gamma_A = 0.5+0.8+0.5 = 1.8$
$D_4 = \{ (u_1,v_1), (u_2,v_3), (u_1,v_4)\}$, $\gamma_A = 0.5+0.8+0.6 = 1.9$
$D_5 = \{(u_1,v_2), (u_1,v_4), (u_2,v_1), (u_2,v_3\}$, $\gamma_A = 0.6 +0.6 +0.5+0.8= 2.5$
Therefore, $\gamma_A = 2.1$

Remark: In the above example, the dominating set D_5 has 4 vertices with maximum cardinality as 2.5 but we cannot consider that the set D_5 as a minimal dominating set. Because the vertices in D_5, dominate only the minimum number of vertices $V \backslash D_5$ in G_A and maximum number vertices dominate itself only.

In D_1, most of the vertices dominate atmost two vertices in $V \backslash D$ and with the minimum number of vertices it has maximum fuzzy cardinality. Hence D_1 is a minimal dominating set.

Theorem 5.3: Let G_{A_1} and G_{A_2} be two regular anti fuzzy graphs and G_A be the cartesian product of anti fuzzy graphs of G_{A_1} and G_{A_2}. Then

i. G_A is an regular anti fuzzy graph.
ii. $\gamma (G_A) \neq \gamma (G_{A_1}) \times \gamma (G_{A_2})$
iii. $\gamma (G_A) = (|V(G_{A_1})| \vee |V(G_{A_2})|) \times (\gamma (G_{A_1}) \vee \gamma (G_{A_2}))$
iv. $\gamma(G_{A_1} \times G_{A_2}) \geq \gamma(G_{A_1}) \times \gamma(G_{A_2})$

Remark: An anti fuzzy graph G_A (= $G_{A_1} \times G_{A_2}$) satisfies vizing's conjecture.

Theorem 5.4: Let G_{A_1} and G_{A_2} be two anti fuzzy graphs and $G_A = G_{A_1} \times G_{A_2}$ be the cartesian product of anti fuzzy graphs of G_{A_1} and G_{A_2}. If H_A be a subgraph of anti fuzzy graphs G_A then
$\gamma(H_A) \geq \gamma(G_A)$

Theorem 5.5: If G_A is an anti cartesian product of anti fuzzy graphs then $\overline{G_A}$ does not have any isolated vertices where $G_A = G_{A_1} \times G_{A_2}$, G_{A_1} and G_{A_2} are simple connected anti fuzzy graphs.

Theorem 5.6: Let G_{A_1} and G_{A_2} be any two strong anti fuzzy graphs. $G_A = (G_{A_1} \times G_{A_2})$ be anti cartesian product of anti fuzzy graphs then

i. $\gamma (G_A) \neq \gamma (\overline{G_A})$

Results 5.7: For the figure 5, the minimal dominating set is obtained by using row maxima algorithm and column maxima algorithm. For applying the algorithms we need to construct a strong adjacency matrix for the resulting anti fuzzy graph.

The strong adjacency matrix $(M_\mu)'$ is given as

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$(M_\mu)' =$

	(u_1,v_1)	(u_1,v_2)	(u_1,v_3)	(u_1,v_4)	(u_2,v_1)	(u_2,v_2)	(u_2,v_3)	(u_2,v_4)	ΣR1(u_i)	ΣR2	ΣR3	ΣR4
(u_1,v_1)	0.5	0.6	0	0	0	0	0	0.6	1.7	1.1	1.1	→
(u_1,v_2)	0	0.6	0.8	0	0	0	0	0	1.4	0	0	0
(u_1,v_3)	0	0.8	0.8	0.8	0	0.8	0	0	3.2	→	→	→
(u_1,v_4)	0	0	0.8	0.6	0	0	0	0	1.4	0	0	0
(u_2,v_1)	0	0	0	0	0.5	0.8	0	0	1.3	0.5	0	0
(u_2,v_2)	0	0	0.8	0	0.8	0.8	0.8	0	3.2	1.6	→	→
(u_2,v_3)	0	0	0	0	0	0.8	0.5	0	1.3	0.5	0	0
(u_2,v_4)	0.6	0	0	0	0	0	0	0.5	1.1	1.1	1.1	0
Σ C1(u_i)	1.1	2.0	3.2	1.4	1.3	3.2	1.3	1.1				
Σ C2(u_i)	1.1	1.2	1.6	0.6	0	↓	0	1.1				
Σ C3(u_i)	1.1	0.6	↓	0	0	↓	0	1.1				
Σ C4(u_i)	↓	0	↓	0	0	↓	0	0				

By using row maxima algorithm,
In Σ R1(u_i), the maximum value is 3.2, but there exists a tie. The vertices (u_1, v_3) and (u_2, v_2) have same sum of strong neighbourhood values. Check the σ(u_1,v_3) and σ(u_2,v_2). Here also exists a tie. So arbitrarily we choose (u_1,v_3) and D={(u_1,v_3)}. Cross their corresponding column (u_1,v_2), (u_1,v_3), (u_1,v_4) and (u_2,v_2).

In the reduced matrix, compute the row sum values (say Σ R2). In Σ R2, The vertex (u_2,v_2) has maximum value and choose that vertex and get D ={(u_1,v_3), (u_2,v_2) }. Cross their corresponding column (u_2, v_1) and (u_2, v_3).

In the reduced matrix, compute the row sum values (say Σ R3). In Σ R3, The vertex (u_1, v_1), (u_2, v_4) has same sum of strong neighbourhood values. Check the σ(u_1,v_1) and σ(u_2,v_4). Here also exists a tie. So arbitrarily we choose the vertex (u_1, v_1) and get D={(u_1,v_3), (u_2,v_2), (u_1,v_1)}. Cross their corresponding column (u_1, v_1) and (u_2, v_4).

In the reduced matrix, compute the row sum values (say Σ R4). In Σ R4, all entries are zero.

Therefore, the minimal dominating set is D ={(u_1,v_3), (u_2,v_2), (u_1,v_1)}, γ_A=2.1

By using column maxima algorithm,
In Σ C1(u_i), the maximum value is 3.2, but there exists a tie. The vertices (u_1, v_3) and (u_2, v_2) have same sum of strong neighbourhood values. Check the σ(u_1, v_3) and σ(u_2, v_2). Here also exists a tie. So arbitrarily we choose (u_2,v_2) and D={(u_2, v_2)}. Cross their corresponding rows (u_1, v_3), (u_2, v_1), (u_2, v_2) and (u_2, v_4).

In the reduced matrix, compute the column sum values (say Σ C2). In Σ C2, The vertex (u_1,v_3) has maximum value and choose that vertex and get D={(u_1,v_3), (u_2,v_2)}. Cross their corresponding rows (u_1,v_2) and (u_1,v_4).

In the reduced matrix, compute the column sum values (say Σ C3). In Σ C3, The vertex (u_1,v_1), (u_2,v_4) has same sum of strong neighbourhood values. Check the σ(u_1,v_1) and σ(u_2,v_4). Here also exists a tie. So arbitrarily we choose the vertex (u_1, v_1) and get D={(u_1, v_3), (u_2, v_2), (u_1, v_1)}. Cross their corresponding rows (u_1, v_1) and (u_2, v_4).

In the reduced matrix, compute the column sum values (say Σ C4). In Σ C4, all entries are zero. Therefore, the minimal dominating set is D={(u_1,v_3), (u_2,v_2), (u_1,v_1)}, γ_A=2.1

Observation 5.8: When we apply an anti cartesian product on $G_{A_1} \times G_{A_2}$. Only in rare case, some edges are not effective edges in G_A. These edges are considered as weakest edge.

Observation 5.9: Anti Cartesian product of two strong anti fuzzy graph is always strong. If μ(u_2,v_2) in G_{A_2} is greater than σ(u_1) in G_{A_1} then the edge μ[(u_1,u_2),(u_1,v_2)] is not effective edge. In such cases, some edges in the resulting graph $G_{A_1} \times G_{A_2}$, are not effective edges. Such type of anti fuzzy graph is called as partially strong anti fuzzy graph.

Note: In the above said example, μ((u_1, v_1), (u_2, v_1)) > σ (u_1, v_1) ∨ σ (u_2, v_1). Hence $G_{A_1} \times G_{A_2}$ is called partially strong anti fuzzy graph. Such edge is called as weakest edge. Even though G_A is an anti fuzzy graph, it is called as partially strong anti fuzzy graph.

Theorem 5.10: Let $G_{A_1} \times G_{A_2}$ be an anti cartesian product of anti fuzzy paths of G_{A_1} and G_{A_2} then the following conditions hold.

i. Removal of all weak edges in $G_{A_1} \times G_{A_2}$ remains connected say $(G_{A_1} \times G_{A_2})'$.
ii. $\gamma(G_{A_1} \times G_{A_2}) \leq \gamma(G_{A_1} \times G_{A_2})'$.

Proof: By theorem [4.1], anti cartesian product of anti fuzzy graph is an anti fuzzy graph. Let e be a weakest edge in $G_{A_1} \times G_{A_2}$. Consider $(G_{A_1} \times G_{A_2})' = (G_{A_1} \times G_{A_2}) \setminus e$. To prove that $(G_{A_1} \times G_{A_2})'$ is connected. Suppose $(G_{A_1} \times G_{A_2})'$ is disconnected anti fuzzy graph. ie., $G_{A_1} \times G_{A_2}$ contains pendent vertices and there is no path between (u_i, v_j) and (u_{i+1}, v_j) except e which is a contradiction to the structure of $G_{A_1} \times G_{A_2}$. Every vertices in $G_{A_1} \times G_{A_2}$ contains atleast two neighbours. Hence $(G_{A_1} \times G_{A_2})'$ is connected. Case (ii) obvious from result (i).

Theorem 5.11: Let G_A be an anti cartesian product of anti fuzzy cycles G_{A_1} and G_{A_2}. If G_{A_1} and G_{A_2} are strong anti fuzzy cycle then all edges in G_A are effective edges, where $\mu(G_{A_2}) \leq \sigma(G_{A_1})$.

Proof: Let G_{A_1} and G_{A_2} be two strong anti fuzzy cycle and consider that $\sigma(u_i)$ in G_{A_1} is greater than $\mu(v_i, w_i)$ in G_{A_2}. Consider that $G_A(=G_{A_1} \times G_{A_2})$ is an anti cartesian product of anti fuzzy cycles G_{A_1} and G_{A_2}. By the definition of anti cartesian product, all the edges should attain maximum fuzzy values to their corresponding vertices with incident edges. ie., $(\mu_1 \times \mu_2)((u_1, u_2), (u_1, v_2)) = \max\{\sigma_1(u_1), \mu_2(u_2, v_2)\}$ for all $u_1 \in V_1$ and $(u_2, v_2) \in E_2$. Therefore, all edges in G_A are effective edges.

Theorem 5.12: If G_{A_1} and G_{A_2} is a complete anti fuzzy graph on m and n vertices and $G_{A_1} \times G_{A_2}$ be an anti fuzzy graph on mn vertices then $\gamma(G_{A_1} \times G_{A_2}) = (m \wedge n) \times \sigma(u_i)$ where $u_i \in V(G_{A_1})$ or $V\, G_{A_2})$ and which has maximum fuzzy value.

Proof: By the definition of complete anti fuzzy graph each pair of vertices is joined by an edge and edges has maximum fuzzy values of their incident vertices. By applying the anti cartesian product on complete anti fuzzy graphs G_{A_1} and G_{A_2} with m and n vertices (say G_A).

To construct a anti dominating set in G_A, choose the vertex which has maximum fuzzy value in G_{A_1} or G_{A_2} (say $u_i \in G_{A_2}$). Consider that m>n, $G_A(=G_{A_1} \times G_{A_2})$ has n component and each component contains m vertices. Let $v_j \in G_{A_2}$, each component in G_A must have the vertex (u_i, v_j) for j=1 to n, with maximum fuzzy value. This vertex is adjacent to all vertices within that component. Therefore, in dominating set D contains (u_i, v_j) for i= 1 to n. Hence |D|=n and all vertices in D has maximum fuzzy values of u_i. Therefore, $\gamma(G_A) = n \times \sigma(u_i)$.

Hence $\gamma(G_{A_1} \times G_{A_2}) = (m \wedge n) \times \sigma(u_i)$.

VI. CONCLUSION

In this paper, domination number is defined on anti fuzzy graphs. The definition of domination number is applied on various types of anti fuzzy graph and obtained the bounds on them. The relation between them is examined. Row maxima and column maxima algorithms are characterized to find the minimal dominating set for given anti fuzzy graph. Some properties of fuzzy matrix is illustrated. The domination number concept is applied on anti cartesian product of anti fuzzy graphs such as anti fuzzy path, anti fuzzy cycle and complete anti fuzzy graph and obtained the bounds on them.

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