International Journal of Mathematical Archive-9(6), 2018, 23-32
Available online through www.ijma.info ISSN 2229 – 5046

VISCOSITY VARIATION EFFECT ON THE COUPLE STRESS SQUEEZE FILM CHARACTERISTICS OF SHORT POROUS JOURNAL BEARINGS

SIDDANGOUDA A[1] AND S. B. PATIL[2*]

[1]Department of Mathematics,
Shree Guru Vidya Peetha Degree College of Science,
Khanadal-585102, Dist: Kalaburagi, India.

[2]Department of Mathematics,
Govt. College (Autonomous), Kalaburagi-585105, India.

(Received On: 14-03-18; Revised & Accepted On: 05-05-18)

ABSTRACT

The purpose of this paper is to study the effect of viscosity variation and non-Newtonian couple stress fluid on the squeeze film lubrication of short porous journal bearing. The problem is analyzed on smooth and porous bearing. The modified Reynold's equation for short porous journal bearing accounting for the viscosity variation of couple stress fluid is mathematically modelled to obtain solution of squeeze film pressure, load carrying capacity and squeeze film time. The results obtained are comparable with standard results and are shown in terms of graphs. The squeeze film pressure , load carrying capacity and squeeze film time significantly enhances for couple stress lubricants as compared to the corresponding Newtonian case. On increasing value of viscosity variation parameter signify a decrease in viscosity, which may be consequence of temperature rise.

***Key Words**: Short Journal bearing, Couple stress fluid, Porous, Squeeze film, Viscosity Variation.*

1. INTRODUCTION

Today, the self-lubricating porous bearings have been studied because of their machine manufacturing and industrial applications. These bearings have self contained oil reservoir and hence do not require continuous lubrication. Most porous bearings have interconnecting pores which store the lubricating oil. Hence, when normal load is applied, the fluid is supplied through the interconnected pores to the fluid film region to support the load, and when the load is removed from the loaded part of the bearing, fluid is reabsorbed by capillary action. Since these can operate without additional lubricant for longer period, porous bearings are widely used, where relubrication would be difficult. Thus, porous metal bearings are used in manufacturing small motors, home appliances, instruments and construction equipments. Because of these practical aspects in daily life, there have been numerous studies on the performance characteristics of such bearing [1-4]. But these studies were confined to Newtonian lubricants.

Recently, the technology of squeeze film phenomenon are widely used in many engineering applications, such as machine tools, disk clutches, dampers, gears, aircraft engines and human joints. The squeeze film behaviour arises from the phenomenon of two lubricated surfaces approaching each other with a normal velocity. Newtonian lubricants are used in the squeeze film bearings [5-6], with the advancement of modern machine equipments, the increasing use of lubricants containing microstructures, such as additives suspensions, granular matter has received great interest. These kinds of fluids exhibit the rheological behaviours of non-Newtonian fluids. Several micro continuum theories have been developed to explain the peculiar behaviour of fluids containing a structure such as polymeric fluids [7, 8]. The micro continuum theory derived by Stokes [9] is the simplest generalization of the classical theory of fluids, which allows for the polar effects such as the presence of couple stress and body couples. By applying this Stokes couple stress fluid model, a number of researches in various squeeze film problems have been presented. The typical studies are the squeeze film behaviour between finite plates of various shapes [10], the squeeze film configuration with reference to ankle joints [11], the squeeze film partial journal bearings [12] and squeeze film performance between a sphere and flat plate [13]. Generally speaking, a higher film pressure and larger load carrying capacity as well as long response time are obtained for the squeeze films by the use of fluids with couple stress.

Corresponding Author: S. B. Patil[2], [2]Department of Mathematics,
Govt. College (Autonomous), Kalaburagi-585105, India.*

Earlier theories were based on the assumptions that the viscosity is constant, although it is a function of both temperature and pressure. The variation in viscosity with temperature is important in many practical applications, where lubricants are required to function over a wide range of temperature [14]. To study the effect of viscosity variation, one has to consider a typical viscosity film thickness relation with thermodynamic problems [15-16]. The effect of viscosity variation on the squeeze film performance of narrow journal bearing with couple stress fluid is studied by Reddy *et.al.* [17] by assuming bearing surfaces are smooth.

In this paper an attempt has been made to study the combined effect of viscosity variation and porous surface on the couple stress squeeze film lubrication of short journal bearing.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The basic equations derived by Stokes [9] for the motion of an incompressible couple stress fluid, in the absence of body forces and body couples are

$$\nabla.\vec{V} = 0 \tag{1}$$

$$\rho\frac{d\vec{V}}{dt} = -\nabla p + \rho\vec{F} + \frac{1}{2}\rho\nabla\times\vec{C} + \mu\nabla^2\vec{V} - \eta\nabla^4\vec{V} \tag{2}$$

Where the vectors $\vec{V}, \vec{F}$ and $\vec{C}$ represents the velocity vector, body force per unit mass and body couple per unit mass, ρ is the density, p is the pressure, μ is the shear viscosity and η is the new material constant responsible for the couple stress fluid property.

Figure 1 shows the physical configuration of a squeeze film short porous journal bearing. The shaft of radius R approaches the bearing surface with velocity $\left(\frac{\partial H}{\partial t}\right)$. The lubricant in the system is taken to be Stokes couple stress fluid. Further, $\theta = x/R$ with R being the radius of the journal. With the usual assumptions of hydrodynamic lubrication applicable to thin films, equations of motion (1) and (2) take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

$$\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4} \tag{4}$$

$$\frac{\partial p}{\partial y} = 0 \tag{5}$$

$$\frac{\partial p}{\partial z} = \mu\frac{\partial^2 w}{\partial y^2} - \eta\frac{\partial^4 w}{\partial y^4} \tag{6}$$

where u, v and w denote the velocity components in the x, y, and z directions respectively.

The boundary conditions at the bearing surface are

$$u(x,o,z) = w(x,o,z) = 0 \tag{7a}$$

$$v(x,o,z) = -v^* \tag{7b}$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=o} = \left(\frac{\partial^2 w}{\partial y^2}\right)_{y=o} = 0 \tag{7c}$$

and at the journal surface are

$$u(x,H,z) = w(x,H,z) = 0 \tag{8a}$$

$$v(x,H,z) = \frac{\partial H}{\partial t} \tag{8b}$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=H} = \left(\frac{\partial^2 w}{\partial y^2}\right)_{y=H} = 0 \tag{8c}$$

where *H* is the fluid film thickness and v^* is the modified Darcy velocity component in the *y*-direction in the porous region. The flow of couplestress fluid in the porous region is governed by modified form of Darcy's law [18] which accounts for the polar effects in porous region given by

$$u^* = \frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial x} \tag{9}$$

$$v^* = \frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial y} \tag{10}$$

$$w^* = \frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial z} \tag{11}$$

where k is the permeability of porous matrix and β (=(η/μ)/k) represents the ratio of microstructure size to the pore size. If $\left(\eta/\mu\right)^{1/2} \approx \sqrt{k}$. i.e. $\beta \approx 1$, then the microstructure additives present in the lubricant block the pores in the porous layer and thus reduce the Darcy flow through the porous matrix. When the microstructure size is very small compared to the pore size, i.e. β <<1, the additives percolate in to the porous matrix. The pressure p^* in the porous region, due to continuity, satisfies the Laplace equation

$$\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} + \frac{\partial^2 p^*}{\partial z^2} = 0 \tag{12}$$

The solution of equations (4) and (6) subjected to the relevant boundary conditions given in equations (8) and (9) is obtained in the form

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left\{ y(y-H) + 2l^2\left[1 - \frac{\cosh((2y-H)/2l)}{\cosh(H/2l)}\right]\right\} \tag{13}$$

and $$w = \frac{1}{2\mu}\frac{\partial p}{\partial z}\left\{ y(y-H) + 2l^2\left[1 - \frac{\cosh((2y-H)/2l)}{\cosh(H/2l)}\right]\right\} \tag{14}$$

where $l = \sqrt{\eta/\mu}$

Integrating equation (12) with respect to y over the porous layer thickness, H_0 and using the boundary conditions of solid backing $\left(\frac{\partial p^*}{\partial y} = 0\right)$ at $y = -H_0$, we obtain,

$$\left.\frac{\partial p^*}{\partial y}\right|_{y=0} = -\int_{-H_0}^{0}\left(\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial z^2}\right)dy \tag{15}$$

Assuming that, the porous layer thickness, H_0 is very small and using the pressure continuity condition $\left(p = p^*\right)$ all the interface $\left(y = 0\right)$ of porous matrix and fluid film, equation (15) reduces to

$$\left.\frac{\partial p^*}{\partial y}\right|_{y=0} = -H_0\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2}\right) \tag{16}$$

Then, the velocity component of the modified Darcy's velocity V^* at the interface $\left(y = 0\right)$ is given by

$$\left.v^*\right|_{y=0} = \frac{kH_0}{\left(1-\beta\right)}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2}\right) \tag{17}$$

Using the expressions (13) and (14) for velocity components u and w in the continuity equation (3) and integrating with respect to y and the use of boundary conditions (7a-7c) and (8a-8c), the modified Reynolds's equation is obtained in the form

$$\frac{\partial}{\partial x}\left\{\left(f(H,l) + \frac{12kH_0}{1-\beta}\right)\frac{\partial p}{\partial x}\right\} + \frac{\partial}{\partial z}\left\{\left(f(H,l) + \frac{12kH_0}{1-\beta}\right)\frac{\partial p}{\partial z}\right\} = 12\mu\frac{\partial H}{\partial t} \tag{15}$$

where

$$f(H,l) = H^3 - 12l^2H - 24l^3\tanh(H/2l)$$

Now it is assumed that, the Newtonian viscosity μ is varying along the fluid film thickness H according to the relation given below [15].

$$\mu = \mu_1\left(\frac{H}{h_1}\right)^Q \tag{19}$$

where μ_1 is the inlet viscosity at $H = h_1 = c(1+\varepsilon)$. The exponent Q may be determined using the relation

$$Q = \frac{\log(\mu_1/\mu_2)}{\log(h_1/h_2)} \tag{20}$$

where μ_2 is the outlet viscosity with film thickness h_2.

The parameter $Q(0 \le Q \le 1)$ depends on the particular lubricant used, for perfect Newtonian fluids $Q = 0$, whereas for perfect gases $Q = 1$. For mathematical simplicity, the couple stress parameter l is assumed to be independent of viscosity variation, this can be done by assuming that η is varying in the same way as μ.

2.2 Short bearing approximation

In order to simplify the problem and to obtain a closed form solution for the fluid pressure, a narrow bearing approximation is assumed. That is the circumferential variation of pressure can be neglected as compared to the axial variation, then the modified Reynold's equation (15)reduces to

$$\frac{\partial}{\partial z}\left\{\left(f(H,l) + \frac{12kH_0}{1-\beta}\right)\frac{\partial p}{\partial z}\right\} = 12\mu\frac{\partial H}{\partial t} \tag{21}$$

Substituting $\mu = \mu_1\left(\frac{H}{h_1}\right)^Q$ in the above equation, to obtain

$$\frac{\partial}{\partial z}\left\{\left(f(H,l) + \frac{12kH_0}{1-\beta}\right)\frac{\partial p}{\partial z}\right\} = 12\mu_1\left(\frac{H}{h_1}\right)^Q\frac{\partial H}{\partial t} \tag{22}$$

Integrating twice with respect to z and applying the following boundary conditions

$$p = 0 \quad \text{at } z = \pm\frac{L}{2} \quad \text{and} \quad \frac{dp}{dz} = 0 \quad \text{at } z = 0 \tag{23}$$

The fluid film pressure is given by

$$p = \frac{6\mu_1}{\left(f(H,l) + \frac{12kH_0}{1-\beta}\right)}\left(\frac{H}{h_1}\right)^Q\left(\frac{dH}{dt}\right)\left(z^2 - \frac{L^2}{4}\right) \tag{24}$$

Introducing the non-dimensional variables

$$\lambda = \frac{L}{2R},\ z^* = \frac{z}{L},\ l^* = \frac{l}{c},\ \frac{dH}{dt} = c\cos\theta\frac{d\varepsilon}{dt}, \psi = \frac{kH_0}{c}$$

$$H^* = \frac{H}{c} = 1 + \varepsilon\cos\theta,\ P = \frac{pc^2}{\mu_1 R^2\left(\frac{d\varepsilon}{dt}\right)} \tag{25}$$

$$\text{and } F(H^*, l^*) = H^{*3} - 12l^{*2}H^* + 24l^{*3}\tanh\left(\frac{H^*}{2l^*}\right)$$

Into above equation the non-dimensional fluid film pressure is given in a closed form is obtained as

$$P = \frac{24\lambda^2\cos\theta H^{*Q}}{\left(F(H^*, l^*) + \frac{12\psi}{1-\beta}\right)(1+\varepsilon)^Q}\left(z^{*2} - \frac{1}{4}\right) \tag{26}$$

2.3 Load carrying capacity

The load carrying capacity is evaluated by integrating the squeeze film pressure acting on the journal shaft is given by

$$w = -2R \int_{z=0}^{z=L/2} \int_{\theta=\pi/2}^{\theta=3\pi/2} p\cos\theta \, d\theta \, dz \tag{27}$$

Introducing the non-dimensional quantity

$$W = \frac{wc^2}{\mu_1 R^2 L(d\varepsilon / dt)} \tag{28}$$

The load carrying capacity can be expressed in non-dimensional form as

$$W = \frac{4\lambda^2}{(1+\varepsilon)^Q} \int_{\theta=\pi/2}^{\theta=3\pi/2} \frac{(1+\varepsilon\cos\theta)^Q}{\left(F(H^*, l^*) + \dfrac{12\psi}{1-\beta}\right)} \cos^2\theta \, d\theta \tag{29}$$

The non-dimensional load carrying capacity W in the above equation (29) cannot be obtained by direct integration. It can be numerically evaluated by the method of Gaussian quadrature.

2.4 Squeeze time eccentricity ratio relationship

For constant load *w*, the time taken by the journal to move from $\varepsilon=0$ to $\varepsilon=\varepsilon_1$ can be obtained by integrating equation (28) with respect to time gives

$$\frac{1}{W} t = \frac{\mu_1 r^3 L\varepsilon}{wc^2} \tag{30}$$

Introducing the non-dimensional response time

$$t^* = \frac{wc^2}{\mu_1 R^3 L} t \tag{31}$$

Using this equation (31) becomes

$$\varepsilon = \frac{t^*}{W}.$$

Then, the journal centre velocity is obtained by solving the equation

$$\frac{d\varepsilon}{dt^*} = \frac{1}{W} \tag{32}$$

with initial condition of $\varepsilon = 0$ at $t^* = 0$. In the limiting case of $\psi \to 0$ equations (26), (28) and (32) reduce to that of non-porous case studied by Reddy *et. al*. [17].

3. RESULTS AND DISCUSSIONS

This paper presents the combined influence of viscosity variation and couple stresses on the squeeze film characteristics of porous journal bearings. These effects are analyzed on the basis of various dimensionless parameters such as the viscosity variation parameter *Q*, couple stress parameter l^*, the permeability parameter ψ and the eccentricity ratio parameter ε.

In the present analysis, we choose the parameters, $\lambda = 0.5$ (length to diameter ratio), since in practice the eccentricity ratio ranges from 0.4 to 0.6. Couple stress fluid is characterized by the non-dimensional parameter l^*, the value of this couple stress parameter depends upon the characteristic material length of the polar suspensions *l* and the radial clearance *c*. Hence the values of l^* are chosen as 0.0, 0.2, 0.4.,0.6. Viscosity variation parameter *Q* lies between 0 and 1. Numerical values of 0, 0.25, 0.5, 0.75 and 1 are assumed for *Q* in order to discuss the effect of viscosity variation in the present analysis and the permeability parameter ψ are 0.0,0.01,0.001,0.0001 chosen for the discussion.

3.1 Squeeze film pressure

Figure 2 shows the variation of non-dimensional pressure P as a function of circumferential coordinate θ (in degrees) on the mid-plane $z^* = 0$ at the eccentricity ratio ε =0.6, couple stress parameter l^* = 0.0 (Newtonian)-0.4 (Non-Newtonian) and permeability parameter ψ =0.001. On comparing with Newtonian fluid the effect of couple stress parameter increases the squeeze film pressure. Further, it is observed that, as viscosity variation parameter increases the squeeze film pressure decreases rapidly for couple stress fluid than Newtonian fluid. Hence, the viscosity variation parameter is enhances for couple stress fluid. Figure 3 shows the variation of non-dimensional pressure P as a function of circumferential coordinate θ (in degrees) on the mid-plane $z^* = 0$ at the eccentricity ratio ε =0.6, viscosity variation parameter Q =0.6 and permeability parameter ψ =0.001. It is observed that the effect of couple stress parameter increases the squeeze film pressure as compared to Newtonian fluid. The variation of non-dimensional pressure P as a function of circumferential coordinate θ (in degrees) on the mid-plane $z^* = 0$ at the eccentricity ratio ε =0.6, viscosity variation parameter Q =0.6 and couple stress parameter l^* =0.4 is depicted in Figure 4. It is observed that, the effect of permeability parameter ψ is to decrease the squeeze film pressure as compared to corresponding solid case. The adverse effect of the porous facing on the bearing surface can be compensated with the selection of microstructure additives.

3.2 Load carrying capacity

The variation of dimensionless load carrying capacity W with the eccentricity ratio ε for different values of viscosity variation factor *Q* is presented in Figure 5. It is observed that, as eccentricity ratio parameter increases the load carrying capacity increases as compared to Newtonian fluid. Figure 6 presents the variation of dimensionless load carrying capacity W with the eccentricity ratio ε for different values of couple stress parameter l^*. It is observed that, the load carrying capacity enhances for couple stress parameter as compared to Newtonian fluid. The variation of dimensionless load carrying capacity W with the eccentricity ratio ε for different values of permeability parameter ψ is depicted in Figure 7. It is observed that, the permeability parameter decreases the load carrying capacity as compared to solid case $(\psi = 0)$.

3.3 Squeeze time eccentricity ratio relationship

Figure 8 shows the variation of dimensionless squeeze film time t^* with eccentricity ratio, ε, for different values of the viscosity variation parameter *Q*, It is observed that, the effect of variation of viscosity variation is to decrease the squeeze film time. The variation of dimensionless squeeze film time t^* with eccentricity ratio, ε, for different values of the couple stress parameter l^* is depicted in Figure 9. It is observed that, the presence of couple stress parameter is to increase the squeeze film time as compared to corresponding Newtonian case. Figure 10 shows the variation of dimensionless squeeze film time t^* with eccentricity ratio, ε, for different values of the permeability parameter ψ. It is observed that, the permeability parameter decreases the squeeze film time as compared to solid case $(\psi = 0)$.

4. CONCLUSIONS

Based on the Stokes micro-continuum theory of couple stress fluids and viscosity variation parameter, the modified Reynold's equation is derived for the squeeze film pressure, the load carrying capacity and squeeze film time. According to the results presented in the above section the following conclusions can be drawn;

1. The effect of viscosity variation is to decreases the squeeze film pressure, load carrying capacity and squeeze film time, which may be a consequence of temperature rise.
2. The effect of couple stresses is to increases load carrying capacity and to lengthen the squeeze film time as compared to the corresponding Newtonian case.
3. The bearing lubricated with the couple stress fluid provides the journal contact and which results in a longer bearing life.
4. The presence of porous facing on the bearing surface affects the performance of the bearing.
5. The adverse effects of the porous facing on the bearing surface can be compensated with the selection of the lubricants with proper microstructure additives.

REFERENCES

1. Murti, P.R.K, (1974), "Analysis of porous slider bearings", Wear, Vol.28, pp.131-134.
2. Kumar, V., (1980), "Friction of a plane porous slider of optimum profile", Wear, Vol.62, pp.417-418.
3. Rouleau, W.T. and Steiner, L.T., (1974), "Hydrodynamic porous journal bearing: part-I Finite full bearings," Transactions of ASME Journal of Lubrication Technology, Vol.96 No.5, pp.346-353.
4. Cameron, A., Morgon,V.T., Stainshy, A.E., Critical conditions for hydrodynamic Lubrication of porous metal bearings. Proc.Inst.Mech.Engg. London, Vol.176 (1962), pp.761-768.
5. Pinkus O., Sternlicht B., (1961), "Theory of hydrodynamic lubrication", McGraw-Hill, New York.
6. Prakash J., Vij S.K., (1972), "Squeeze film in porous metal bearings", Journal of Lubrication Technology, Vol.94, pp.302-305.
7. Ariman T.T., Sylvester N.D., (1973), "Micro continuum fluid mechanic, a review", International Journal of Engineering Science, Vol.11, pp.905-930.
8. Ariman T.T., Sylvester N.D., (1974), "Applications of Micro continuum fluid mechanic", International Journal of Engineering Science, Vol.12, pp.273-293.
9. Stokes V.K., (1966), "Couple stresses in fluids", Journal Physics of Fluids, Vol.9, 1709-1715.
10. Ramanaih G., (1979), "Squeeze film between finite plates lubricated by fluids with couple stress", Wear, Vol.54, pp.315-320.
11. Albert E. Y. and Ali Amar Al-allaq, (2013), "The hydrodynamic squeeze film lubrication of the ankle joints", International Journal of Mechanical Engineering and Applications, Vol.1.No.2, pp.34-42.
12. Lin J.R., (1997), "Squeeze film characteristics of long partial journal bearings lubricated with couple stress fluids", Tribology International, Vol.30, pp.53-58.
13. Lin J.R., (2000), "Squeeze film characteristics between sphere and a flat plate couple stress fluid model", Computers and Structures, Vol.75, pp.73-80.
14. Freeman P., (1962), "Lubrication and Friction", Ch.2, Pitman, London.
15. Taipei N.,(1962),"Theory of lubrication", ch.3, Stanford university press, Stanford, CA.
16. Tipie N., (1967), "On the field of temperature in lubricating films", Journal Basic Engineering, Vol.89, pp.483-486.
17. Jaya Chandra Reddy G., Eshwara Reddy C., Ramakrishna Prasad K., (2008), "Effect of viscosity variation on the squeeze film performance of a narrow hydrodynamic journal bearings operating with couple stress fluid", Part-J, Proc. Inst. Mech. Engg. Journal of Engineering Tribology, Vol.222, pp.141-150.
18. Agrawal V.K. and Bhat S.B., (1980), "Porous pivoted slider bearings lubricated with a micropolar fluid," Wear, Vol.61, pp.1–8.

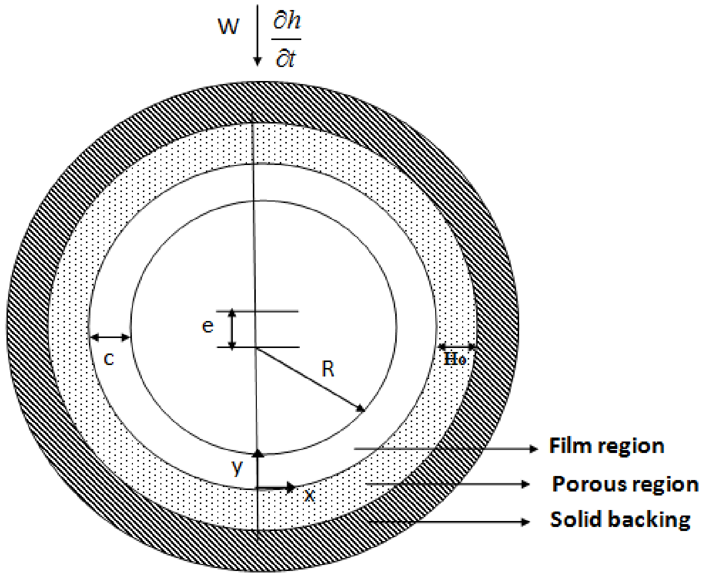

Figure-1: Physical configuration of a short porous journal bearing

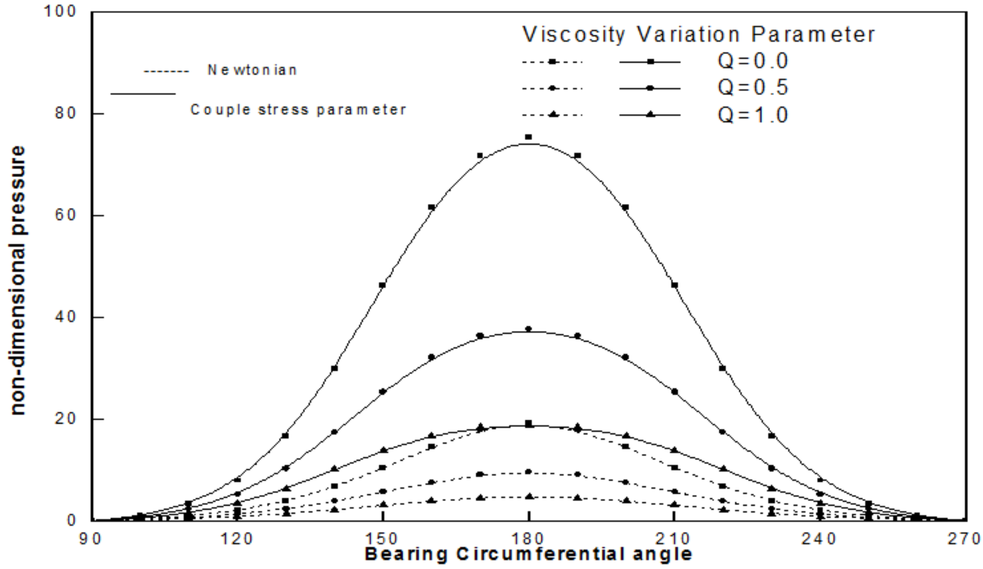

Fig.2. Non-dimensional pressure versus bearing circumferential angle for different viscosity variation parameter

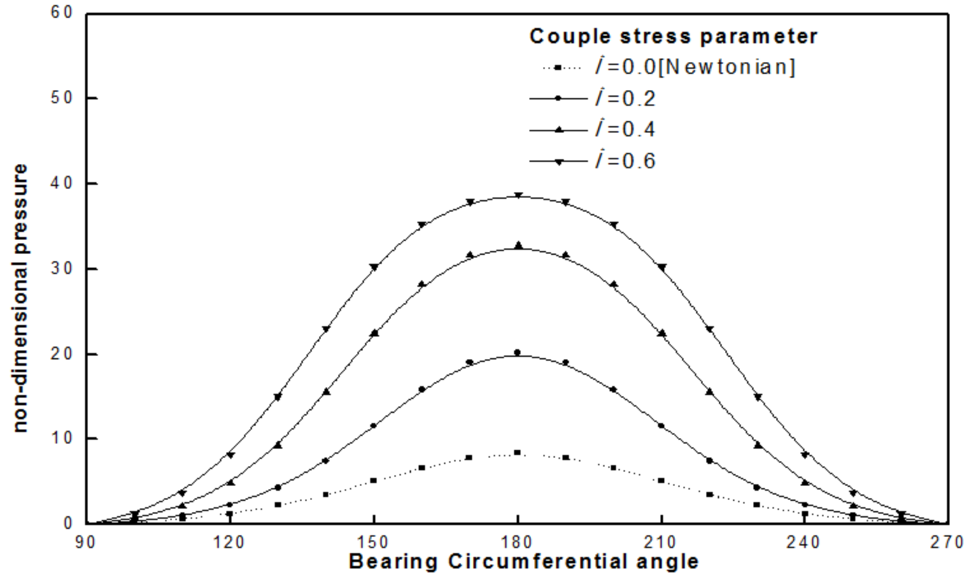

Fig.3. Non-dimensional pressure versus bearing circumferential angle for different Couple stress parameter

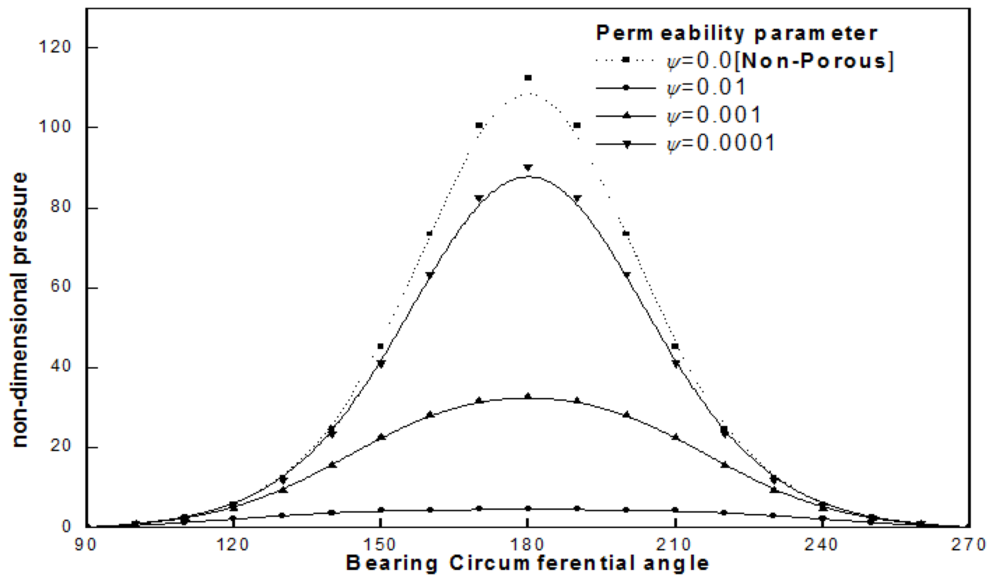

Fig.4. Non-dimensional pressure versus bearing circumferential angle for different permeability parameter

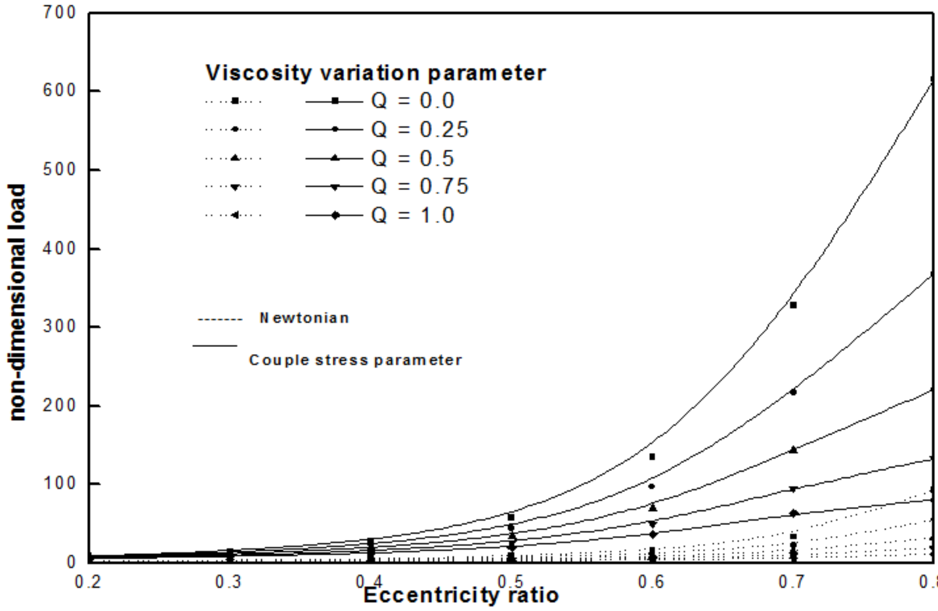

Fig.5. Non-dimensional load versus eccentricity ratio for different viscosity variation parameter

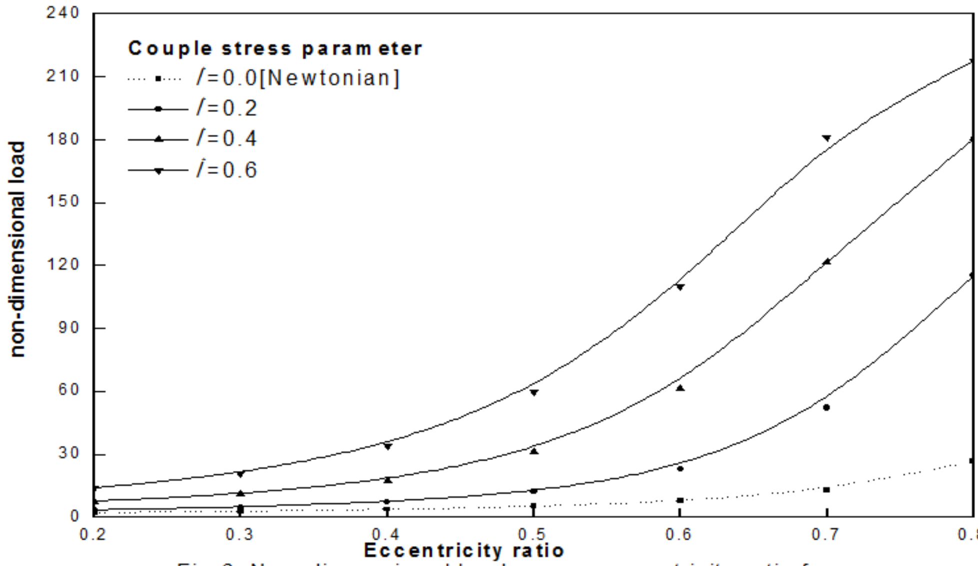

Fig.6. Non-dimensional load versus eccentricity ratio for different couple stress parameter

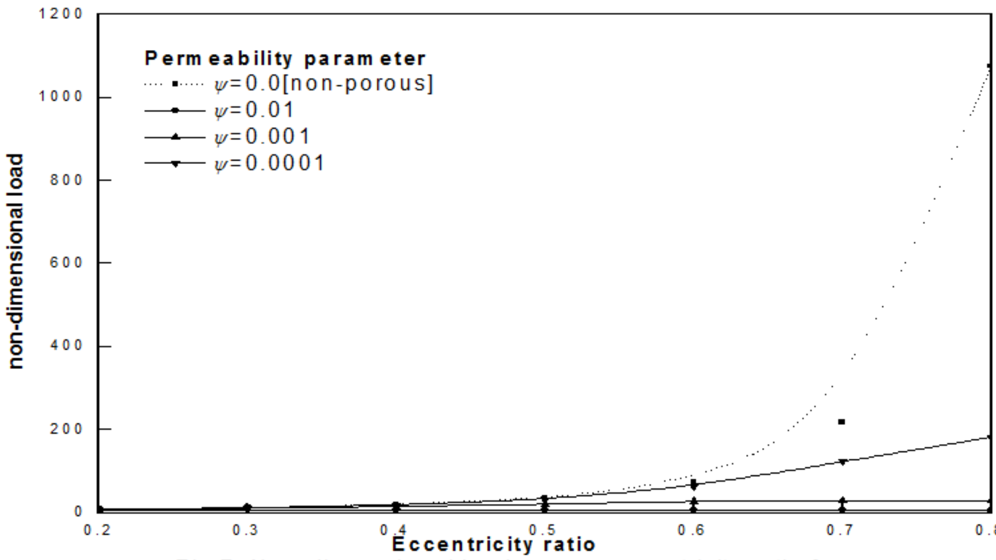

Fig.7. Non-dimensional load versus eccentricity ratio for different permeability parameter

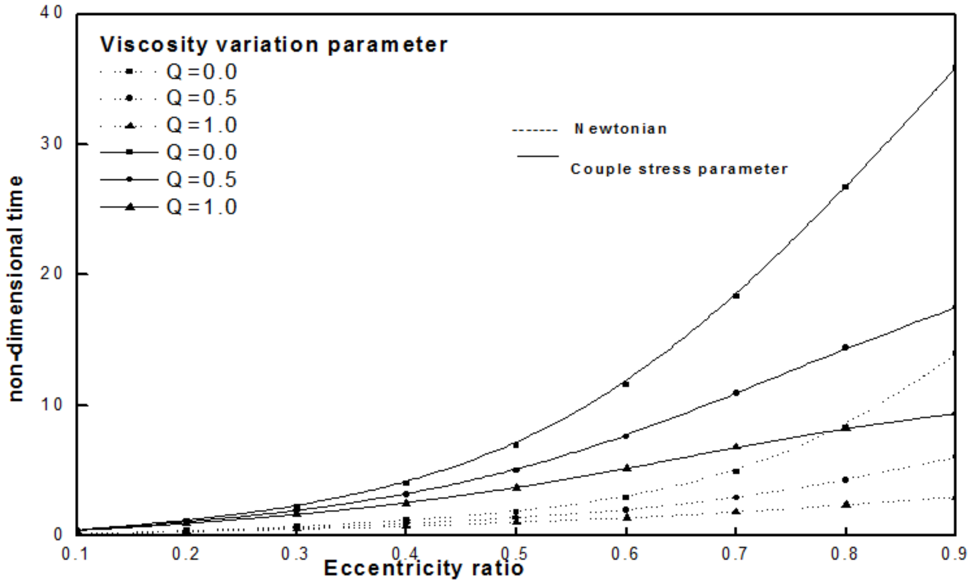

Fig.8. Non-dimensional time versus eccentricity ratio for different viscosity variation parameter

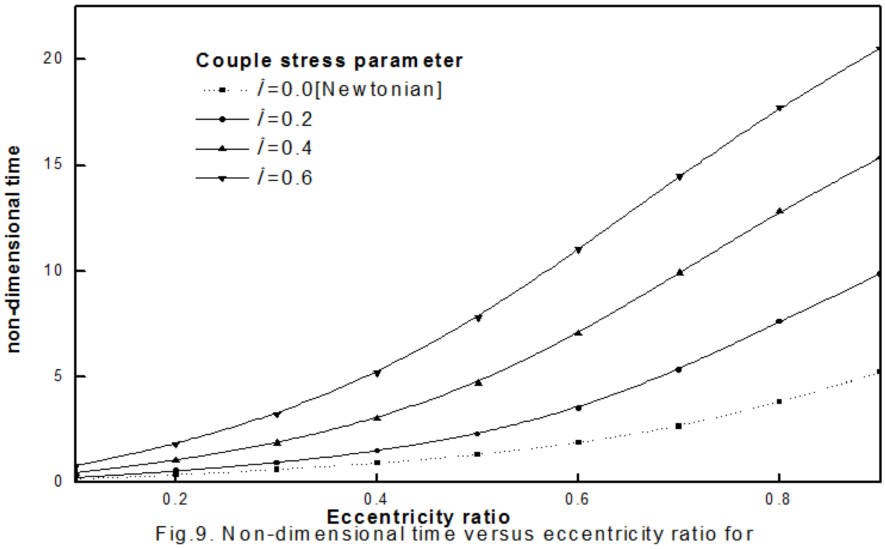

Fig.9. Non-dimensional time versus eccentricity ratio for different couple stress parameter

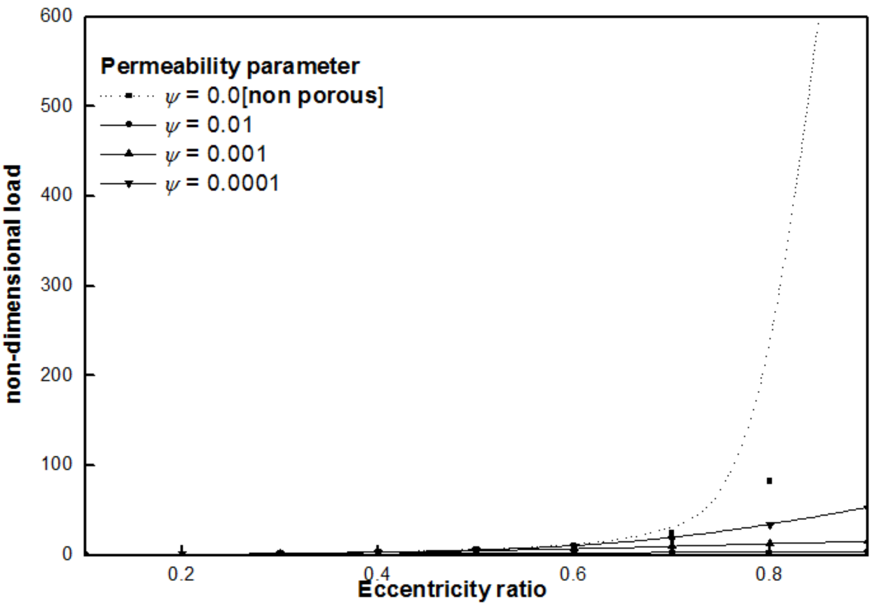

Fig.10. Non-dimensional time versus eccentricity ratio for different permeability parameter