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BAYESIAN ESTIMATION OF THE GENERALIZED POWER WEIBULL DISTRIBUTION PARAMETERS BASED ON PROGRESSIVE CENSORING SCHEMES

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ABSTRACT

In this paper, we discussed the method of obtaining the optimal type-II progressive censoring schemes. We compared between Bayesian estimations of loss function binary, square error, Entropy loss function and linex loss function for the shape parameters α and θ of the generalized power Weibull (GPW) distribution based on progressive type-II censoring scheme. This comparison was done by using simulation study and application on real life of data.

Keywords: *Generalized Power Weibull, Progressive Type-II Censoring, Binary Loss Function, Square Error Loss Function, Linex Loss Function, Entropy Loss Function.*

1. INTRODUCTION

Pham, and Lai, (2007) introduced, the generalized power Weibull (GPW) distribution as another extension of the Weibull family. Nikulin and Haghighi (2007), introduced the cumulative distribution function of the generalized power Weibull (GPW) family is

$$F(x;\theta,\alpha) = 1 - e^{1-(1+x^{\alpha})^{\theta}}; \quad x \geq 0, \alpha, \theta > 0 \tag{1}$$

and its the corresponding probability density function is

$$f(x;\theta,\alpha) = \theta\alpha x^{\alpha-1}(1+x^{\alpha})^{\theta-1}e^{1-(1+x^{\alpha})^{\theta}} \tag{2}$$

the quantile function of the generalized power Weibull family is

$$x_u = \left((1-\ln(1-u))^{\frac{1}{\theta}} - 1\right)^{\frac{1}{\alpha}}; 0 < u < 1 \tag{3}$$

Kim and Han (2009) discussed, progressively type II censored sampling is an important method of obtaining data in lifetime studies. Balakrishnan and Chan (2004) introduced, a progressively type II censored sample as follows. For more examples, see Dey *et al.* (2014), Dey *et al.* (2016), see for instance the book by Balakrishnan and Aggrawalla (2000), and an excellent review article by Balakrishnan *et al.* (2007). Cho *et al.* [2015] discussed the Bayesian estimation for the entropy of the Weibull distribution based on the symmetric and asymmetric loss functions, such as the squared error, linex and general entropy loss functions. Almongy and Almetwalt [2018] discussed the Bayesian estimates for the GPW parameters are obtained based on binary loss function and squared error loss function. Soliman (2000) observed that most of the Bayesian inference procedures have been developed under square error loss function which is symmetrical and associates equal importantance to the losses due to over estimation and under estimation of equal magnitude.

2. PROGRESSIVE TYPE-II CENSORING SCHEME

Dey *et al.* (2016) discussed Progressive Type-II censoring scheme can be described as follows: Suppose n units are placed on a life test and the experimenter decides beforehand the quantity m, the number of failures to be observed. Now at the time of the first failure, R_1 of the remaining $n-1$ surviving units are randomly removed from the experiment. At the time of the second failure, R_2 of the remaining $n - R_1 - 1$ units are randomly removed from the experiment. Finally, at the time of the m-th failure, all the remaining surviving units $R_m = n - m - R_1 - \cdots - R_{m-1}$ are removed from the experiment.

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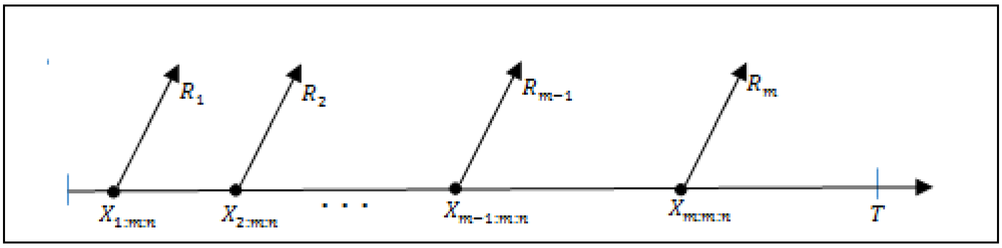

Figure-1: Plot the progressive type-II censoring scheme

Therefore, a progressive Type-II censoring scheme consists of m, and $R_1,\cdots,R_m$, such that $R_1+\cdots+R_m = n-m$. We observed the data $\{(x_1,R_1),\ldots,(x_m,R_m)\}$ in a progressive censoring scheme. Although we have included $R_1,\ldots,R_m$as part of the data, these are known in advance. Based on the observed sample $x_1<\cdots<x_m$ from a progressive Type-II censoring scheme, $R_1,\ldots,R_m$ the likelihood function can be written as

$$L = A\prod_{i=1}^{m} f(x_i;\,\theta,\alpha)\,(1-F(x_i;\,\theta,\alpha)^{R_i};\;\; \theta,\alpha>0 \tag{5}$$

where

$$A = n(n-R_1-1)\ldots\left(n-\sum_{i=1}^{m-1} R_i-(m-1)\right)$$

The likelihood function can be written as

$$L = A\;\theta^m\alpha^m e^{\sum_{i=1}^{m}(1-(1+x_i{}^{\alpha})^{\theta})^{R_i+1}}\prod_{i=1}^{m}\left[x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right] \tag{6}$$

Almetwalt and Almongy [2018] discussed the complete censoring is a special case of the progressive type-II censoring scheme that can be obtained simply by taking $R_1 = \cdots = R_m = 0$, and note that the usual type II censoring scheme is a special case of the progressive type II censoring scheme that can be obtained simply by taking $R_1 = \cdots = R_{m-1} = 0$.

3. BAYESIAN ESTIMATION OF THE GPW DISTRIBUTION

In this section we consider the Bayesian estimation for the parameters of the GPW distribution under the assumption that the random variables α and θ have an independent gamma distribution is a conjugate prior to the GPW distributions. Assumed that $\theta \sim Gamma(a,b)$ and $\alpha \sim Gamma(c,d)$, then, the joint prior density of α and θ can be written as

$$g(\alpha,\theta) \propto \theta^{a-1}\,e^{-\theta b}\,\alpha^{c-1}\,e^{-\alpha d};\;\; a,b,c,and\ d>0 \tag{7}$$

here all the hyper parameters $a,b,c\ and\ d$ are known and non-negative.

Bayesian Estimation in Progressive Type-II Censoring Scheme

Based on the likelihood function (6) and the joint prior density (7), the joint posterior of Progressive (P) Type-II censored of α and θ is

$$g(\alpha,\theta|x) = K\theta^{m+a-1}\alpha^{m+c-1}e^{-\alpha d-\theta b}\,e^{\sum_{i=1}^{m}(1-(1+x_i{}^{\alpha})^{\theta})^{R_i+1}}\prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right) \tag{8}$$

where the normalizing constant K is

$$K = \left[\int_0^\infty\int_0^\infty \theta^{m+a-1}\alpha^{m+c-1}e^{-\alpha d-\theta b}\,e^{\sum_{i=1}^{m}(1-(1+x_i{}^{\alpha})^{\theta})^{R_i+1}}\prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right)\,d\alpha\,d\theta\right]^{-1} \tag{9}$$

3.1. Binary Loss Function

under Binary (B) loss function it should be the estimation of all parameters are obtained by differentiating the posterior function in (8) with respect to θ and α and equating them to zero.

$$\hat{\alpha}_B = \frac{\partial g(\alpha,\theta|x)}{\partial\alpha},\qquad \hat{\theta}_B = \frac{\partial g(\alpha,\theta|x)}{\partial\theta} \tag{10}$$

3.2. Square Error Loss Function

A very well-known symmetric loss function is square error (SE) loss function which define as $L(\hat{\delta}_{SE},\delta_{SE}) = (\hat{\delta}-\delta)^2$, with δ is vector of barometer (θ,α), and $\hat{\delta}$ being an estimate of δ. For this situation, the Bayes estimate, say $\hat{\delta}_{SE}$, is given by the posterior mean of δ. The estimation of parameters are obtained by

$$\hat{\theta}_{SE} = \int_0^\infty \int_0^\infty K\theta^{m+a}\alpha^{m+c-1}e^{-\alpha d-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}} \prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right) d\alpha\, d\theta \tag{11}$$

and

$$\hat{\alpha}_{SE} = \int_0^\infty \int_0^\infty K\theta^{m+a-1}\alpha^{m+c}e^{-\alpha d-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}} \prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right) d\alpha\, d\theta \tag{12}$$

3.3. Linex Loss Function

Varian (1975) introduced The Linex loss function which is asymmetric. One of the most commonly-used asymmetric loss functions is the linex loss

$$L(\hat{\delta}_L, \delta_L) = e^{h(\hat{\delta}-\delta)} - h(\hat{\delta}-\delta) - 1, \qquad h \neq 0$$

Cho *et al.* (2015) discussed The sign and magnitude of the shape parameter h reflects the direction and degree of asymmetry, respectively. For $h > 0$, the overestimation is more serious than underestimation, for $h < 0$, the underestimation is more serious than the overestimation, and for h closed to zero, the Linex loss is approximately squared error loss and therefore almost symmetric. Zellner (1986) introduced the Bayes estimator of δ_L, denoted by $\hat{\delta}_L$under the Linex loss is the value $\hat{\delta}$ which minimizes the above equation, is given by:

$$\hat{\delta}_L = \frac{-1}{h}\ln E_\delta\left(e^{-h\,\delta}\right) \tag{13}$$

For this situation, the Bayes estimate, say $\hat{\delta}_L$, is given by the posterior mean of δ_L. If $h = 1$, the estimation of parameters are obtained by

$$\hat{\theta}_L = \frac{-1}{h}\ln\left(\int_0^\infty \int_0^\infty K\,\theta^{m+a-1}\alpha^{m+c-1}e^{-\alpha d-\theta(b+h)}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}} \prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right)\, d\alpha\, d\theta\right) \tag{14}$$

and

$$\hat{\alpha}_L = \frac{-1}{h}\ln\left(\int_0^\infty \int_0^\infty K\,\theta^{m+a-1}\alpha^{m+c-1}e^{-\alpha(d+h)-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}} \prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right)\, d\alpha\, d\theta\right) \tag{15}$$

3.4. Entropy Loss Function

Another commonly-used asymmetric loss function is the general entropy (E) loss function given by:

$$L(\hat{\delta}_E, \delta_E) \propto \left(\frac{\delta}{\hat{\delta}}\right)^q - q\ln\left(\frac{\delta}{\hat{\delta}}\right) - 1, \qquad q \neq 0$$

For $q > 0$, a positive error has a more serious effect than a negative error, and for $q < 0$, a negative error has a more serious effect than a positive error. Note that for $q = -1$, the Bayes estimate coincides with the Bayes estimate under the SEL function. In this case, the Bayes estimate of θ is obtained as:

$$\hat{\delta}_E = \left(E_\delta\left(\hat{\delta}^{\,-q}\right)\right)^{\frac{-1}{q}} \tag{16}$$

For this situation, the Bayes estimate, say $\hat{\delta}_E$, is given by the posterior mean of δ_E. If $q = 1$, the estimation of parameters are obtained by

$$\hat{\theta}_E = \left(\int_0^\infty \int_0^\infty K\,\theta^{m+a-q-1}\alpha^{m+c-1}e^{-\alpha d-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}} \prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right)\, d\alpha\, d\theta\right)^{\frac{-1}{q}} \tag{17}$$

and

$$\hat{\alpha}_E = \left(\int_0^\infty \int_0^\infty K\,\theta^{m+a-1}\alpha^{m+c-q-1}e^{-\alpha d-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}} \prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right)\, d\alpha\, d\theta\right)^{\frac{-1}{q}} \tag{18}$$

But the equation (10, 11, 12, 14, 15, 17 and 18) has to be performed numerically using a nonlinear optimization algorithm.

4. SIMULATION STUDY

In this section; Monte Carlo simulation is done for comparison between estimation methods based on Progressive Type-II Censoring Scheme. The data- generating form GPW Distribution, where $\delta = (2, 1.5)$, and the parameters of prior distribution are $(a, b) = (1.1, 2)$ and $(c, d) = (1.2, 1.5)$. We restricted the number of repeated-samples to 1000.

The Balakrishnan and Aggarwala (2000) introduced optimal censoring schemes: we could define the best scheme as the scheme which minimizes the mean squared error (MSE) of the estimator.

$$MSE = Mean(\hat{\delta} - \delta)^2,\ Bias = \hat{\delta} - \delta \qquad (19)$$

Table-1: Bayesian Estimation to Progressive Type-II Censoring Scheme when n=30

(n, m)	scheme		Binary		Square Error		Linex		Entropy	
			bias	MSE	bias	MSE	bias	MSE	bias	MSE
(30,0)	complete	$\hat{\theta}$	-0.0514	0.0624	-0.6665	0.4897	-0.0785	0.0514	-0.0793	0.0531
		$\hat{\alpha}$	-0.0266	0.0341	-0.4661	0.2612	-0.0244	0.0429	-0.0323	0.0443
(30,5)	(0*4,25) type 2	$\hat{\theta}$	-1.3104	1.7323	-1.3591	1.8589	-1.1150	1.2605	-1.2414	1.5558
		$\hat{\alpha}$	-0.5681	0.3508	-0.7901	0.6447	-0.4451	0.2287	-0.4967	0.2781
	(25,0*4)	$\hat{\theta}$	-0.5608	0.3398	-0.8454	0.7212	-0.4444	0.2121	-0.5415	0.3123
		$\hat{\alpha}$	-0.5094	0.2903	-0.8079	0.6733	-0.4412	0.2284	-0.4694	0.2538
	(5*5)	$\hat{\theta}$	-1.0822	1.1774	-1.1862	1.4122	-0.8906	0.7997	-1.0176	1.0501
		$\hat{\alpha}$	-0.5234	0.3013	-0.7829	0.6334	-0.4213	0.2078	-0.4623	0.2442
(30,10)	(0*9,20),type 2	$\hat{\theta}$	-0.8181	0.7092	-1.0767	1.1925	-0.6992	0.5312	-0.7562	0.6156
		$\hat{\alpha}$	-0.3740	0.1769	-0.6867	0.5037	-0.2934	0.1249	-0.3223	0.1441
	(20,0*9)	$\hat{\theta}$	0.7781	0.6368	-0.0961	0.0221	0.7061	0.5108	0.8022	0.6608
		$\hat{\alpha}$	-0.3099	0.1338	-0.6582	0.4691	-0.2777	0.1183	-0.2922	0.1273
	(2*10)	$\hat{\theta}$	-0.2797	0.1102	-0.6807	0.4898	-0.1969	0.0662	-0.2230	0.0820
		$\hat{\alpha}$	-0.2861	0.1191	-0.6330	0.4373	-0.2202	0.0871	-0.2413	0.0980
	(5*4,0*6)	$\hat{\theta}$	0.4309	0.2020	-0.2462	0.0748	0.4078	0.1770	0.4603	0.2264
		$\hat{\alpha}$	-0.2537	0.1018	-0.6104	0.4113	-0.2048	0.0815	-0.2202	0.0889
(30,15)	(0*14,15) type 2	$\hat{\theta}$	-0.4846	0.2982	-0.8830	0.8414	0.3491	0.2357	-0.4404	0.2649
		$\hat{\alpha}$	-0.2470	0.1066	-0.6009	0.4110	0.6360	0.4909	-0.2077	0.0922
	(15,0*14)	$\hat{\theta}$	1.2408	1.6628	0.1931	0.0983	0.2626	0.7084	1.2741	1.7295
		$\hat{\alpha}$	-0.1609	0.0701	-0.4842	0.3086	0.1469	0.2307	-0.1351	0.0666
	(5*3,0*12)	$\hat{\theta}$	1.2239	1.5868	0.2155	0.1082	1.1202	1.3152	1.2463	1.6384
		$\hat{\alpha}$	-0.1087	0.0584	-0.4268	0.2640	-0.0687	0.0530	-0.0789	0.0557
	(0*12,5*3)	$\hat{\theta}$	-0.3850	0.2172	-0.7950	0.6995	-0.3179	0.1699	-0.3356	0.1887
		$\hat{\alpha}$	-0.2137	0.0914	-0.5692	0.3773	-0.1540	0.0709	-0.1728	0.0788
(30,20)	(0*19,10) type 2	$\hat{\theta}$	-0.2660	0.1424	-0.7594	0.6564	-0.2371	0.1300	-0.2439	0.1393
		$\hat{\alpha}$	-0.1551	0.0729	-0.5365	0.3505	-0.1129	0.0631	-0.1272	0.0681
	(10,0*19)	$\hat{\theta}$	1.1047	1.3882	0.2801	0.2879	1.0508	1.2426	1.1282	1.4467
		$\hat{\alpha}$	-0.0303	0.0539	-0.4181	0.2602	0.0083	0.0538	-0.0008	0.0553
	(1*10,0*10)	$\hat{\theta}$	1.0344	1.2159	0.3420	0.3521	0.9770	1.0747	1.0569	1.2701
		$\hat{\alpha}$	0.0748	0.0632	-0.2892	0.2068	0.1113	0.0704	0.1039	0.0707
	(0*10,1*10)	$\hat{\theta}$	0.1456	0.1268	-0.3738	0.2785	0.16723	0.12690	0.1811	0.1468
		$\hat{\alpha}$	-0.0223	0.0546	-0.3893	0.2462	0.02188	0.05537	0.0106	0.0570
(30,25)	(0*24,5) type 2	$\hat{\theta}$	-0.1342	0.0798	-0.6955	0.5517	-0.13244	0.08060	-0.1344	0.0850
		$\hat{\alpha}$	-0.0868	0.0561	-0.4923	0.3088	-0.05989	0.05281	-0.0706	0.0555
	(5,0*24)	$\hat{\theta}$	0.6167	0.5052	-0.0619	0.1828	0.60590	0.48662	0.6316	0.5347
		$\hat{\alpha}$	0.0409	0.0566	-0.3389	0.2245	0.07395	0.06233	0.0664	0.0630
	(0*20,1*5)	$\hat{\theta}$	-0.0052	0.0755	-0.5654	0.4141	-0.00220	0.07576	-0.0003	0.0817
		$\hat{\alpha}$	-0.0288	0.0534	-0.4232	0.2644	0.00029	0.05310	-0.0096	0.0548
	(1*5,0*20)	$\hat{\theta}$	0.6282	0.5225	-0.0287	0.1964	0.61440	0.49970	0.6412	0.5503
		$\hat{\alpha}$	0.0714	0.0620	-0.3016	0.2140	0.10310	0.06945	0.0961	0.0699

In the table 1, the average Bias and MSE for the loss function of Bayesian estimation (Binary, Square Error, Linex and Entropy) is obtained, to estimate unknown parameters of GPW(θ, α), when the initial $\theta = 2$ and $\alpha = 1.5$ and sample size $(n = 30)$ in different sampling schemes. The Bayesian estimation is working quite well, in terms of Linex loss function. In special case complete and type II, the corresponding to loss function of Linex are Binary, Square Error, and Entropy loss functions, where it is accompanied by the least Bias and MSE. Followed by Entropy Loss Function and then Binary Loss function in terms of Bias and MSE. In another cases the Linex loss function is the best loss function to estimate parameters of GPW where it is the least Bias and MSE.

Table-2: Bayesian Estimation to Progressive Type-II Censoring Scheme when n=25

(n, m)	scheme		Binary		Square Error		Linex		Entropy	
			Bias	MSE	Bias	MSE	Bias	MSE	bias	MSE
(25,0)	complete	$\hat{\theta}$	-0.0716	0.0562	-0.6750	0.5044	-0.0944	0.0598	-0.0955	0.0621
		$\hat{\alpha}$	-0.0455	0.0485	-0.4686	0.2835	-0.0323	0.0469	-0.0419	0.0487
(25,5)	(0*4,20) type 2	$\hat{\theta}$	-1.2637	1.6112	-1.3308	1.7834	-1.0760	1.1752	-1.1965	1.4468
		$\hat{\alpha}$	-0.5633	0.3479	-0.7830	0.6368	-0.4413	0.2280	-0.4946	0.2788
	(20,0*4)	$\hat{\theta}$	-0.5162	0.2884	-0.8191	0.6762	-0.4096	0.1806	-0.4991	0.2658
		$\hat{\alpha}$	-0.5056	0.2893	-0.7983	0.6616	-0.4340	0.2253	-0.4640	0.2521
	(5*4,0)	$\hat{\theta}$	-0.9369	0.8849	-1.0858	1.1834	-0.7623	0.5870	-0.8832	0.7859
		$\hat{\alpha}$	-0.5032	0.2833	-0.7712	0.6190	-0.4081	0.1998	-0.4476	0.2339
	(0,5*4)	$\hat{\theta}$	-1.0995	1.2179	-1.2012	1.4500	-0.9111	0.8394	-1.0361	1.0824
		$\hat{\alpha}$	-0.5198	0.2998	-0.7687	0.6150	-0.4123	0.2027	-0.4578	0.2426
(25,10)	(0*9,15) type 2	$\hat{\theta}$	-0.7381	0.5882	-1.0290	1.0969	1.3385	1.8483	-0.6835	0.5152
		$\hat{\alpha}$	-0.3568	0.1687	-0.6687	0.4865	1.9026	3.6932	-0.3078	0.1398
	(15,0*9)	$\hat{\theta}$	0.7281	0.5581	-0.1133	0.0337	0.4436	0.1987	0.7416	0.5732
		$\hat{\alpha}$	-0.2953	0.1311	-0.6364	0.4500	0.4221	0.1809	-0.2659	0.1180
	(3*5,0*5)	$\hat{\theta}$	0.3383	0.1383	-0.2826	0.1033	0.3301	0.1263	0.3696	0.1595
		$\hat{\alpha}$	-0.2281	0.0955	-0.5780	0.3840	-0.1724	0.0745	-0.1895	0.0819
	(0*5,3*5)	$\hat{\theta}$	-0.5170	0.3094	-0.8460	0.7522	-0.4160	0.2126	-0.4569	0.2527
		$\hat{\alpha}$	-0.2940	0.1281	-0.6198	0.4283	-0.2190	0.0911	-0.2449	0.1048
(25,15)	(0*14,10) type 2	$\hat{\theta}$	-0.4038	0.2295	-0.8375	0.7697	-0.3564	0.1958	-0.3725	0.2135
		$\hat{\alpha}$	-0.2221	0.0999	-0.5759	0.3915	-0.1672	0.0804	-0.1868	0.0892
	(10,0*14)	$\hat{\theta}$	0.9873	1.0905	0.0944	0.1023	0.9193	0.9357	1.0051	1.1295
		$\hat{\alpha}$	-0.1255	0.0697	-0.4662	0.3046	-0.0789	0.0608	-0.0911	0.0647
	(1*10,0*5)	$\hat{\theta}$	0.6810	0.5542	0.0028	0.1009	0.6472	0.4914	0.7119	0.6003
		$\hat{\alpha}$	-0.0279	0.0569	-0.3675	0.2405	0.0190	0.0571	0.0075	0.0589
	(0*5,1*10)	$\hat{\theta}$	0.0649	0.0873	-0.4344	0.2822	0.0961	0.0845	0.1065	0.1002
		$\hat{\alpha}$	-0.0918	0.0622	-0.4434	0.2822	-0.0381	0.0562	-0.0532	0.0598
(25,20)	(0*19,5) type 2	$\hat{\theta}$	-0.1960	0.1040	-0.7262	0.6001	-0.1878	0.1028	-0.1922	0.1093
		$\hat{\alpha}$	-0.1232	0.0678	-0.5082	0.3317	-0.0882	0.0614	-0.1020	0.0658
	(5,0*19)	$\hat{\theta}$	0.6352	0.5361	-0.0688	0.1649	0.6069	0.4893	0.6405	0.5517
		$\hat{\alpha}$	0.0011	0.0604	-0.3645	0.2485	0.0370	0.0622	0.0272	0.0637
	(1*5,0*15)	$\hat{\theta}$	0.6423	0.5468	-0.0173	0.1988	0.6107	0.4949	0.6458	0.5603
		$\hat{\alpha}$	0.0411	0.0647	-0.3233	0.2343	0.0751	0.0690	0.0662	0.0700
	(0*15,1*5)	$\hat{\theta}$	-0.0421	0.0838	-0.5731	0.4309	-0.0328	0.0831	-0.0315	0.0908
		$\hat{\alpha}$	-0.0585	0.0604	-0.4324	0.2807	-0.0209	0.0581	-0.0335	0.0609

Table-3: Bayesian Estimation to Progressive Type-II Censoring Scheme when n=20

(n, m)	scheme		Binary		Square Error		Linex		Entropy	
			bias	MSE	bias	MSE	bias	MSE	bias	MSE
(20,0)	complete	$\hat{\theta}$	-0.1004	0.0685	-0.6829	0.5273	-0.1259	0.0743	-0.1282	0.0779
		$\hat{\alpha}$	-0.0648	0.0629	-0.4629	0.3179	-0.0471	0.0598	-0.0593	0.0631
(20,5)	(0*4,15) type 2	$\hat{\theta}$	-1.1922	1.4382	-1.2874	1.6715	-1.0184	1.0559	-1.1318	1.2976
		$\hat{\alpha}$	-0.5519	0.3407	-0.7693	0.6247	-0.4327	0.2265	-0.4877	0.2782
	(15,0*4)	$\hat{\theta}$	-0.4703	0.2378	-0.7905	0.6295	-0.3753	0.1502	-0.4568	0.2209
		$\hat{\alpha}$	-0.4933	0.2838	-0.7796	0.6434	-0.4193	0.2197	-0.4518	0.2481
	(3*5)	$\hat{\theta}$	-0.9575	0.9252	-1.1100	1.2392	-0.7910	0.6335	-0.9048	0.8261
		$\hat{\alpha}$	-0.5058	0.2922	-0.7585	0.6101	-0.4050	0.2039	-0.4498	0.2428
(20,10)	(0*9,10) type 2	$\hat{\theta}$	-0.6418	0.4604	-0.9715	0.9934	-0.5598	0.3648	-0.5996	0.4141
		$\hat{\alpha}$	-0.3319	0.1621	-0.6384	0.4749	-0.2568	0.1199	-0.2868	0.1386
	(10,0*9)	$\hat{\theta}$	0.6121	0.4154	-0.1541	0.0730	0.5557	0.3389	0.6201	0.4253
		$\hat{\alpha}$	-0.2596	0.1236	-0.5927	0.4327	-0.2057	0.1004	-0.2234	0.1100
	(1*10)	$\hat{\theta}$	-0.1253	0.0623	-0.5752	0.3840	-0.0758	0.0469	-0.0848	0.0560
		$\hat{\alpha}$	-0.2341	0.1095	-0.5620	0.4012	-0.1676	0.0841	-0.1902	0.0946
	(2*5,0*5)	$\hat{\theta}$	0.3432	0.1516	-0.2624	0.1180	0.3305	0.1354	0.3679	0.1694
		$\hat{\alpha}$	-0.1947	0.0944	-0.5297	0.3775	-0.1372	0.0769	-0.1555	0.0844
(20,15)	(0*14,5) types 2	$\hat{\theta}$	-0.3010	0.1560	-0.7795	0.6879	-0.2804	0.1467	-0.2908	0.1582

		$\hat{\alpha}$	-0.1826	0.0933	-0.5336	0.3908	-0.1352	0.0793	-0.1543	0.0874
	(5,0*14)	$\hat{\theta}$	0.5900	0.4698	-0.1097	0.1730	0.5544	0.4136	0.5956	0.4842
		$\hat{\alpha}$	-0.0733	0.0741	-0.4134	0.3404	-0.0273	0.0698	-0.0407	0.0737
	(1*5,0*10)	$\hat{\theta}$	0.5707	0.4412	-0.0837	0.1841	0.5342	0.3864	0.5757	0.4551
		$\hat{\alpha}$	-0.0229	0.0723	-0.3541	0.3344	0.0206	0.0723	0.0084	0.0751
	(0*10,1*5)	$\hat{\theta}$	-0.1158	0.0966	-0.6024	0.4764	-0.0953	0.0912	-0.0974	0.1012
		$\hat{\alpha}$	-0.1121	0.0774	-0.4525	0.3468	-0.0621	0.0694	-0.0793	0.0749

Table-4: Bayesian Estimation to Progressive Type-II Censoring Scheme when n=50

(n, m)	scheme		Binary		Square Error		Linex		Entropy	
			bias	MSE	bias	MSE	bias	MSE	bias	MSE
(50,0)	complete	$\hat{\theta}$	-0.0272	0.0324	-0.0231	0.0337	-0.0401	0.0330	-0.0403	0.0337
		$\hat{\alpha}$	-0.0119	0.0259	0.0080	0.0268	-0.0059	0.0258	-0.0104	0.0262
(50,5)	(0*4,45) type 2	$\hat{\theta}$	-1.4312	2.0597	-1.0986	1.2298	-1.2105	1.4795	-1.3534	1.8430
		$\hat{\alpha}$	-0.5864	0.3654	-0.4114	0.1959	-0.4518	0.2280	-0.4990	0.2735
	(45,0*4)	$\hat{\theta}$	-0.6865	0.5015	-0.2669	0.0988	-0.5424	0.3148	-0.6606	0.4631
		$\hat{\alpha}$	-0.5331	0.3071	-0.4394	0.2223	-0.4593	0.2374	-0.4834	0.2597
(50,10)	(0*9,40) type 2	$\hat{\theta}$	-1.0257	1.0867	-1.2033	1.4745	-0.8696	0.7939	-0.9477	0.9353
		$\hat{\alpha}$	-0.4183	0.2024	-0.7300	0.5519	-0.3315	0.1385	-0.3590	0.1585
	(40,0*9)	$\hat{\theta}$	0.7769	0.6709	-0.1062	0.0163	0.7316	0.5541	0.8426	0.7354
		$\hat{\alpha}$	-0.3389	0.1435	-0.7712	0.6119	-0.3396	0.1452	-0.3512	0.1535
(50,15)	(0*14,35) type 2	$\hat{\theta}$	-0.7069	0.5531	-0.4813	0.3108	-0.6100	0.4259	-0.6513	0.4809
		$\hat{\alpha}$	-0.3098	0.1271	-0.2128	0.0802	-0.2413	0.0902	-0.2607	0.1009
	(35,0*14)	$\hat{\theta}$	1.7146	3.0981	1.6550	2.7537	1.5519	2.4442	1.7642	3.1679
		$\hat{\alpha}$	-0.2672	0.1008	-0.2622	0.0993	-0.2321	0.0858	-0.2408	0.0904
(50,20)	(0*19,30) type 2	$\hat{\theta}$	-0.4743	0.2938	-0.8812	0.8359	-0.4159	0.2430	-0.4359	0.2655
		$\hat{\alpha}$	-0.2251	0.0837	-0.6078	0.4011	-0.1738	0.0650	-0.1885	0.0713
	(30,0*19)	$\hat{\theta}$	1.6262	3.5168	0.4849	0.2685	1.8865	3.6705	2.0964	4.5553
		$\hat{\alpha}$	-0.0880	0.0281	-0.6395	0.4316	-0.1264	0.0496	-0.1339	0.0522
(50,25)	(0*24,25) type 2	$\hat{\theta}$	-0.3262	0.1814	-0.1935	0.1377	-0.2864	0.1554	-0.2955	0.1662
		$\hat{\alpha}$	-0.1719	0.0667	-0.1038	0.0520	-0.1248	0.0526	-0.1365	0.0565
	(25,0*24)	$\hat{\theta}$	-0.9995	0.9991	1.8761	3.5915	1.8858	3.7296	2.0514	3.7440
		$\hat{\alpha}$	-0.5001	0.2501	-0.0692	0.0370	-0.0274	0.0368	-0.0339	0.0380
50,30)	(0*29,20) type 2	$\hat{\theta}$	-0.2144	0.1142	-0.7426	0.6185	-0.1914	0.1047	0.6267	0.5743
		$\hat{\alpha}$	-0.1228	0.0493	-0.5385	0.3286	-0.0858	0.0447	-0.0277	0.0551
	(20,0*29)	$\hat{\theta}$	-1.0000	0.9999	0.5750	0.4359	1.6870	3.0377	1.7998	3.4805
		$\hat{\alpha}$	-0.5000	0.2500	-0.4389	0.2318	0.0572	0.0423	0.0516	0.0425
(50,35)	(0*34,15) type 2	$\hat{\theta}$	-0.1864	0.0998	-0.0856	0.0826	-0.1326	0.0770	-0.1343	0.0806
		$\hat{\alpha}$	-0.1140	0.0480	-0.0477	0.0412	-0.0593	0.0393	-0.0673	0.0410
	(15,0*34)	$\hat{\theta}$	-0.9999	0.9999	1.4443	2.2510	1.3519	2.0023	1.4165	2.2124
		$\hat{\alpha}$	-0.5000	0.2500	0.1112	0.0533	0.1152	0.0544	0.1105	0.0542
(50,40)	(0*39,10) type 2	$\hat{\theta}$	-0.3371	0.2378	-0.6827	0.5111	-0.0891	0.0551	-0.0898	0.0571
		$\hat{\alpha}$	-0.1862	0.0763	-0.5003	0.2853	-0.0369	0.0347	-0.0435	0.0358
	(10,0*39)	$\hat{\theta}$	-1.0000	0.9999	0.1903	0.1965	0.9375	1.0071	0.9667	1.0768
		$\hat{\alpha}$	-0.5000	0.2500	-0.3009	0.1562	0.1383	0.0597	0.1340	0.0594
(50,45)	(0*44,5) type 2	$\hat{\theta}$	-0.7269	0.6611	-0.6712	0.4848	-0.0670	0.0402	-0.0675	0.0413
		$\hat{\alpha}$	-0.3815	0.1752	-0.4894	0.2740	-0.0233	0.0302	-0.0290	0.0310
	(5,0*44)	$\hat{\theta}$	-0.9999	0.9998	-0.1957	0.1253	0.4765	0.2977	0.4852	0.3099
		$\hat{\alpha}$	-0.5000	0.2500	-0.3340	0.1685	0.1133	0.0491	0.1091	0.0488

5. APPLICATION

In this section, we have given an application of Bayesian estimation of GPW distribution using real data set to illustrate that GPW distribution provides significant improvements by different loss functions. These data are from Soliman et al (2013) concerning the data on 19 times to breakdown of an insulating fluid between electrodes at a voltage of 34 k.v. (minutes). The result of One-sample Kolmogorov-Smirnov test sine test = 0.1531, and p-value=0.709, hence, the data follow GPW distribution.

Table-5: Bayesian Estimation of GPW based on Progressive Type-II Censoring of Real Data

			Binary	Square Error	Linex	Entropy
(19,0)	complete	$\hat{\theta}$	0.2520	0.2498	0.3440	0.2682
		$\hat{\alpha}$	1.2707	1.1601	1.2858	1.2652
(19,5)	(0*4,14) type 2	$\hat{\theta}$	0.1930	0.2070	0.2492	0.1865
		$\hat{\alpha}$	0.9804	0.9951	1.0034	0.9150
	(14,0*4)	$\hat{\theta}$	0.6247	0.4939	0.9070	0.7721
		$\hat{\alpha}$	1.6240	1.1322	1.7189	1.8684
(19,9)	(0*8,10) type 2	$\hat{\theta}$	0.2292	0.2329	0.3128	0.2366
		$\hat{\alpha}$	1.2048	1.1222	1.2077	1.1641
	(10,0*8)	$\hat{\theta}$	0.4294	0.3784	0.6415	0.5168
		$\hat{\alpha}$	1.5297	1.1925	1.5872	1.6601
(19,10)	(0*9,9) type2	$\hat{\theta}$	0.2277	0.2317	0.3057	0.2338
		$\hat{\alpha}$	1.1927	1.1194	1.1955	1.1520
	(9,0*9)	$\hat{\theta}$	0.4017	0.3601	0.5985	0.4779
		$\hat{\alpha}$	1.5075	1.1996	1.5588	1.6187
(19,10)	(0*14,4) type 2	$\hat{\theta}$	0.2250	0.2291	0.3033	0.2340
		$\hat{\alpha}$	1.2053	1.1273	1.2064	1.1686
	(4,0*14)	$\hat{\theta}$	0.3035	0.2900	0.4355	0.3391
		$\hat{\alpha}$	1.3862	1.2021	1.4127	1.4214

CONCLUSION

In this paper, we discussed the Bayesian estimation under the non-informative independent gamma priors of the GPW distribution based on progressive type-II censoring scheme by different loss functions. A comparison had been done between the proposed loss functions (Binary, Square Error, Linex, and Entropy) on the basis of Monte Carlo Simulation study. The performance of the different estimators optimal censoring schemes is compared based on simulation study to determine the optimal censoring schemes by using MSE and Bias. A real data set has been considered to illustrate the practical utility of the paper and show how the scheme works in practice. The Linex loss function is the best loss function to estimate parameters of GPW where it is the least Bias and MSE. Finally, the Bayesian estimation under Linex loss function can be applied to estimate parameters any other distributions.

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