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SHORTEST DISTANCE FROM SPANNING TREE

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ABSTRACT

In this paper, verify minimum duration using spanning tree. In duration of Program Evaluate and Review Technique Method (PERT) problem compare with Prim's algorithm and Kruskal's algorithm.

Keywords: *Network, PERT, Shortest spanning tree, Connected Graph.*

I. INTRODUCTION

Operations Research (OR) started just before World War II in Britain with the establishment of teams of scientists to study the strategic and tactical problems involved in military operations. It was in 1917, when A.K.ERLANG, a Danish mathematician, published his work on the problem of congestion of telephone traffic.Leonhard Euler's paper on "Seven Bridges of Konigsberg" published in 1736. It took 200 years before the first book on graph theory was written. Routes between the cities can be represented using graphs.PERT was developed primarily to simplify the planning and scheduling of large and complex projects. It was developed for the U.S.NAVY special projects office in 1957 to support the U.S.Navy's Polaris nuclear submarine project.

II. PRELIMINARIES

2.1 Network: [3] A network is a graphic representation of a project's operations and is composed of activities and events that must be completed to reach the end objective of a project, showing the planning sequence of their accomplishments, their dependence and inter-relationships.

2.2 Activity: [3] An activity is a task, or item of work to be done, that consumes time, effort, money or other resources. It lies between two events, called the 'preceding' and 'succeeding' ones. An activity is represented by an arrow with its head indicating the sequence in which the events are to occur.

2.3 Event: [3] An event represents the start (beginning) or completion (end) off some activity and as such it consumes no time. It has no time duration and does not consume any resources. An event is nothing but a node and is generally represented on the network by a circle, rectangle, hexagon or some other geometric shape.

2.5 Pert: [3] Program Evaluation and Review Technique.

2.6 Graph: [1] A Graph G consists of a pair {V (G), X (G)} where V (G) is a non-empty finite set whose elements are called points (or) vertices. X (G) is a set of unordered pairs of distinct element of V (G). The elements of X (G) are called lines (or) edges of the graph G.

2.8 Spanning Tree: [4] A tree T is said to be a spanning tree of a connected graph G if T is a sub graphof G and T contains all vertices of G.

R.C. PRIM'S ALGORITHM (minimum spanning tree algorithm): [5]

Step-1: Define a weight matrix $A=[a_{ij}]$ of size $p\times p$ for a given graph G of order p, whose rows and columns corresponding to the vertices of G, by taking ij^{th} entry a_{ij} as;

$a_{ij}= w(v_i, v_j)$, if $v_iv_j\in E(G)$ and $i\neq j$

$= \infty$, if $v_iv_j\notin E(G)$ and $i\neq j$ =empty if $i = j$.

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National Conference March 1st - 2018, On "Recent Advances in Pure and Applied Mathematics", Organized by Department of Mathematics, Arul Anandar College (Autonomous), Madurai. Tamilnadu, India.

Step-2: Choose ijth entry of the matrix A such that a_{ij}=min {$a_{ij:}$ a_{ij}∈A}. Let T be a tree with vertex set V'= {v_i,v_j} and edge set E'= {v_iv_j}.

Step-3: Obtain a matrix A' by A by discarding the columns corresponding to the vertices that are already chosen.

Step-4: If A' contains no columns, the STOP, the last tree is the required minimum spanning tree. Otherwise, choose the ijth entry of the matrix A' such that a_{ij}=min {a_{ij} : a_{ij}∈ A' and v_i∈V'}. Let T' be a tree with vertex set V'∪{v_i, v_j} and edge set E'∪{v_iv_j}. Return to step 3.

J.B. KRUSKAL'S ALGORITHM (minimum spanning tree algorithm): [5]

Step-1: Select an edge *e* with minimum weight in the graph G, label the end points of *e* by v_1 and v_2. Let T' be the tree with edge set E'= {*e*} and vertex set V'= {v_1, v_2}.

Step-2: Check whether V'=V (G), the vertex set of G. If V'=V (G), stop, T' is the required spanning tree. Otherwise go to step 3.

Step-3: Then there exists a vertex v_i∈V' and a neighbouring vertex v_k of v_i not in V'. Among all possible such vertices v_k not in V' choose a vertex v_k and a vertex v_i∈V' such that w(v_k, v_i) is minimum. Include the vertex v_k to the tree T' along with edge v_kv_i of minimum weight and call the tree so obtained as T'. Replace V' by V'∪ {v_k} and E' by E'∪ {v_kv_i}. Return to step 2.

III. NUMERICAL APPLICATION

Problem: [3] Draw the PERT Network:

Activity	A	B	C	D	E	F	G	H
Immediate Predecessor	-	-	A	B	D,A	B	C,E,F	G
Optimistic	4	8	4	1	2	4	10	18
Most likely	5	12	5	3	2	5	14	20
Pessimistic	6	16	12	5	2	6	18	34

Solution:

Table-I

Activity	t_0	t_m	t_p	$t_e = \dfrac{t_0 + 4t_m + t_p}{6}$
A	4	5	6	5
B	8	12	16	12
C	4	5	12	6
D	1	3	5	3
E	2	2	2	2
F	4	5	6	5
G	10	14	18	14
H	18	20	34	22
I	0	0	0	0
J	0	0	0	0

NETWORK:

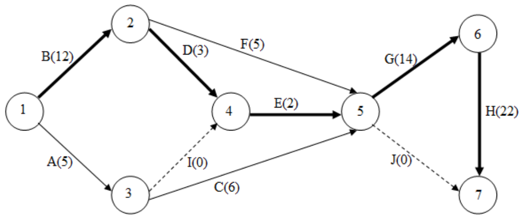

Fig.-1

Critical Path: 1-2-4-5-6-7 (or) B-D-E-G-H; Project Duration: 53 days

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GRAPH THEORY CALCULATIONS:

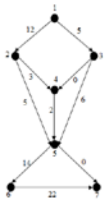


Fig.-2

Graph

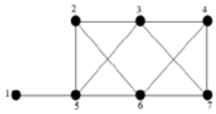
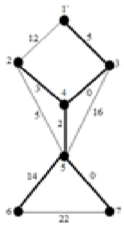


Fig.-3

mplement Graph

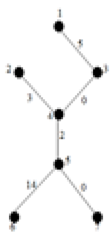
SHORTEST SPANNING TREE IN A WEIGHTED GRAPH:



$$\begin{bmatrix} - & 12 & 5 & \infty & \infty & \infty & \infty \\ 12 & - & \infty & 3 & 5 & \infty & \infty \\ 5 & \infty & - & 0 & 6 & \infty & \infty \\ \infty & 3 & 0 & - & 2 & \infty & \infty \\ \infty & 5 & 6 & 2 & - & 14 & 0 \\ \infty & \infty & \infty & \infty & 14 & - & 22 \\ \infty & \infty & \infty & \infty & 0 & 22 & - \end{bmatrix}$$

Fig.4

R.C.PRIM'S ALGORITHM:



$$\begin{bmatrix} - & 12 & 5 & \infty & \infty & \infty & \infty \\ 12 & - & \infty & 3 & 5 & \infty & \infty \\ 5 & \infty & - & 0 & 6 & \infty & \infty \\ \infty & 3 & 0 & - & 2 & \infty & \infty \\ \infty & 5 & 6 & 2 & - & 14 & 0 \\ \infty & \infty & \infty & \infty & 14 & - & 22 \\ \infty & \infty & \infty & \infty & 0 & 22 & - \end{bmatrix}$$

Fig.5

The weighted graph of the tree W (T) = 24days

J.B.KRUSKAL'S ALGORITHM:

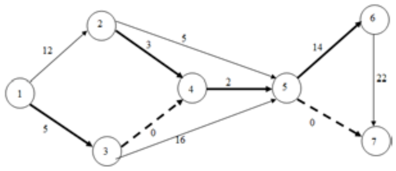


Fig.-6

The Weighted graph of the tree W (T) = Sum of Weights on its edges
= 24 days

Table-II

Critical Path	53 days
Connected Graph	Exists
Directed Graph (or) Digraph	Exists
Spanning Tree (2^n); where n=7	128(Exists)
Fundamental Circuit	Exists
The Weighted graph of the tree W (T)	24 days

RESULT

Shortest Spanning Tree in a Weighted Graph < Critical Path (24 < 53).

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V. CONCLUSION

To identify which algorithm works efficiently we have taken some numerical examples. All these results confirm that the value obtained by shortest spanning tree in a weighted graph will be less than critical path. Hence we can conclude that the shortest spanning tree in a weighted graph will be more efficient than the network problems.

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