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INTUITIONISTIC ANTI - FUZZY HX BI-IDEAL OF A HX RING

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ABSTRACT

In this paper, we define the notion of intuitionistic anti- fuzzy HX bi-ideal of a HX ring and some of their related properties are investigated. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti- fuzzy HX bi-ideal of a HX ring and discuss some of its properties. We discuss the concept of an image, pre-image of an intuitionistic anti-fuzzy HX bi-ideal of a HX ring and homomorphic, anti homomorphic properties of an intuitionistic anti- fuzzy HX bi-ideal of a HX ring are discussed.

Keywords: *intuitionistic fuzzy set, fuzzy HX ring, intuitionistic ant- fuzzy HX bi-ideal, homomorphism and anti homomorphism of an intuitionistic anti- fuzzy HX bi-ideal, anti-image and pre-image of an intuitionistic anti- fuzzy HX bi-ideal.*

INTRODUCTION

In 1965, Zadeh [13] introduced the concept of fuzzy subset μ of a set X as a function from X into the closed unit interval [0, 1] and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic, set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [11] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [5] introduced the concept of HX group. In 1982 Wang-jin Liu [7] introduced the concept of fuzzy ring and fuzzy ideal. With the successful upgrade of algebraic structure of group many researchers considered the algebraic structure of some other algebraic systems in which ring was considered as first. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2, 3] gave the structures of HX ring on a class of ring. R.Muthuraj *et.al* [10]. Introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of an intuitionistic anti fuzzy HX bi-ideal of a HX ring and investigate some related properties. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti-fuzzy HX bi-ideal and discuss some of its properties. Also we introduce the anti image and pre-image of an intuitionistic anti-fuzzy HX bi-ideal and discuss some of its properties.

PRELIMINARY

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R, +, $\cdot$) is a Ring, e is the additive identity element of R and xy, we mean x.y

2.1 Definition [6]: Let R be a ring. In 2^R - {ϕ}, a non-empty set $\vartheta \subset 2^R$ - {ϕ} with two binary operation '+' and '.' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by

i. A + B = {a + b / a $\in$ A and b $\in$ B}, which its null element is denoted by Q, and the negative element of A is denoted by – A.
ii. AB = {ab / a $\in$ A and b $\in$ B},
iii. A (B + C) = AB + AC and (B + C) A = BA + CA.

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2.2 Definition: Let R be a ring. Let H = $\{\langle x, \mu(x), \eta(x)\rangle / x\in R\}$ be an intuitionistic fuzzy set defined on a ring R, where $\mu : R\to[0,1]$, $\eta : R\to[0,1]$ such that $0 \le \mu(x) + \eta(x) \le 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. An intuitionistic fuzzy subset $\lambda^H = \{\langle A, \lambda^\mu(A), \lambda^\eta(A)\rangle / A\in\Re$ and $0\le \lambda^\mu(A) + \lambda^\eta(A) \le 1\}$ of $\Re$ is said to be an intuitionistic fuzzy HX bi-ideal or intuitionistic fuzzy bi-ideal induced by H of a HX ring $\Re$ if the following conditions are satisfied. For all A, B, C$\in\Re$,

i. $\lambda^\mu(A-B) \ge \min\{\lambda^\mu(A), \lambda^\mu(B)\}$
ii. $\lambda^\mu(AB) \ge \min\{\lambda^\mu(A), \lambda^\mu(B)\}$
iii. $\lambda^\mu(ABC) \ge \min\{\lambda^\mu(A), \lambda^\mu(C)\}$
iv. $\lambda^\eta(A-B) \le \max\{\lambda^\eta(A), \lambda^\eta(B)\}$
v. $\lambda^\eta(AB) \le \max\{\lambda^\eta(A), \lambda^\eta(B)\}$
vi. $\lambda^\eta(ABC) \le \max\{\lambda^\eta(A), \lambda^\eta(C)\}$

where $\lambda^\mu(A) = \max\{\mu(x) / \text{for all } x\in A \subseteq R\}$ and $\lambda^\eta(A) = \min\{\eta(x) / \text{for all } x\in A \subseteq R\}$.

3.1 Intuitionistic anti-fuzzy HX bi-ideal of a HX ring

In this section we define the concept of an intuitionistic anti-fuzzy HX bi-ideal of a HX ring and discuss some related results.

3.1.1 Definition: Let R be a ring. Let H = $\{\langle x, \mu(x), \eta(x)\rangle / x\in R\}$ be an intuitionistic fuzzy set defined on a ring R, where $\mu : R\to[0,1]$, $\eta : R\to[0,1]$ such that $0 \le \mu(x) + \eta(x) \le 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. An intuitionistic fuzzy subset $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle / A\in\Re$ and $0 \le \lambda_\mu(A) + \lambda_\eta(A) \le 1\}$ of $\Re$ is called an intuitionistic anti-fuzzy HX bi-ideal of $\Re$ or an intuitionistic anti-fuzzy bi-ideal induced by H if the following conditions are satisfied. For all A, B $\in\Re$,

i. $\lambda_\mu(A-B) \le \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,
ii. $\lambda_\mu(AB) \le \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,
iii. $\lambda_\mu(ABC) \le \max\{\lambda_\mu(A), \lambda_\mu(C)\}$,
iv. $\lambda_\eta(A-B) \ge \min\{\lambda_\eta(A), \lambda_\eta(B)\}$,
v. $\lambda_\eta(AB) \ge \min\{\lambda_\eta(A), \lambda_\eta(B)$,
vi. $\lambda_\eta(ABC) \ge \min\{\lambda_\eta(A), \lambda_\eta(C)$.

where $\lambda_\mu(A) = \min\{\mu(x) / \text{for all } x\in A \subseteq R\}$ and $\lambda_\eta(A) = \max\{\eta(x) / \text{for all } x\in A \subseteq R\}$.

3.1.2 Theorem: If H is an intuitionistic anti-fuzzy bi-ideal of a ring R then the intuitionistic fuzzy subset λ_H is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$.

Proof: Let H be an intuitionistic anti-fuzzy bi-ideal of R.

i.
$$\begin{aligned}
\max\{\lambda_\mu(A), \lambda_\mu(B)\} &= \max\{\min\{\mu(x) /\text{for all } x\in A \subseteq R\}, \min\{\mu(y)/ \text{for all } y\in B \subseteq R\}\}\\
&= \max\{\mu(x_0), \mu(y_0)\}\\
&\ge \mu(x_0 - y_0)\\
&\ge \min\{\mu(x-y) / \text{for all } x-y \in A-B \subseteq R\}\\
&\ge \lambda_\mu(A-B)\\
\lambda_\mu(A-B) &\le \max\{\lambda_\mu(A), \lambda_\mu(B)\}.
\end{aligned}$$

ii.
$$\begin{aligned}
\max\{\lambda_\mu(A), \lambda_\mu(B)\} &= \max\{\min\{\mu(x) /\text{for all } x\in A\subseteq R\}, \min\{\mu(y) / \text{for all } y\in B \subseteq R\}\}\\
&= \max\{\mu(x_0), \mu(y_0)\}\\
&\ge \mu(x_0 y_0)\\
&\ge \min\{\mu(xy) / \text{for all } xy \in AB \subseteq R\}\\
&\ge \lambda_\mu(AB)\\
\lambda_\mu(AB) &\le \max\{\lambda_\mu(A), \lambda_\mu(B)\}.
\end{aligned}$$

iii.
$$\begin{aligned}
\max\{\lambda_\mu(A), \lambda_\mu(C)\} &= \max\{\min\{\mu(x) /\text{for all } x\in A\subseteq R\}, \min\{\mu(z) / \text{for all } z\in C \subseteq R\}\}\\
&= \max\{\mu(x_0), \mu(z_0)\}\\
&\ge \mu(x_0 y_0 z_0)\\
&\ge \min\{\mu(xyz) / \text{for all } xyz \in ABC \subseteq R\}\\
&\ge \lambda_\mu(ABC)\\
\lambda_\mu(ABC) &\le \max\{\lambda_\mu(A), \lambda_\mu(C)\}.
\end{aligned}$$

iv.
$$\begin{aligned}
\min\{\lambda_\eta(A), \lambda_\eta(B)\} &= \min\{\max\{\eta(x)/\text{for all } x\in A\subseteq R\}, \max\{\eta(y)/\text{for all } y\in B\subseteq R\}\}
\end{aligned}$$

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$= \min\{\eta(x_0), \eta(y_0)\}$

$\leq \eta(x_0 - y_0)$

$\leq \max\{\eta(x-y) \,/\, \text{for all } x-y \in A-B \subseteq R\}$

$\leq \lambda_\eta(A - B)$

$\lambda_\eta(A-B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$.

v. $\lambda_\eta(AB) = \min\{\eta(xy) \,/\, \text{for all } x \in A \subseteq R \text{ and } y \in B \subseteq R\}$

$= \eta(x_0 y_0)$, for $x_0 \in A$ and $y_0 \in B$.

$\geq \min\{\eta(x_0), \eta(y_0)\}$

$= \min\{\{\eta(x) \,/\, \text{for all } x \in A \subseteq R\}, \{\eta(y) \,/\, \text{for all } x \in B \subseteq R\}\}$

$\geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$.

$\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$.

vi. $\lambda_\eta(ABC) = \min\{\eta(xyz) \,/\, \text{for all } x \in A \subseteq R,\ y \in B \subseteq R \text{ and } z \in C \subseteq R\}$

$= \eta(x_0 y_0 z_0)$, for $x_0 \in A$, $y_0 \in B$ and $z_0 \in C$.

$\geq \min\{\eta(x_0), \eta(z_0)\}$

$= \min\{\{\eta(x) \,/\, \text{for all } x \in A \subseteq R\}, \{\eta(z) \,/\, \text{for all } z \in C \subseteq R\}\}$

$\geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}$.

$\lambda_\eta(ABC) \geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}$.

Hence, λ_H is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.

3.1.3 Remark

i. If H is not an intuitionistic anti-fuzzy bi-ideal of R then the intuitionistic fuzzy subset λ_H of $\mathfrak{R}$ is an intuitionistic anti-fuzzy HX bi-ideal of $\mathfrak{R}$, provided $|X| \geq 2$ for all $X \in \mathfrak{R}$.

ii. If H is an intuitionistic fuzzy subset of a ring R and λ_H be an intuitionistic anti-fuzzy HX bi-ideal on $\mathfrak{R}$, such that $\lambda_\mu(A) = \max\{\mu(x) \,/\, \text{for all } x \in A \subseteq R\}$ and $\lambda_\eta(A) = \min\{\eta(x) \,/\, \text{for all } x \in A \subseteq R\}$, then H may or may not be an intuitionistic anti-fuzzy bi-ideal of R.

3.1.4 Theorem: Let H be an intuitionistic fuzzy subset on R. Let $\mathfrak{R} \subset 2^R - \{\phi\}$ be a HX ring. If λ_H is an intuitionistic anti-fuzzy HX right ideal of a HX ring $\mathfrak{R}$ then λ_H is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.

Proof: Let λ_H be an intuitionistic anti-fuzzy HX right ideal of a HX ring $\mathfrak{R}$.
Then for all A, B $\in \mathfrak{R}$.

i. $\lambda_\mu(A - B) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,

ii. $\lambda_\mu(AB) \leq \lambda_\mu(A)$

iii. $\lambda_\eta(A - B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$

iv. $\lambda_\eta(AB) \geq \lambda_\eta(A)$.

Therefore , $\lambda_\mu(AB) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$

$\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$

Let A , B , C $\in \mathfrak{R}$,

$\lambda_\mu(ABC) = \lambda_\mu(A(BC))$

$\leq \lambda_\mu(A)$,

$\leq \max\{\lambda_\mu(A), \lambda_\mu(C)\}$.

$\lambda_\mu(ABC) \leq \max\{\lambda_\mu(A), \lambda_\mu(C)\}$.

$\lambda_\eta(ABC) = \lambda_\eta(A(BC))$

$\geq \lambda_\eta(A)$

$\geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}$

$\lambda_\eta(ABC) \geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}$.

Hence, λ_H is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.

3.1.5 Theorem: Let H be an intuitionistic fuzzy subset on R. Let $\mathfrak{R} \subset 2^R - \{\phi\}$ be a HX ring. If λ_H is an intuitionistic anti-fuzzy HX left ideal of a HX ring $\mathfrak{R}$ then λ_H is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.

Proof: It is clear.

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3.1.6 Remark: Every intuitionistic anti-fuzzy HX (right or left) ideal of a HX ring $\Re$ is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$.

3.1.7 Theorem: Let G and H be any two intuitionistic fuzzy sets on R. Let γ_G and λ_H be any two intuitionistic anti-fuzzy HX bi-ideals of a HX ring $\Re$ then their intersection, $\gamma_G \cap \lambda_H$ is also an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$.

Proof: Let G = $\{\langle x, \alpha(x), \beta(x)\rangle / x\in R\}$ and H = $\{\langle x, \mu(x), \eta(x)\rangle / x\in R\}$ be any two intuitionistic fuzzy sets defined on a ring R.
Then, $\gamma_G = \{\langle A, \gamma_\alpha(A), \gamma_\beta(A)\rangle / A\in\Re\}$ and $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle / A\in\Re\}$ be any two intuitionistic anti-fuzzy HX bi-ideals of a HX ring $\Re$.
$\gamma_G \cap \lambda_H = \{\langle A, (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\beta \cap \lambda_\eta)(A)\rangle / A\in\Re\}$

Let A,B,C $\in \Re$.

i.
$$\begin{aligned}
(\gamma_\alpha \cap \lambda_\mu)(A-B) &= \min\{\gamma_\alpha(A-B), \lambda_\mu(A-B)\}\\
&\le \min\{\max\{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max\{\lambda_\mu(A), \lambda_\mu(B)\}\}\\
&= \max\{\min\{\gamma_\alpha(A), \lambda_\mu(A)\}, \min\{\gamma_\alpha(B), \lambda_\mu(B)\}\}\\
&= \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}\\
(\gamma_\alpha \cap \lambda_\mu)(A-B) &\le \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}.
\end{aligned}$$

ii.
$$\begin{aligned}
(\gamma_\alpha \cap \lambda_\mu)(AB) &= \min\{\gamma_\alpha(AB), \lambda_\mu(AB)\}\\
&\le \min\{\max\{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max\{\lambda_\mu(A), \lambda_\mu(B)\}\}\\
&= \max\{\min\{\gamma_\alpha(A), \lambda_\mu(A)\}, \min\{\gamma_\alpha(B), \lambda_\mu(B)\}\}\\
&= \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}\\
(\gamma_\alpha \cap \lambda_\mu)(AB) &\le \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}.
\end{aligned}$$

iii.
$$\begin{aligned}
(\gamma_\alpha \cap \lambda_\mu)(ABC) &= \min\{\gamma_\alpha(ABC), \lambda_\mu(ABC)\}\\
&\le \min\{\max\{\gamma_\alpha(A), \gamma_\alpha(C)\}, \max\{\lambda_\mu(A), \lambda_\mu(C)\}\}\\
&= \max\{\min\{\gamma_\alpha(A), \lambda_\mu(A)\}, \min\{\gamma_\alpha(C), \lambda_\mu(C)\}\}\\
&= \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(C)\}\\
(\gamma_\alpha \cap \lambda_\mu)(ABC) &\le \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(C)\}.
\end{aligned}$$

iv.
$$\begin{aligned}
(\gamma_\beta \cup \lambda_\eta)(A-B) &= \max\{\gamma_\beta(A-B), \lambda_\eta(A-B)\}\\
&\ge \max\{\min\{\gamma_\beta(A), \gamma_\beta(B)\}, \min\{\lambda_\eta(A), \lambda_\eta(B)\}\}\\
&= \min\{\max\{\gamma_\beta(A), \lambda_\eta(A)\}, \max\{\gamma_\beta(B), \lambda_\eta(B)\}\}\\
&= \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}\\
(\gamma_\beta \cup \lambda_\mu)(A-B) &\ge \min\{(\gamma_\beta \cup \gamma_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}.
\end{aligned}$$

v.
$$\begin{aligned}
(\gamma_\beta \cup \lambda_\eta)(AB) &= \max\{\gamma_\beta(AB), \lambda_\eta(AB)\}\\
&\ge \max\{\min\{\gamma_\beta(A), \gamma_\beta(B)\}, \min\{\lambda_\eta(A), \lambda_\eta(B)\}\}\\
&= \min\{\max\{\gamma_\beta(A), \lambda_\eta(A)\}, \max\{\gamma_\beta(B), \lambda_\eta(B)\}\}\\
&= \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}\\
(\gamma_\beta \cup \lambda_\mu)(AB) &\ge \min\{(\gamma_\beta \cup \gamma_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}.
\end{aligned}$$

vi.
$$\begin{aligned}
(\gamma_\beta \cup \lambda_\eta)(ABC) &= \max\{\gamma_\beta(ABC), \lambda_\eta(ABC)\}\\
&\ge \max\{\min\{\gamma_\beta(A), \gamma_\beta(C)\}, \min\{\lambda_\eta(A), \lambda_\eta(C)\}\}\\
&= \min\{\max\{\gamma_\beta(A), \lambda_\eta(A)\}, \max\{\gamma_\beta(C), \lambda_\eta(C)\}\}\\
&= \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(C)\}\\
(\gamma_\beta \cup \lambda_\mu)(ABC) &\ge \min\{(\gamma_\beta \cup \gamma_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(C)\}.
\end{aligned}$$

Hence, $\gamma_G \cap \lambda_H$ is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$.

3.1.8 Theorem: Let G and H be any two intuitionistic fuzzy sets on R. Let γ_G be an intuitionistic anti-fuzzy HX (right or left) ideal and λ_H be an intuitionistic anti-fuzzy HX (right or left) ideal of a HX ring $\Re$ then their intersection, $\gamma_G \cap \lambda_H$ is also an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$.

Proof: It is clear.

3.1.9 Theorem: Let G and H be any two intuitionistic fuzzy sets on R. Let γ_G and λ_H be any two intuitionistic anti-fuzzy HX bi-ideals of a HX ring $\Re$ then their union, $\gamma_G \cup \lambda_H$ is also an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$.

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Proof: Let G = { $\langle x, \alpha(x), \beta(x)\rangle$ / $x\in R$} and H = {$\langle x, \mu(x), \eta(x)\rangle$ / $x\in R$} be any two intuitionistic fuzzy sets defined on a ring R.
Then, $\gamma_G = \{\langle A, \gamma_\alpha(A), \gamma_\beta(A)\rangle / A\in\mathfrak{R}\}$ and $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle / A\in\mathfrak{R}\}$ be any two intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.Then,
$\gamma_G\cup\lambda_H = \{\langle A, (\gamma_\alpha\cup\lambda_\mu)(A), (\gamma_\beta\cap\lambda_\eta)(A)\rangle / A\in\mathfrak{R}\}$.

Let A,B $\in\mathfrak{R}$,

i. $(\gamma_\alpha\cup\lambda_\mu)(A-B) = \min\{\gamma_\alpha(A-B), \lambda_\mu(A-B)\}$
$\geq \min\{\max\{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max\{\lambda_\mu(A), \lambda_\mu(B)\}\}$
$= \max\{\min\{\gamma_\alpha(A), \lambda_\mu(A)\}, \min\{\gamma_\alpha(B), \lambda_\mu(B)\}\}$
$= \max\{(\gamma_\alpha\cup\lambda_\mu)(A), (\gamma_\alpha\cup\lambda_\mu)(B)\}$
$(\gamma_\alpha\cup\lambda_\mu)(A-B) \geq \max\{(\gamma_\alpha\cup\lambda_\mu)(A), (\gamma_\alpha\cup\lambda_\mu)(B)\}$.

ii. $(\gamma_\alpha\cup\lambda_\mu)(AB) = \min\{\gamma_\alpha(AB), \lambda_\mu(AB)\}$
$\geq \min\{\max\{\gamma_\alpha(A), \gamma_\alpha(B)\}, \max\{\lambda_\mu(A), \lambda_\mu(B)\}\}$
$= \max\{\min\{\gamma_\alpha(A), \lambda_\mu(A)\}, \min\{\gamma_\alpha(B), \lambda_\mu(B)\}\}$
$= \max\{(\gamma_\alpha\cup\lambda_\mu)(A), (\gamma_\alpha\cup\lambda_\mu)(B)\}$
$(\gamma_\alpha\cup\lambda_\mu)(AB) \geq \max\{(\gamma_\alpha\cup\lambda_\mu)(A), (\gamma_\alpha\cup\lambda_\mu)(B)\}$.

iii. $(\gamma_\alpha\cup\lambda_\mu)(ABC) = \min\{\gamma_\alpha(ABC), \lambda_\mu(ABC)\}$
$\geq \min\{\max\{\gamma_\alpha(A), \gamma_\alpha(C)\}, \max\{\lambda_\mu(A), \lambda_\mu(C)\}\}$
$= \max\{\min\{\gamma_\alpha(A), \lambda_\mu(A)\}, \min\{\gamma_\alpha(C), \lambda_\mu(C)\}\}$
$= \max\{(\gamma_\alpha\cup\lambda_\mu)(A), (\gamma_\alpha\cup\lambda_\mu)(C)\}$
$(\gamma_\alpha\cup\lambda_\mu)(ABC) \geq \max\{(\gamma_\alpha\cup\lambda_\mu)(A), (\gamma_\alpha\cup\lambda_\mu)(C)\}$.

iv. $(\gamma_\beta\cap\lambda_\eta)(A-B) = \max\{\gamma_\beta(A-B), \lambda_\eta(A-B)\}$
$\leq \max\{\min\{\gamma_\beta(A), \gamma_\beta(B)\}, \min\{\lambda_\eta(A), \lambda_\eta(B)\}\}$
$= \min\{\max\{\gamma_\beta(A), \lambda_\eta(A)\}, \max\{\gamma_\beta(B), \lambda_\eta(B)\}\}$
$= \min\{(\gamma_\beta\cap\lambda_\eta)(A), (\gamma_\beta\cap\lambda_\eta)(B)\}$
$(\gamma_\beta\cap\lambda_\eta)(A-B) \leq \min\{(\gamma_\beta\cap\lambda_\eta)(A), (\gamma_\beta\cap\lambda_\eta)(B)\}$.

v. $(\gamma_\beta\cap\lambda_\eta)(AB) = \max\{\gamma_\beta(AB), \lambda_\eta(AB)\}$
$\leq \max\{\min\{\gamma_\beta(A), \gamma_\beta(B)\}, \min\{\lambda_\eta(A), \lambda_\eta(B)\}\}$
$= \min\{\max\{\gamma_\beta(A), \lambda_\eta(A)\}, \max\{\gamma_\beta(B), \lambda_\eta(B)\}\}$
$= \min\{(\gamma_\beta\cap\lambda_\eta)(A), (\gamma_\beta\cap\lambda_\eta)(B)\}$
$(\gamma_\beta\cap\lambda_\eta)(AB) \leq \min\{(\gamma_\beta\cap\lambda_\eta)(A), (\gamma_\beta\cap\lambda_\eta)(B)\}$.

vi. $(\gamma_\beta\cap\lambda_\eta)(ABC) = \max\{\gamma_\beta(ABC), \lambda_\eta(ABC)\}$
$\leq \max\{\min\{\gamma_\beta(A), \gamma_\beta(C)\}, \min\{\lambda_\eta(A), \lambda_\eta(C)\}\}$
$= \min\{\max\{\gamma_\beta(A), \lambda_\eta(A)\}, \max\{\gamma_\beta(C), \lambda_\eta(C)\}\}$
$= \min\{(\gamma_\beta\cap\lambda_\eta)(A), (\gamma_\beta\cap\lambda_\eta)(C)\}$
$(\gamma_\beta\cap\lambda_\eta)(AB) \leq \min\{(\gamma_\beta\cap\lambda_\eta)(A), (\gamma_\beta\cap\lambda_\eta)(C)\}$.

Hence, $\gamma_G\cup\lambda_H$ is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.

3.1.10 Theorem: Let μ be an intuitionistic fuzzy set defined on R. Let λ^μ is an intuitionistic fuzzy HX bi-ideal of $\mathfrak{R}$ if and only if $(\lambda^\mu)^c$ is an intuitionistic anti-fuzzy HX bi-ideal of $\mathfrak{R}$.

Proof: Let λ^μ be an intuitionistic fuzzy HX bi-ideal of $\mathfrak{R}$.

Let A, B, C $\in\mathfrak{R}$

i. $\lambda^\mu(A-B) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$
$\Leftrightarrow 1-\lambda^\mu(A-B) \leq 1-\min\{\lambda^\mu(A), \lambda^\mu(B)\}$
$\Leftrightarrow (\lambda^\mu)^c(A-B) \leq \max\{(1-\lambda^\mu(A)),(1-\lambda^\mu(B))\}$
$\Leftrightarrow (\lambda^\mu)^c(A-B) \leq \max\{(\lambda^\mu)^c(A), (\lambda^\mu)^c(B)\}$.

ii. $\lambda^\mu(AB) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$
$\Leftrightarrow 1-\lambda^\mu(AB) \leq 1-\min\{\lambda^\mu(A), \lambda^\mu(B)\}$
$\Leftrightarrow (\lambda^\mu)^c(AB) \leq \max\{(1-\lambda^\mu(A)),(1-\lambda^\mu(B))\}$ $\Leftrightarrow$
$(\lambda^\mu)^c(AB) \leq \max\{(\lambda^\mu)^c(A), (\lambda^\mu)^c(B)\}$.

iii. $\lambda^\mu(ABC) \geq \min\{\lambda^\mu(A), \lambda^\mu(C)\}$
$\Leftrightarrow 1-\lambda^\mu(ABC) \leq 1-\min\{\lambda^\mu(A), \lambda^\mu(C)\}$

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$\Leftrightarrow$ $(\lambda^\mu)^c(ABC) \leq \max\{(1-\lambda^\mu(A)),(1-\lambda^\mu(C))\}$ $\Leftrightarrow$
$(\lambda^\mu)^c(ABC) \leq \max\{(\lambda^\mu)^c(A), (\lambda^\mu)^c(C)\}$.

iv. $\lambda^\eta(A-B) \leq \max\{\lambda^\eta(A), \lambda^\eta(B)\}$
$\Leftrightarrow$ $1-\lambda^\eta(A-B) \geq 1-\max\{\lambda^\eta(A), \lambda^\eta(B)\}$
$\Leftrightarrow$ $(\lambda^\eta)^c(A-B) \geq \min\{(1-\lambda^\eta(A)),(1-\lambda^\eta(B))\}$
$\Leftrightarrow$ $(\lambda^\eta)^c(A-B) \geq \min\{(\lambda^\eta)^c(A), (\lambda^\eta)^c(B)\}$.

v. $\lambda^\eta(AB) \leq \max\{\lambda^\eta(A), \lambda^\eta(B)\}$
$\Leftrightarrow$ $1-\lambda^\eta(AB) \geq 1-\max\{\lambda^\eta(A), \lambda^\eta(B)\}$
$\Leftrightarrow$ $(\lambda^\eta)^c(AB) \geq \min\{(1-\lambda^\eta(A)),(1-\lambda^\eta(B))\}$ $\Leftrightarrow$
$(\lambda^\eta)^c(AB) \geq \min\{(\lambda^\eta)^c(A), (\lambda^\eta)^c(B)\}$.

vi. $\lambda^\eta(ABC) \leq \max\{\lambda^\eta(A), \lambda^\eta(C)\}$
$\Leftrightarrow$ $1-\lambda^\eta(ABC) \geq 1-\max\{\lambda^\eta(A), \lambda^\eta(C)\}$
$\Leftrightarrow$ $(\lambda^\eta)^c(ABC) \geq \min\{(1-\lambda^\eta(A)), (1-\lambda^\eta(C))\}$ $\Leftrightarrow$
$(\lambda^\eta)^c(ABC) \geq \min\{(\lambda^\eta)^c(A), (\lambda^\eta)^c(C\}$.

Hence, $(\lambda^\mu)^c$ is an intuitionistic anti-fuzzy HX bi-ideal on $\Re$.

3.1.11 Definition: Let G = $\{\langle x, \alpha(x), \beta(x)\rangle / x\in R\}$ and H = $\{\langle x, \mu(x), \eta(x)\rangle / x\in R\}$ be any two intuitionistic fuzzy sets defined on a ring R. Let $\Re_1 \subset 2^{R}{}_1 - \{\phi\}$ and $\Re_2 \subset 2^{R}{}_2 - \{\phi\}$ be any two HX rings. Let $\gamma_G = \{\langle A, \gamma_\alpha(A), \gamma_\beta(A)\rangle / A\in\Re\}$ and $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle / A\in\Re\}$ be any two intuitionistic fuzzy subsets of a HX ring $\Re$, then the anti product of γ_G and λ_H is defined as

$$(\gamma_G \times \lambda_H) = \{\langle(A,B), (\gamma_\alpha \cup \lambda_\mu)(A,B), (\gamma_\beta \cap \lambda_\eta)(A, B)\rangle /(A, B) \in \Re_1 \times \Re_2\},$$

where, $(\gamma_\alpha \cup \lambda_\mu)(A, B) = \min\{\gamma_\alpha(A), \lambda_\mu(B)\}$, for all $(A, B) \in \Re_1 \times \Re_2$,
$(\gamma_\beta \cap \lambda_\eta)(A, B) = \max\{\gamma_\beta(A), \lambda_\eta(B)\}$, for all $(A, B) \in \Re_1 \times \Re_2$.

3.1.12 Theorem: Let G and H be any two intuitionistic fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^{R}{}_1 - \{\phi\}$ and $\Re_2 \subset 2^{R}{}_2 - \{\phi\}$ be any two HX rings. If γ_G and λ_H are any two intuitionistic anti-fuzzy HX bi-ideals of $\Re_1$ and $\Re_2$ respectively then, $\gamma_G \times \lambda_H$ is also an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re_1 \times \Re_2$.

Proof: Let G = $\{\langle x, \alpha(x), \beta(x)\rangle / x\in R_1\}$ and H=$\{\langle x, \mu(x), \eta(x)\rangle / x\in R_2\}$ be any two intuitionistic fuzzy sets defined on a ring R_1 and R_2 respectively.
Then, $\gamma_G = \{\langle A, \gamma_\alpha(A), \gamma_\beta(A)\rangle / A\in\Re_1\}$ and $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle / A\in\Re_2\}$ be any two intuitionistic anti- fuzzy HX bi-ideal of a HX ring $\Re_1$ and $\Re_2$ respectively. Then,
$(\gamma_G \times \lambda_H) = \{\langle(A,B), (\gamma_\alpha\cap \lambda_\mu)(A, B), (\gamma_\beta \cup \lambda_\eta)(A, B)\rangle /(A, B) \in \Re_1 \times \Re_2\}$,
where $(\gamma_\alpha\cap \lambda_\mu)(A, B) = \max\{\gamma_\alpha(A), \lambda_\mu(B)\}$, for all $(A, B) \in \Re_1 \times \Re_2$,
$(\gamma_\beta \cup \lambda_\eta)(A, B) = \min\{\gamma_\beta(A), \lambda_\eta(B)\}$, for all $(A, B) \in \Re_1 \times \Re_2$.

Let A, B,I $\in \Re_1 \times \Re_2$, where A = (C,D), B = (E,F) and I = (J, K),

i.
$$\begin{aligned}
(\gamma_\alpha \cap \lambda_\mu)(A-B) &= \max\{\gamma_\alpha(C-E), \lambda_\mu(D-F)\}\\
&\geq \max\{\max\{\gamma_\alpha(C), \gamma_\alpha(E)\}, \max\{\lambda_\mu(D), \lambda_\mu(F)\}\}\\
&= \max\{\max\{\gamma_\alpha(C), \lambda_\mu(D)\},\max\{\gamma_\alpha(D), \lambda_\mu(E)\}\}\\
&= \max\{(\gamma_\alpha \cap \lambda_\mu)(C, D), (\gamma_\alpha \cap \lambda_\mu)(E, F)\}\\
(\gamma_\alpha \cap \lambda_\mu)(A-B) &\geq \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}.
\end{aligned}$$

ii.
$$\begin{aligned}
(\gamma_\alpha \cap \lambda_\mu)(AB) &= (\gamma_\alpha \cap \lambda_\mu)((C, D) \cdot (E, F))\\
&= (\gamma_\alpha \cap \lambda_\mu)(CE, DF)\\
&= \max\{\gamma_\alpha(CE), \lambda_\mu(DF)\}\\
&\geq \max\{\max\{\gamma_\alpha(C), \gamma_\alpha(E)\}, \max\{\lambda_\mu(D), \lambda_\mu(F)\}\}\\
&= \max\{\max\{\gamma_\alpha(C), \lambda_\mu(D)\},\max\{\gamma_\alpha(E), \lambda_\mu(F)\}\}\\
&= \max\{(\gamma_\alpha \cap \lambda_\mu)(C, D), (\gamma_\alpha \cap \lambda_\mu)(E, F)\}\\
(\gamma_\alpha \cap \lambda_\mu)(AB) &\geq \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(B)\}.
\end{aligned}$$

iii.
$$\begin{aligned}
(\gamma_\alpha \cap \lambda_\mu)(ABI) &= (\gamma_\alpha \cap \lambda_\mu)((C, D) \cdot (E, F) \cdot (J,K))\\
&= (\gamma_\alpha \cap \lambda_\mu)(CEJ, DFK)\\
&= \max\{\gamma^\alpha(CEJ), \lambda^\mu(DFK)\}\\
&\geq \max\{\max\{\gamma_\alpha(C), \gamma_\alpha(J)\}, \max\{\lambda_\mu(D), \lambda_\mu(K)\}\}\\
&= \max\{\max\{\gamma_\alpha(C), \lambda_\mu(D)\},\max\{\gamma_\alpha(J), \lambda_\mu(K)\}\}\\
&= \max\{(\gamma_\alpha \cap \lambda_\mu)(C, D), (\gamma_\alpha \cap \lambda_\mu)(J, K)\}
\end{aligned}$$

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$$\begin{array}{lllll}
 & (\gamma_\alpha \cap \lambda_\mu)(ABI) & \geq & \max\{(\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\alpha \cap \lambda_\mu)(I)\}. \\
\text{iv.} & (\gamma_\beta \cup \lambda_\eta)(A-B) & = & \min\{\gamma_\beta(C-E), \lambda_\eta(D-F)\} \\
 & & \leq & \min\{\min\{\gamma_\beta(C), \gamma_\beta(E)\}, \min\{\lambda_\eta(D), \lambda_\eta(F)\}\} \\
 & & = & \min\{\min\{\gamma_\beta(C), \lambda_\eta(D)\}, \min\{\gamma_\beta(E), \lambda_\eta(F)\}\} \\
 & & = & \min\{(\gamma_\beta \cup \lambda_\eta)(C, D), (\gamma_\beta \cup \lambda_\eta)(E, F)\} \\
 & (\gamma_\beta \cup \lambda_\eta)(A-B) & \leq & \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}. \\
\text{v.} & (\gamma_\beta \cup \lambda_\eta)(AB) & = & (\gamma_\beta \cup \lambda_\eta)((C, D)\cdot(E, F)) \\
 & & = & (\gamma_\beta \cup \lambda_\eta)(CE, DF) \\
 & & = & \min\{\gamma_\beta(CE), \lambda_\eta(DF)\} \\
 & & \leq & \min\{\min\{\gamma_\beta(C), \gamma_\beta(E)\}, \min\{\lambda_\eta(D), \lambda_\eta(F)\}\} \\
 & & = & \min\{\min\{\gamma_\beta(C), \lambda_\eta(D)\}, \min\{\gamma_\beta(E), \lambda_\eta(F)\}\} \\
 & & = & \min\{(\gamma_\beta \cup \lambda_\eta)(C, D), (\gamma_\beta \cup \lambda_\eta)(E, F)\} \\
 & (\gamma_\beta \cup \lambda_\eta)(AB) & \leq & \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(B)\}. \\
\text{vi.} & (\gamma_\beta \cup \lambda_\eta)(ABI) & = & (\gamma_\beta \cup \lambda_\eta)((C, D)\cdot(E, F)\cdot(J, K)) \\
 & & = & (\gamma_\beta \cup \lambda_\eta)(CEJ, DFK) \\
 & & = & \min\{\gamma_\beta(CEJ), \lambda_\eta(DFK)\} \\
 & & \leq & \min\{\min\{\gamma_\beta(C), \gamma_\beta(J)\}, \min\{\lambda_\eta(D), \lambda_\eta(K)\}\} \\
 & & = & \min\{\min\{\gamma_\beta(C), \lambda_\eta(D)\}, \min\{\gamma_\beta(J), \lambda_\eta(K)\}\} \\
 & & = & \min\{(\gamma_\beta \cup \lambda_\eta)(C, D), (\gamma_\beta \cup \lambda_\eta)(J, K)\} \\
 & (\gamma_\beta \cup \lambda_\eta)(ABI) & \leq & \min\{(\gamma_\beta \cup \lambda_\eta)(A), (\gamma_\beta \cup \lambda_\eta)(I)\}.
\end{array}$$

Hence, $\gamma^G \times \lambda^H$ is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.

3.1.13 Definition: Let H be an intuitionistic fuzzy set of R. Let $\mathfrak{R} \subset 2^R - \{\phi\}$ be a HX ring. Let λ_H be an intuitionistic fuzzy set of $\mathfrak{R}$. We define the following "necessity" and possibility" operations:

$\square\lambda_H = \{\langle A, \lambda_\mu(A), 1-\lambda_\mu(A)\rangle / A \in \mathfrak{R}\}$

$\Diamond\lambda_H = \{\langle A, 1-\lambda_\eta(A), \lambda_\eta(A)\rangle / A \in \mathfrak{R}\}$.

3.1.14 Theorem: Let H be an intuitionistic fuzzy set on R. Let λ_H be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$ then $\square\lambda_H$ is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.

Proof: Let λ_H be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$. Then,

$$\begin{array}{llll}
\text{i.} & \lambda_\mu(A-B) & \leq & \max\{\lambda_\mu(A), \lambda_\mu(B)\}, \\
\text{ii.} & \lambda_\mu(AB) & \leq & \max\{\lambda_\mu(A), \lambda_\mu(B)\}, \\
\text{iii.} & \lambda_\mu(AB) & \leq & \max\{\lambda_\mu(A), \lambda_\mu(C)\}, \\
\text{iv.} & \lambda_\eta(A-B) & \geq & \min\{\lambda_\eta(A), \lambda_\eta(B)\}, \\
\text{v.} & \lambda_\eta(AB) & \geq & \min\{\lambda_\eta(A), \lambda_\eta(B)\}, \\
\text{vi.} & \lambda_\eta(ABC) & \geq & \min\{\lambda_\eta(A), \lambda_\eta(C)\}.
\end{array}$$

$$\begin{array}{llll}
\text{Now,} & \lambda_\mu(A-B) & \leq & \max\{\lambda_\mu(A), \lambda_\mu(B)\} \\
 & 1-\lambda_\mu(A-B) & \geq & 1-\max\{\lambda_\mu(A), \lambda_\mu(B)\} \\
 & & \geq & \min\{1-\lambda_\mu(A), 1-\lambda_\mu(B)\}. \\
\text{That is,} & 1-\lambda_\mu(A-B) & \geq & \min\{1-\lambda_\mu(A), 1-\lambda_\mu(B)\}.
\end{array}$$

$$\begin{array}{llll}
\text{We have,} & \lambda_\mu(AB) & \leq & \max\{\lambda_\mu(A), \lambda_\mu(B)\} \\
 & 1-\lambda_\mu(AB) & \geq & 1-\max\{\lambda_\mu(A), \lambda_\mu(B)\} \\
 & & \geq & \min\{1-\lambda_\mu(A), 1-\lambda_\mu(B)\}. \\
\text{That is,} & 1-\lambda_\mu(AB) & \geq & \min\{1-\lambda_\mu(A), 1-\lambda_\mu(B)\}.
\end{array}$$

$$\begin{array}{llll}
\text{We have,} & \lambda_\mu(ABC) & \leq & \max\{\lambda_\mu(A), \lambda_\mu(C)\} \\
 & 1-\lambda_\mu(ABC) & \geq & 1-\max\{\lambda_\mu(A), \lambda_\mu(C)\} \\
 & & \geq & \min\{1-\lambda_\mu(A), 1-\lambda_\mu(C)\}. \\
\text{That is,} & 1-\lambda_\mu(ABC) & \geq & \min\{1-\lambda_\mu(A), 1-\lambda_\mu(C)\}.
\end{array}$$

Hence, $\square\lambda_H$ is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}$.

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3.1.15 Theorem: Let H be an intuitionistic fuzzy set on R. Let λ_H be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$ then $\Diamond\lambda_H$ is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$.

Proof: Let λ_H be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$. Then,

i. $\lambda_\mu(A - B) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,
ii. $\lambda_\mu(AB) \leq \max\{\lambda_\mu(A), \lambda_\mu(B)\}$,
iii. $\lambda_\mu(AB) \leq \max\{\lambda_\mu(A), \lambda_\mu(C)\}$,
iv. $\lambda_\eta(A - B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$,
v. $\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$,
vi. $\lambda_\eta(ABC) \geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}$.

Now,
$$\lambda_\eta(A - B) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$$
$$1 - \lambda_\eta(A - B) \leq 1 - \min\{\lambda_\eta(A), \lambda_\eta(B)\}$$
$$\leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$
That is,
$$1 - \lambda_\eta(A - B) \leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$

We have,
$$\lambda_\eta(AB) \geq \min\{\lambda_\eta(A), \lambda_\eta(B)\}$$
$$1 - \lambda_\eta(AB) \leq 1 - \min\{\lambda_\eta(A), \lambda_\eta(B)\}$$
$$\leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$
That is,
$$1 - \lambda_\eta(AB) \leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(B)\}.$$

We have,
$$\lambda_\eta(ABC) \geq \min\{\lambda_\eta(A), \lambda_\eta(C)\}$$
$$1 - \lambda_\eta(ABC) \leq 1 - \min\{\lambda_\eta(A), \lambda_\eta(C)\}$$
$$\leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(C)\}.$$
That is,
$$1 - \lambda_\eta(ABC) \leq \max\{1 - \lambda_\eta(A), 1 - \lambda_\eta(C)\}.$$

Hence, $\Diamond\lambda_H$ is an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re$.

4.2 Homomorphism and anti homomorphism

In this section, we introduce the concept of an image, pre-image of an intuitionistic fuzzy HX bi-ideal of a HX ring and discuss its properties under homomorphism and anti homomorphism.

4.2.1 Definition: Let R_1 and R_2 be any two rings. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings defined on R_1 and R_2 respectively. Let H and G be any two intuitionistic fuzzy subsets in R_1 and R_2 respectively. Let λ_H and γ_G be any two intuitionistic anti-fuzzy HX bi-ideals of HX rings $\Re_1$ and $\Re_2$ respectively induced by H and G. Let f: $\Re_1 \rightarrow \Re_2$ be a mapping then the anti image of λ_H denoted as f (λ_H) is an intuitionistic fuzzy subset of $\Re_2$ and is defined as for each $U \in \Re_2$,

$$(f(\lambda_\mu))(U) = \begin{cases} \inf\{\lambda_\mu(X) : X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 0 & , \text{otherwise} \end{cases}$$

$$(f(\lambda_\eta))(U) = \begin{cases} \sup\{\lambda_\eta(X) : X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 1 & , \text{otherwise} \end{cases}$$

Also the pre-image of γ_G denoted as $f^{-1}(\gamma_G)$ under f is an intuitionistic fuzzy subset of $\Re_1$ defined as for each $X \in \Re_1$, $(f^{-1}(\gamma_\alpha))(X) = \gamma_\alpha(f(X))$ and $(f^{-1}(\gamma_\beta))(X) = \gamma_\beta(f(X))$.

4.2.2 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f: $\Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings. Let H be an intuitionistic fuzzy subset of R_1. Let λ_H be an intuitionistic anti-fuzzy HX bi-ideal of $\Re_1$ then f (λ_H) is an intuitionistic anti-fuzzy HX bi-ideal of $\Re_2$, if λ_H has an infimum property and λ_H is f-invariant.

Proof: Let $H = \{\langle x, \mu(x), \eta(x)\rangle / x \in R_1\}$ be an intuitionistic fuzzy sets defined on a ring R_1.
Then, $\lambda_H = \{\langle X, \lambda_\mu(X), \lambda_\eta(X)\rangle / X \in \Re_1\}$ be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re_1$.
Then, $f(\lambda_H) = \{\langle f(X), f(\lambda_\mu)(f(X)), f(\lambda_\eta)(f(X))\rangle / X \in \Re_1\}$.

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There exist $X, Y \in \Re_1$ such that $f(X)$, $f(Y) \in \Re_2$,

i. $$\begin{aligned} (f(\lambda_\mu))(f(X)-f(Y)) &= (f(\lambda_\mu))(f(X-Y)), \\ &= \lambda_\mu(X-Y) \\ &\le \max\{\lambda_\mu(X), \lambda_\mu(Y)\} \\ &= \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\ (f(\lambda_\mu))(f(X)-f(Y)) &\le \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \end{aligned}$$

ii. $$\begin{aligned} (f(\lambda_\mu))(f(X)\,f(Y)) &= (f(\lambda_\mu))(f(XY)) \\ &= \lambda_\mu(XY) \\ &\le \max\{\lambda_\mu(X), \lambda_\mu(Y)\} \\ &= \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\ (f(\lambda_\mu))(f(X)f(Y)) &\le \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}. \end{aligned}$$

iii. $$\begin{aligned} (f(\lambda_\mu))(f(X)\,f(Y)f(Z)) &= (f(\lambda_\mu))(f(XYZ)) \\ &= \lambda_\mu(XYZ) \\ &\le \max\{\lambda_\mu(X), \lambda_\mu(Z)\} \\ &= \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Z))\} \\ (f(\lambda_\mu))(f(X)f(Y)f(Z)) &\le \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Z))\}. \end{aligned}$$

iv. $$\begin{aligned} (f(\lambda_\eta))(f(X)-f(Y)) &= (f(\lambda_\eta))(f(X-Y)), \\ &= \lambda_\eta(X-Y) \\ &\ge \min\{\lambda_\eta(X), \lambda_\eta(Y)\} \\ &= \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\ (f(\lambda_\eta))(f(X)-f(Y)) &\ge \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \end{aligned}$$

v. $$\begin{aligned} (f(\lambda_\eta))(f(X)\,f(Y)) &= (f(\lambda_\eta))(f(XY)) \\ &= \lambda_\eta(XY) \\ &\ge \min\{\lambda_\eta(X), \lambda_\eta(Y)\} \\ &= \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\ (f(\lambda_\eta))(f(X)f(Y)) &\ge \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}. \end{aligned}$$

vi. $$\begin{aligned} (f(\lambda_\eta))(f(X)\,f(Y)f(Z)) &= (f(\lambda_\eta))(f(XYZ)) \\ &= \lambda_\eta(XYZ) \\ &\ge \min\{\lambda_\eta(X), \lambda_\eta(Z)\} \\ &= \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\} \\ (f(\lambda_\eta))(f(X)f(Y)f(Z)) &\ge \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\}. \end{aligned}$$

Hence, $f(\lambda_H)$ is an intuitionistic anti-fuzzy HX bi-ideal of $\Re_2$.

4.6.3 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f: \Re_1 \to \Re_2$ be a homomorphism on HX rings. Let G be an intuitionistic fuzzy subset of R_2. Let γ_G be an intuitionistic anti-fuzzy HX bi-ideal of $\Re_2$, then $f^{-1}(\gamma_G)$ is an intuitionistic anti-fuzzy HX bi-ideal of $\Re_1$.

Proof: Let $G = \{\langle y, \alpha(y), \beta(y)\rangle \,/\, y\in R_2\}$ be an intuitionistic fuzzy sets defined on a ring R_2.
Then, $\gamma_G = \{\langle Y, \gamma_\alpha(Y), \gamma_\beta(Y)\rangle \,/\, Y\in\Re_2\}$ be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re_2$.
Then, $f^{-1}(\gamma_G) = \{\langle X, f^{-1}(\gamma_\alpha)(X), f^{-1}(\gamma_\beta)(X)\rangle \,/\, X\in\Re_1\}$.

For any $X,Y \in \Re_1$, $f(X)$, $f(Y) \in \Re_2$,

i. $$\begin{aligned} (f^{-1}(\gamma_\alpha))(X-Y) &= \gamma_\alpha(f(X-Y)) \\ &= \gamma_\alpha(f(X)-f(Y)) \\ &\le \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\} \\ &= \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\ (f^{-1}(\gamma_\alpha))(X-Y) &\le \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}. \end{aligned}$$

ii. $$\begin{aligned} (f^{-1}(\gamma_\alpha))(XY) &= \gamma_\alpha(f(XY)) \\ &= \gamma_\alpha(f(X)\,f(Y)) \\ &\le \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\} \\ &= \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \\ (f^{-1}(\gamma_\alpha))(XY) &\le \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\} \end{aligned}$$

iii. $$\begin{aligned} (f^{-1}(\gamma_\alpha))(XYZ) &= \gamma_\alpha(f(XYZ)) \\ &= \gamma_\alpha(f(Z)\,f(Y)\,f(X)) \\ &\le \max\{\gamma_\alpha(f(Z)), \gamma_\alpha(f(X))\} \end{aligned}$$

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$$
\begin{array}{rll}
 & = & \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Z))\} \\
 & = & \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Z)\} \\
(f^{-1}(\gamma_\alpha))(XYZ) & \leq & \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Z)\} \\
\text{iv.}\quad (f^{-1}(\gamma_\beta))(X-Y) & = & \gamma_\beta(f(X-Y)) \\
 & = & \gamma_\beta(f(X)-f(Y)) \\
 & \geq & \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\} \\
 & = & \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\} \\
(f^{-1}(\gamma_\beta))(X-Y) & \geq & \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}. \\
\text{v.}\quad (f^{-1}(\gamma_\beta))(XY) & = & \gamma_\beta(f(XY)) \\
 & = & \gamma_\beta(f(X)\,f(Y)) \\
 & \geq & \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\} \\
 & = & \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\} \\
(f^{-1}(\gamma_\beta))(XY) & \geq & \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}. \\
\text{vi.}\quad (f^{-1}(\gamma_\beta))(XYZ) & = & \gamma_\beta(f(XYZ)) \\
 & = & \gamma_\beta(f(Z)\,f(Y)\,f(X)) \\
 & \geq & \min\{\gamma_\beta(f(Z)), \gamma_\beta(f(X))\} \\
 & = & \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Z))\} \\
 & = & \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Z)\} \\
(f^{-1}(\gamma_\beta))(XYZ) & \geq & \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Z)\}.
\end{array}
$$

Hence, $f^{-1}(\gamma_G)$ is an intuitionistic anti-fuzzy HX bi-ideal of $\Re_1$.

4.6.4 Theorem: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let H be an intuitionistic fuzzy subset of R_1. Let λ_H be an intuitionistic anti-fuzzy HX bi-ideal of $\Re_1$ then $f(\lambda_H)$ is an intuitionistic anti-fuzzy HX bi-ideal of $\Re_2$, if λ_H has an infimum property and λ_H is f-invariant.

Proof: Let $H = \{\langle x, \mu(x), \eta(x)\rangle / x \in R_1\}$ be an intuitionistic fuzzy sets defined on a ring R_1.
Then, $\lambda_H = \{\langle X, \lambda_\mu(X), \lambda_\eta(X)\rangle / X \in \Re_1\}$ be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\Re_1$.
Then, $f(\lambda_H) = \{\langle f(X), f(\lambda_\mu)(f(X)), f(\lambda_\eta)(f(X))\rangle / X \in \Re_1\}$.
There exist X, $Y \in \Re_1$ such that $f(X)$, $f(Y) \in \Re_2$,

$$
\begin{array}{rll}
\text{i.}\quad (f(\lambda_\mu))(f(X)-f(Y)) & = & (f(\lambda_\mu))(f(X-Y)), \\
 & = & \lambda_\mu(X-Y) \\
 & \leq & \max\{\lambda_\mu(X), \lambda_\mu(Y)\} \\
 & = & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\
(f(\lambda_\mu))(f(X)-f(Y)) & \leq & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\
\text{ii.}\quad (f(\lambda_\mu))(f(X)\,f(Y)) & = & (f(\lambda_\mu))(f(YX)) \\
 & = & \lambda_\mu(YX) \\
 & \leq & \max\{\lambda_\mu(Y), \lambda_\mu(X)\} \\
 & = & \max\{\lambda_\mu(X), \lambda_\mu(Y)\} \\
 & = & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\} \\
(f(\lambda_\mu))(f(X)f(Y)) & \leq & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Y))\}. \\
\text{iii.}\quad (f(\lambda_\mu))(f(X)\,f(Y)f(Z)) & = & (f(\lambda_\mu))(f(ZYX)) \\
 & = & \lambda_\mu(ZYX) \\
 & \leq & \max\{\lambda_\mu(Z), \lambda_\mu(X)\} \\
 & = & \max\{\lambda_\mu(X), \lambda_\mu(Z)\} \\
 & = & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Z))\} \\
(f(\lambda_\mu))(f(X)f(Y)f(Z)) & \leq & \max\{(f(\lambda_\mu))(f(X)), (f(\lambda_\mu))(f(Z))\}. \\
\text{iv.}\quad (f(\lambda_\eta))(f(X)-f(Y)) & = & (f(\lambda_\eta))(f(X-Y)), \\
 & = & \lambda_\eta(X-Y) \\
 & \geq & \min\{\lambda_\eta(X), \lambda_\eta(Y)\} \\
 & = & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\
(f(\lambda_\eta))(f(X)-f(Y)) & \geq & \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\} \\
\text{v.}\quad (f(\lambda_\eta))(f(X)\,f(Y)) & = & (f(\lambda_\eta))(f(YX)) \\
 & = & \lambda_\eta(YX) \\
 & \geq & \min\{\lambda_\eta(Y), \lambda_\eta(X)\}
\end{array}
$$

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$$\begin{aligned}
& && = && \min\{\lambda_\eta(X), \lambda_\eta(Y)\}\\
& && = && \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}\\
& (f(\lambda_\eta))(f(X)f(Y)) && \geq && \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Y))\}.\\
\text{vi.}\quad & (f(\lambda_\eta))(f(X)f(Y)f(Z)) && = && (f(\lambda_\eta))(f(ZYX))\\
& && = && \lambda_\eta(ZYX)\\
& && \geq && \min\{\lambda_\eta(Z), \lambda_\eta(X)\}\\
& && = && \min\{\lambda_\eta(X), \lambda_\eta(Z)\}\\
& && = && \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\}\\
& (f(\lambda_\eta))(f(X)f(Y)f(Z)) && \geq && \min\{(f(\lambda_\eta))(f(X)), (f(\lambda_\eta))(f(Z))\}.
\end{aligned}$$

Hence, $f(\lambda_H)$ is an intuitionistic anti-fuzzy HX bi-ideal of $\mathfrak{R}_2$.

4.6.5 Theorem: Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on R_1 and R_2 respectively. Let f: $\mathfrak{R}_1 \to \mathfrak{R}_2$ be an anti homomorphism on HX rings. Let G be an intuitionistic fuzzy subset of R_2. Let γ_G be an intuitionistic anti-fuzzy HX bi-ideal of $\mathfrak{R}_2$, then $f^{-1}(\gamma_G)$ is an intuitionistic anti-fuzzy HX bi-ideal of $\mathfrak{R}_1$.

Proof: Let $G = \{\langle y, \alpha(y), \beta(y)\rangle \,/\, y \in R_2\}$ be an intuitionistic fuzzy sets defined on a ring R_2.
Then, $\gamma_G = \{\langle Y, \gamma_\alpha(Y), \gamma_\beta(Y)\rangle \,/\, Y \in \mathfrak{R}_2\}$ be an intuitionistic anti-fuzzy HX bi-ideal of a HX ring $\mathfrak{R}_2$.
Then, $f^{-1}(\gamma_G) = \{\langle X, f^{-1}(\gamma_\alpha)(X), f^{-1}(\gamma_\beta)(X)\rangle \,/\, X \in \mathfrak{R}_1\}$.

For any X, Y $\in \mathfrak{R}_1$, f(X) , f(Y) $\in \mathfrak{R}_2$,

$$\begin{aligned}
\text{i.}\quad & (f^{-1}(\gamma_\alpha))(X-Y) && = && \gamma_\alpha(f(X-Y))\\
& && = && \gamma_\alpha(f(X)-f(Y))\\
& && \leq && \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\}\\
& && = && \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}\\
& (f^{-1}(\gamma_\alpha))(X-Y) && \leq && \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}.\\
\text{ii.}\quad & (f^{-1}(\gamma_\alpha))(XY) && = && \gamma_\alpha(f(XY))\\
& && = && \gamma_\alpha(f(Y)f(X))\\
& && \leq && \max\{\gamma_\alpha(f(Y)), \gamma_\alpha(f(X))\}\\
& && = && \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Y))\}\\
& && = && \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}\\
& (f^{-1}(\gamma_\alpha))(XY) && \leq && \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Y)\}\\
\text{iii.}\quad & (f^{-1}(\gamma_\alpha))(XYZ) && = && \gamma_\alpha(f(XYZ))\\
& && = && \gamma_\alpha(f(Z)f(Y)f(X))\\
& && \leq && \max\{\gamma_\alpha(f(Z)), \gamma_\alpha(f(X))\}\\
& && = && \max\{\gamma_\alpha(f(X)), \gamma_\alpha(f(Z))\}\\
& && = && \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Z)\}\\
& (f^{-1}(\gamma_\alpha))(XYZ) && \leq && \max\{(f^{-1}(\gamma_\alpha))(X), (f^{-1}(\gamma_\alpha))(Z)\}\\
\text{iv.}\quad & (f^{-1}(\gamma_\beta))(X-Y) && = && \gamma_\beta(f(X-Y))\\
& && = && \gamma_\beta(f(X)-f(Y))\\
& && \geq && \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\}\\
& && = && \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}\\
& (f^{-1}(\gamma_\beta))(X-Y) && \geq && \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}.\\
\text{v.}\quad & (f^{-1}(\gamma_\beta))(XY) && = && \gamma_\beta(f(XY))\\
& && = && \gamma_\beta(f(Y)f(X))\\
& && \geq && \min\{\gamma_\beta(f(Y)), \gamma_\beta(f(X))\}\\
& && = && \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Y))\}\\
& && = && \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}\\
& (f^{-1}(\gamma_\beta))(XY) && \geq && \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Y)\}.\\
\text{vi.}\quad & (f^{-1}(\gamma_\beta))(XYZ) && = && \gamma_\beta(f(XYZ))\\
& && = && \gamma_\beta(f(Z)f(Y)f(X))\\
& && \geq && \min\{\gamma_\beta(f(Z)), \gamma_\beta(f(X))\}\\
& && = && \min\{\gamma_\beta(f(X)), \gamma_\beta(f(Z))\}\\
& && = && \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Z)\}\\
& (f^{-1}(\gamma_\beta))(XY) && \geq && \min\{(f^{-1}(\gamma_\beta))(X), (f^{-1}(\gamma_\beta))(Z)\}.
\end{aligned}$$

Hence, $f^{-1}(\gamma_G)$ is an intuitionistic anti-fuzzy HX bi-ideal of $\mathfrak{R}_1$.

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REFERENCES

1. Atanassov.K.T, intuitionistic fuzzy sets, Fuzzy Sets and System 20 (1986), pp 87-96.
2. Bing-xueYao and Yubin-Zhong, Upgrade of algebraic structure of ring, Fuzzy information and Engineering (2009) 2:219-228.
3. Bing-xueYao and Yubin-Zhong, The construction of power ring, Fuzzy information and Engineering (ICFIE), ASC 40, pp.181-187, 2007.
4. Dheena.P and Mohanraaj.G, On intuitionistic fuzzy k ideals of semi rings, International Journal of computational cognition, Volume 9, No.2, 45-50, June 2011.
5. Li Hong Xing, HX group, BUSEFAL, 33(4), 31-37, October 1987.
6. Li Hong Xing, HX ring, BUSEFAL, 34(1) 3-8, January 1988.
7. Liu.W.J., Fuzzy invariant subgroups and fuzzy ideals, Fuzzy sets and systems, 8, 133 - 139.
8. Muthuraj.R, Manikandan K.H., Sithar Selvam.P.M., Intuitionistic Q-fuzzy normal HX group, International Journal of Mathematical Archive, Vol.2, no10. pp 1828-1832.
9. Muthuraj.R and Manikandan K.H, Pseudo Fuzzy Cosets of a HX Group, Applied Mathematical Science, Hikari Ltd,2013,Volume 7,no.86,pp.4259-4271.
10. Muthuraj.R, Manikandan K.H., Intuitionistic Q-fuzzy HX subgroup and $< \alpha, \beta >$ level sub HX groups, CIIT International DOI – FS 122011010.
11. Rosenfeld. A., Fuzzy groups, J.Math.Anal., 35(1971),512-517.
12. Sharma.P.K, On Intuitionistic Anti-Fuzzy Ideal and Quotient Ring, IJAMP, 2012, Volume 4, 2, pp 109-119.
13. Zadeh.L.A., Fuzzy sets, Information and control,8,338-353.

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