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# NON-NEWTONIAN EFFECTS ON SQUEEZE FILM LUBRICATION BETWEEN POROUS STEPPED CIRCULAR PLATES

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***ABSTRACT***

*The purpose of this paper is to study the effect of non-Newtonian fluid on the squeeze film lubrication between porous stepped circular plates. The problem is analyzed on smooth and porous plates. The modified Reynolds equation is derived for porous stepped circular plates and applied to obtain solution of squeeze film characteristics. The present paper traces out the comparison of classical Newtonian lubricant case with non-Newtonian micropolar fluids and are found to enhance the load carrying capacity and lengthen the approaching the time of porous stepped circular plates. The load capacity decreases as the step height increases. The results obtained are comparable with standard results and are shown in terms of graphs. The squeeze film time is lengthened for the micropolar lubricants as compared to the corresponding Newtonian case.*

***Key Words***: *Stepped circular Plates, Micropolar fluid, Porous, Squeeze film.*

## 1. INTRODUCTION

Self lubricated porous bearings, in which pores are impregnated with oil, are extensively used in industrial applications where their low cost makes them economically viable as well as periodic lubrication is restricted or impractical. Porous bearings are usually made from compressed and sintered metal powders (bronze, iron and stainless steel). The sintering process produces a porous structure that can absorb lubricating oil. With the rotation of the shaft, the oil in the pores comes out to lubricate the bearings and return into pores when it stops. Because of these practical aspects, there have been numerous studies on the performance characteristics of such bearing [1-3]. But these studies were confined to Newtonian lubricants.

Recently, applications of squeeze-film technology shows great importance in many areas of applied science and industrial engineering, such as machine elements, automotive components, animal joints, matching gears, wet-clutch plates. In general, research of squeeze film characteristics concentrates attentions on the use of Newtonian lubricants. For example, by Hays [4], Hamrock [5], Abell and Ames[6], Rashidi *et.al*.[7].

The classical continuum theory could not justify the whole description of flow behaviour of fluids with additives which lead to the development of microcontinuum theory for fluids. “micropolar fluids” is the one such theory attributed to Eringen [8,9]. The main characteristic of these theory micropolar fluids is the presence of suspended rigid micro structured particles. Two independent kinematic vector fields are introduced in this theory, namely the vector field representing the translation velocity of the fluid particles and secondly the vector field representing the angular velocities of the particles i.e. the micro rotation vector. This theory leads to provide a model for lubricants containing suspended additive particles.

Several investigators used the miropolar fluid theory for the study of several bearing systems such as slider bearing[10], journal bearings [11,12], squeeze film bearings [13,14] and porous bearings [15,16] and have found some advantages of micropolar fluids over the Newtonian fluids such as increased load carrying capacity and increased time of approach for squeeze film bearings.

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Hence, in this paper an attempt has been made to study of micropolar fluid effect on squeeze film characteristics between stepped circular plates.

## 2. MATHEMATICAL FORMULATION OF THE PROBLEM

The physical configuration of the problem is as shown in the figure 1. where the upper plate approaching the lower with a normal velocity V. The lubricant in the film region and that in the porous region is modelled as Eringen's [9] micropolar fluid. The assumptions of hydrodynamic lubrication are made:

a) the lubricant is micropolar fluid;
b) the bearing surface (y=0) is porous.

The basic equations governing the flow of micropolar lubricants[9] under the usual assumptions of lubrication theory for thin films [12] are

Conservation of linear momentum:

$$\left(\mu+\frac{\chi}{2}\right)\frac{\partial^2 u}{\partial y^2}+\chi\frac{\partial v_3}{\partial y}-\frac{\partial p}{\partial r}=0\ , \tag{1}$$

$$\frac{\partial p}{\partial y}=0\,. \tag{2}$$

Conservation of angular momentum:

$$\gamma\frac{\partial^2 v_3}{\partial y^2}-2\,\chi\, v_3-\chi\frac{\partial u}{\partial y}=0\ . \tag{3}$$

Conservation of mass:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial v}{\partial y}=0 \tag{4}$$

Where $(u,v)$ are the velocity components of the lubricant in the *r* and *y* directions, respectively, and $v_3$ is micro rotational velocity component, $\chi$ is the spin viscosity and $\gamma$ is the viscosity coefficient for micropolar fluids and $\mu$ is the Newtonian viscosity coefficient.

The flow of micropolar lubricants in a porous matrix is governed by the modified Darcy's law, which account for the polar effects is given by [16]

$$\vec{q}^{\,*}=\frac{-k}{(\mu+\chi)}\nabla\, p^* \tag{5}$$

Where $\vec{q}^{\,*}=\left(u^*,v^*,w^*\right)$ is the modified Darcy's velocity vector, $k$ is the permeability of the porous matrix and $p^*$ is the pressure in the porous region. Due to continuity of fluid in the porous matrix, $p^*$ satisfies the Laplace Equation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p^*}{\partial r}\right)+\frac{1}{r}\frac{\partial^2 p^*}{\partial\theta^2}+\frac{\partial^2 p^*}{\partial y^2}=0 \tag{6}$$

The relevant boundary conditions are

(a) at the upper surface $(y=h)$

$$u=0,\ v=-V\ ,\ v_3=0 \tag{7a}$$

(b) at the bearing surface $(y=0)$

$$u=0,\ v=v^*\ ,\ v_3=0 \tag{7b}$$

## 3. SOLUTION OF THE PROBLEM

The solution of equations (1) - (3) subject to the corresponding boundary conditions given in the equations (7a) and (7b) is obtained in the form.

$$u = \frac{1}{\mu}\left(\frac{y^2}{2}\frac{\partial p}{\partial r} + A_{11}\, y\right) - \frac{2N^2}{m} \times \left[A_{21}\sinh(my) + A_{31}\cosh(my)\right] + A_{41} \tag{8}$$

$$v_3 = A_{21}\cosh(my) + A_{31}\sinh(my) - \frac{1}{2\mu}\left(y\frac{\partial p}{\partial r} + A_{11}\right) \tag{9}$$

where

$$A_{11} = 2\mu A_{21}\ ,$$

$$A_{21} = \frac{A_{31}\sinh(mh) - \dfrac{h}{2\mu}\dfrac{\partial p}{\partial r}}{1-\cosh(mh)}\ ,$$

$$A_{31} = \frac{h}{2\mu}\frac{\partial p}{\partial r}\left[\frac{h}{2}\left(\cosh(mh)-1\right) + h - \frac{N^2}{m}\sinh(mh)\right] \times \frac{1}{A_5}\ ,$$

$$A_{41} = \frac{2N^2}{m} A_{31}\ ,$$

$$A_5 = \frac{h}{\mu}\left[\sinh(mh) - \frac{2N^2}{mh}\left(\cosh(mh)-1\right)\right],$$

in which

$$m = \frac{N}{l}\ ,\quad N = \left(\frac{\chi}{\chi + 2\mu}\right)^{\frac{1}{2}},\qquad l = \left(\frac{\gamma}{4\mu}\right)^{\frac{1}{2}}.$$

Integrating equation (6) with respect to *y* over the porous layer thickness, $\delta$ and using the boundary conditions of solid backing $\left(\dfrac{\partial p^*}{\partial y} = 0\right)$ at $y = -\delta$, we obtain,

$$\left.\frac{\partial p^*}{\partial y}\right|_{y=0} = -\int_{-\delta}^{0}\left(\frac{\partial^2 p^*}{\partial r^2}\right)dy \tag{10}$$

Assuming that, the porous layer thickness, $\delta$ is very small and using the pressure continuity condition $\left(p = p^*\right)$ all the interface $\left(y=0\right)$ of porous matrix and fluid film, equation (10) reduces to

$$\left.\frac{\partial p^*}{\partial y}\right|_{y=0} = -\delta\frac{\partial^2 p}{\partial r^2} \tag{11}$$

Then, the velocity component of the modified Darcy's velocity $v^*$ at the interface $\left(y=0\right)$ is given by

$$\left.v^*\right|_{y=0} = \frac{k\delta}{(\mu+\chi)}\frac{\partial^2 p}{\partial r^2} \tag{12}$$

The modified Reynolds equation is obtained by integrating the equation of continuity (4) with respect to *y* over the film thickness, $h$ and replacing $u$ in equation (3) by their corresponding expression given in equation (8) and also using the boundary conditions for $v$ given in equations (7a) and (7b) in the form.

$$\frac{d}{dr}\left[\left(f\left(N,l,h\right) + \frac{12\mu k\delta}{(\mu+\chi)}\right)\frac{dp}{dr}\right] = -12 r\mu V \tag{13}$$

Where

$$f\left(N,l,h\right) = h^3 + 12 l^2 h - 6 N l h^2 \coth\left(\frac{Nh}{2l}\right)$$

On integrating Equation (13) using the boundary condition

$$\frac{dp}{dr}=0 \quad \text{at} \quad r=0$$

$$\frac{dp_i}{dr}=-\frac{12\,\mu\,V\,r}{f_i(N,l,h_i)+\dfrac{12\mu k\delta}{(\mu+\chi)}} \tag{14}$$

where

$$h_i=h_1 \;\; for \quad 0\le r\le KR\,;$$
$$\quad = h_2 \;\; for \quad KR\le r\le R.$$
$$f_i\left(N,l,h_i\right)=h_i^3+12\,l^2\,h_i-6\,N\,l\,h_i^2\coth\left(\frac{Nh_i}{2l}\right)$$

The relevant boundary conditions for the pressure are

$$p_1=p_2 \qquad \text{at} \qquad x=KR\ , \tag{15a}$$
$$p_2=0 \qquad \text{at} \qquad x=R\ . \tag{15b}$$

Solution of equation (14) subject to the boundary conditions (15a) and (15b) is

$$p_1=6\,\mu V\left(\frac{K^2R^2-r^2}{f_1(N,l,h_1)+\dfrac{12\mu k\delta}{(\mu+\chi)}}+\frac{R^2(1-K^2)}{f_2(N,l,h_2)+\dfrac{12\mu k\delta}{(\mu+\chi)}}\right) \text{and} \tag{16}$$

$$p_2=\frac{6\,\mu\,V}{f_2(N,l,h_2)+\dfrac{12\mu k\delta}{(\mu+\chi)}}\,(R^2-r^2)\,. \tag{17}$$

The load carrying capacity of the bearing, *w* is defined in the form

$$w=\int_0^{KR}p_1\,2\pi r\;\,dr\;+\;\int_{KR}^{R}p_2\,2\pi r\;\,dr\,. \tag{18}$$

Which in nondimensional form

$$W=\frac{wh_2^3}{8\,\mu\pi V R^3}=\left[\frac{K^4}{f_1(N,l^*,H^*)+12\psi\left\{\dfrac{1-N^2}{1-N^2}\right\}}+\frac{1-K^4}{f_2(N,l^*,1)+12\psi\left\{\dfrac{1-N^2}{1-N^2}\right\}}\right] \tag{19}$$

Where $H^*=\dfrac{h_1}{h_2}$ , $\psi=\dfrac{k\delta}{h_2^3}$, $r^*=\dfrac{r}{R}$ and $l^*=\dfrac{l}{h_2}$

$$f_1(N,l^*,H^*)=H^{*3}+12l^{*2}H^*-6NH^{*2}\,l^*\coth\left(\frac{NH^*}{2l^*}\right)$$

$$f_2(N,l^*,1)=1+12l^{*2}-6N\,l^*\coth\left(\frac{N}{2l^*}\right)$$

Writing $V = -\dfrac{dh_2}{dt}$ in equation (19), the squeezing time for reducing the initial film thickness $h_0$ of $h_2$ to a final thickness $h_f$ of $h_2$ is given by

$$t^* = \frac{w h_0^2 t}{8\mu\pi R^3} = \int_{h_f^*}^{1} \left\{ K^4 \left[ f_1(N, h_s^*, h_2^*, l^*) + 12\psi \left\{ \frac{1-N^2}{1+N^2} \right\} \right] + f_2(N, h_2^*, l^*) \right\} dh_2^* \tag{20}$$

Where $f_1(N,h_s^*,h_2^*,l^*) = \left[ \left( (h_2^* + h_s^*)^3 + 12 l^{*2} (h_2^* + h_s^*) - 6N l^* (h_2^* + h_s^*)^2 \coth\left( \dfrac{N h_2^* \left(1 + \dfrac{h_s^*}{h_2^*}\right)}{2 l^*} \right) \right) \right]^{-1}$,

$$f_2(N, h_2^*, l^*) = \frac{1-K^4}{\left( h_2^{*3} + 12 l^{*2} h_2^* - 6N h_2^{*2} l^* \coth\left( \dfrac{N h_2^*}{2 l^*} \right) \right) + 12\psi \left\{ \dfrac{1-N^2}{1+N^2} \right\}},$$

$$h_f^* = \frac{h_f}{h_0}, \quad h_2^* = \frac{h_2}{h_0}, \quad h_s^* = \frac{h_s}{h_0}, \quad l^* = \frac{l}{h_0}.$$

**4. RESULTS AND DISCUSSIONS**

In this paper, the effect of squeeze film lubrication between porous stepped circular plates with micropolar fluid is presented. The microplar fluid is characterized by two non-dimensional parameters such as the coupling number, $N\left( = \dfrac{\chi}{\chi + 2\mu} \right)^{1/2}$ which characterizes the coupling between the Newtonian and microrotational viscosities, the parameter, $l^*\left( = l/h_2 \right)$ in which $l^*$ has the dimension of length and may be considered as chain length of microstructures additives. The parameter $l^*$, characterizes the interaction of the bearing geometry with the lubricant properties. In the limiting case as $l^* \to 0$ the effect of microstructures becomes negligible. The effect of permeability is observed through the non-dimensional permeability parameter, $\psi\left( = \dfrac{k\delta}{h_2^3} \right)$ and it is to be noted that as $\psi \to 0$ the problem reduces to the corresponding solid case and as $l^*, N \to 0$ it reduces to the corresponding Newtonian case.

**4.1 Load carrying capacity**

The variation of non-dimensional load carrying capacity $W$ with $H^*$ for different values of $N$ with $l^* = 0.15, K = 0.6, \psi = 0.01$ is presented in figure 2. The dotted curve in graph corresponds to Newtonian case. As compared with the corresponding Newtonian case, the load carrying capacity increases with increasing values of coupling number *N*. It is observed that the effect micropolar fluid parameter N enhances the load carrying capacity as compared to the Newtonian case. Figure 3 depicts the variation of non-dimensional load carrying capacity $W$ with $H^*$ for different values of $l^*$ with $N = 0.4, K = 0.6, \psi = 0.01$. It is observed that the increasing values of material length $l^*$ increases the load carrying capacity as compared to the Newtonian case. The variation of non-dimensional load carrying capacity $W$ with $H^*$ for different values of $\psi$ with $l^* = 0.15, K = 0.6, N = 0.4$ is presented in figure 4. The effect of $\psi$ is to decrease the load carrying capacity as compared to corresponding solid case $(\psi = 0)$. The adverse effects of the porous facing on the bearing surface can be compensated with the selection of the lubricants with proper microstructure additives. Figure 5 depicts the variation of non-dimensional load carrying capacity $W$ with $H^*$ for different values of *K* with $l^* = 0.15, N = 0.4, \psi = 0.01$. As the value of *K* increases the load carrying capacity decreases.

 

**4.2 Squeeze film Time-height relationship**

The most important characteristics of the squeeze film bearings is the squeeze film time i.e. the time required for reducing the initial film thickness $h_2$ of $h_0$ to a final value $h_f$. The variation of non-dimensional time of approach $t^*$ with $h_f^*$ for different values for $N$ with $l^* = 0.15, K = 0.6, \psi = 0.01$ is presented in figure 6. It is observed that, the presence of micropolar fluid as lubricant have longer response time as compared to the Newtonian case. Figure 7 depicts the variation of non-dimensional time of approach $t^*$ with $h_f^*$ for different values for $l^*$ with $N = 0.5, K = 0.6, \psi = 0.01$. For increasing values of $l^*$ the squeeze film time increases as compared to the Newtonian case. The variation of non-dimensional time of approach $t^*$ with $h_f^*$ for different values for $\psi$ with $l^* = 0.15, K = 0.6, N = 0.5$ is presented in figure 8. The effect of $\psi$ is to decrease the squeeze film time as compared to the corresponding solid case $(\psi = 0)$ Figure 9 depicts the variation of non-dimensional time of approach $t^*$ with $h_f^*$ for different values for $K$ with $N = 0.5, l^* = 0.15, \psi = 0.01$. It is observed that, the response time increases for decreasing values of *K*.

**5. CONCLUSIONS**

This paper predicts the effect of micropolar fluid on the squeeze film lubrication characteristics between porous stepped circular plates on the basis of Eringen's [9] microplar fliud. According to the results computed The effect of micropolar is to increases the squeeze film pressure and the load carrying capacity as compared to the corresponding Newtonian case.The squeeze film time is lengthened for the micropolar lubricants as compared to the corresponding Newtonian case. The presence of porous facing on the bearing surface affects the performance of the bearing. The adverse effects of the porous facing on the bearing surface can be compensated with the selection of the lubricants with proper microstructure additives.

**6. REFERENCES**

1. Murti, P.R.K, (1974), "Analysis of porous slider bearings", Wear, Vol.28, pp.131-134.
2. Kumar, V., (1980), "Friction of a plane porous slider of optimum profile", Wear, Vol.62, pp.417-418.
3. Rouleau, W.T. and Steiner, L.T., (1974), "Hydrodynamic porous journal bearing: part-I Finite full bearings," Transactions of ASME Journal of Lubrication Technology, Vol.96 No.5, pp.346-353.
4. Hays, D.F., (1963), "Squeeze films for rectangular plates," Journal of Basic Engineering, Vol.85, pp.243-246.
5. Hamrock, B.J., (1994), "Fundamental of fluid film lubrication," McGraw-Hill, New york.
6. Abell, M.L., Ames, W.F., (1992), "Symmetry reduction of Reynolds equation to film Lubrication," Journal of Applied Mechanics, Vol.59, pp.206-210.
7. Rashidi, M.M., Hamed, S., Saeed, D., (2008), "Analytic Approximate Solutions for Unsteady Two-Dimensional and Axisymmetric Squeezing Flows between Parallel Plates," Mathematical Problems in Engineering, 2008, Article ID 935095, 12 pages.
8. Eringen, A.C., (1966), "Theory of microplar fluids," Journal of Mathematics and Mechanics, Vol.16, pp.1-18.
9. Eringen, A.C., (1964), "Simple microfluids," International Journal of Engineering Science, Vol.2, pp.205-217.
10. Datta A.B., (1972), "Pivoted slider bearing with convex pad surface in micropolar fluid," Japanese Journal of Applied Physics, Vol.11, pp.98–102.
11. Das., Guha S. K. and Chattapadhyay A. K., (2004), "Theoretical analysis of Stability characteristics of hydrodynamic journal bearings with micropolar fluid," Proc. Inst. Mech. Enggr. Journal of Engineering Tribology (part-J), Vol.258, pp.45-56.
12. Naduvinamani BN and Kashinath B., (2008), "Surface roughness effects on the static and dynamic behaviour of squeeze film lubrication of short journal bearings with micropolar fluids," Proc. Inst. Mech. Enggr. Journal of Engineering Tribology (part-J), Vol.222, pp.121–131.
13. Prakash J and Sinha P., (1976), "Cyclic squeeze films in micropolar fluid Lubricated journal bearing," ASME Journal of Lubrication Technology, Vol.98, pp.412–417.
14. Prakash J. and Sinha., (1976), "Squeeze film theory for micropolar fluids," Journal of Lubrication Technology, pp.139-144.
15. Verma P.D.S., Agrawal V.K. and Bhat S.K., (1979), "Porous inclined slider bearing lubricated with micropoplar fluid," Wear, Vol.53, pp.101–106.
16. Agrawal V.K. and Bhat S.B., (1980), "Porous pivoted slider bearings lubricated with a micropolar fluid," Wear, Vol.61, pp.1–8.

**NOMENCLATURE**

| | |
|---|---|
| $H^*$ | non-dimensional mean film thickness $\left(=\dfrac{h_1}{h_2}\right)$. |
| $h_1$ | maximum film thickness |
| $h_2$ | minimum film thickness |
| $h_s^*$ | step height $\left(=\dfrac{h_2}{h_0}\right)$. |
| $KR$ | position of the step $0<KR<R$. |
| $k$ | permeability of the porous matrix |
| $l$ | characteristic length of the polar suspension $\left(=\left(\dfrac{\gamma}{4\mu}\right)^{\frac{1}{2}}\right)$ |
| $l^*$ | non-dimensional form of $l\left(=\dfrac{l}{h_2}\right)$ |
| $N$ | coupling number $\left(=\left(\dfrac{\chi}{\chi+2\mu}\right)^{\frac{1}{2}}\right)$ |
| $p$ | pressure in the film region. |
| $p_1$ | fluid film pressure in the region $0\le x\le KR$. |
| $p_2$ | fluid film pressure in the region $KR\le x\le R$. |
| $R$ | radius of the circular plate |
| $t$ | time of approach |
| $t^*$ | non-dimensional time of approach $\left(=\dfrac{w h_0^2 t}{8\mu\pi R^3}\right)$. |
| $V$ | velocity approach. |
| $w$ | load carrying capacity |
| $W$ | non-dimensional load carrying capacity $\left(=\dfrac{W h_2^3}{8\mu\pi V R^3}\right)$. |
| $\eta$ | lubricant couple stress constant |
| $\mu$ | lubricant viscosity |
| $\psi$ | permeability parameter |
| $\chi$ | spin viscosity |
| $\gamma$ | viscosity co-efficient for micropolar fluids |
| $\mu$ | viscosity co-efficient |

 

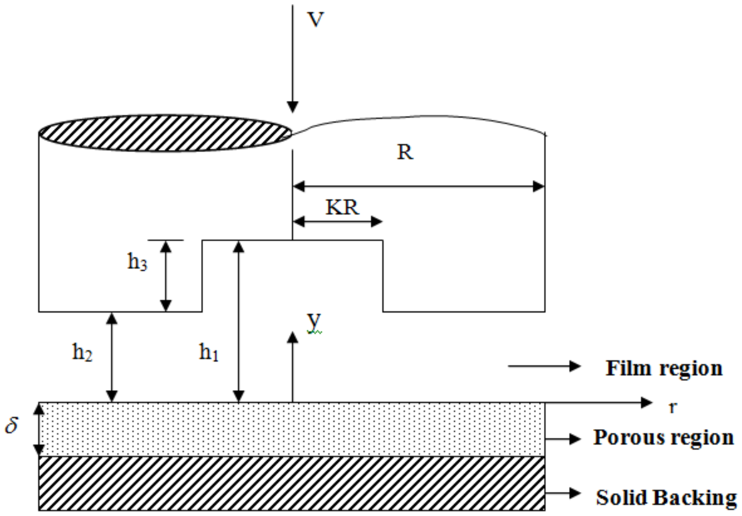


**Figure-1:** Squeeze film between porous stepped circular plates

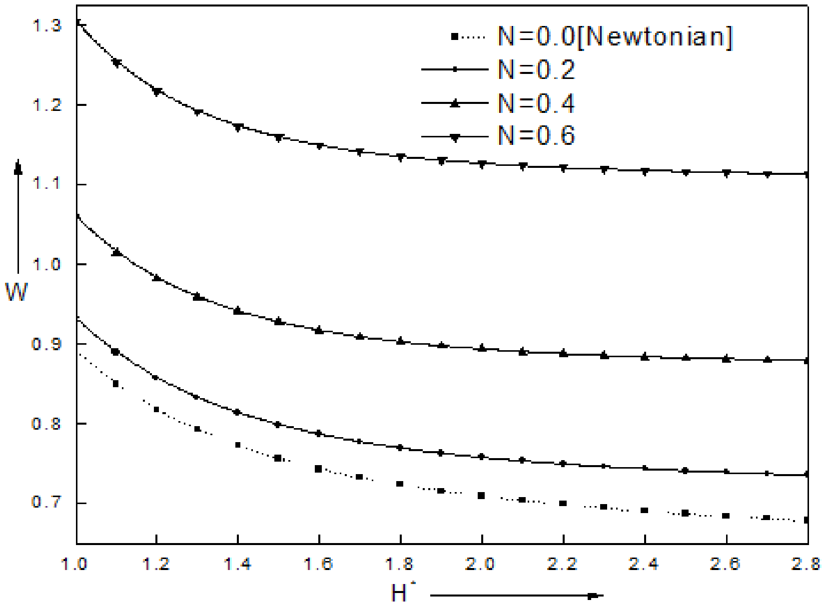


Figure 2. Variation of non-dimensional W with H* for different values of N with $l^* = 0.15$, $\psi = 0.01$, $K = 0.6$.

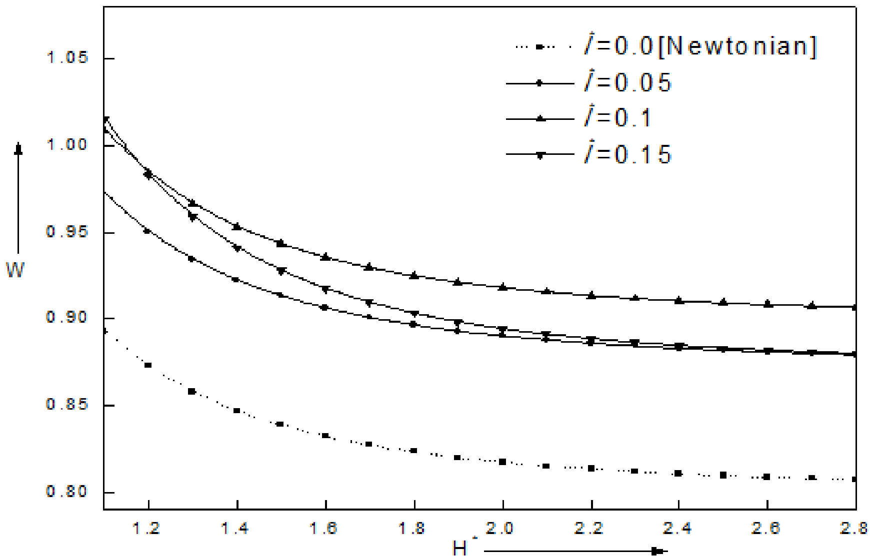


Fig 3. Variation of non-dimenstional W with H* for different values of l* with $\psi = 0.01$, $K = 0.6$, $N = 0.4$.

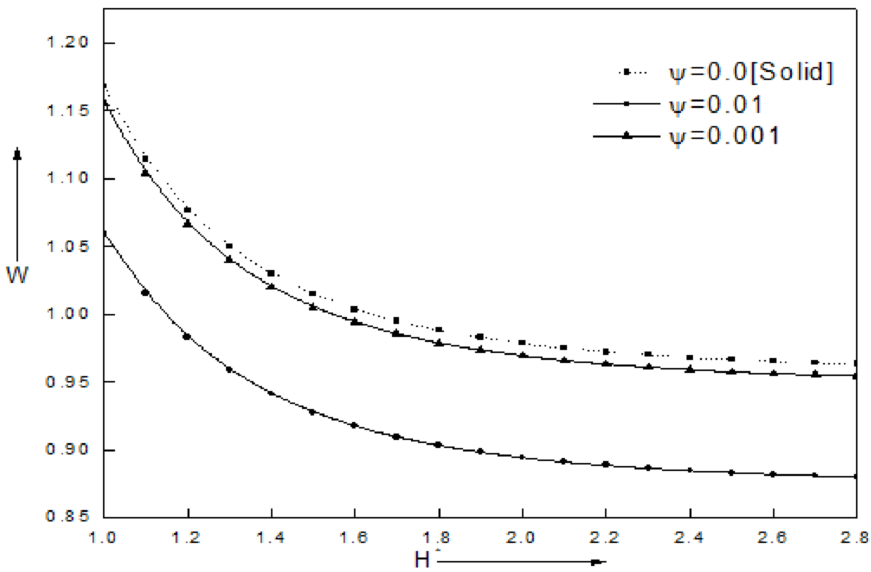


Fig.4 Variation of non-dimensional W with H* for different values of ψ with K = 0.6, N = 0.4, l* = 0.15.

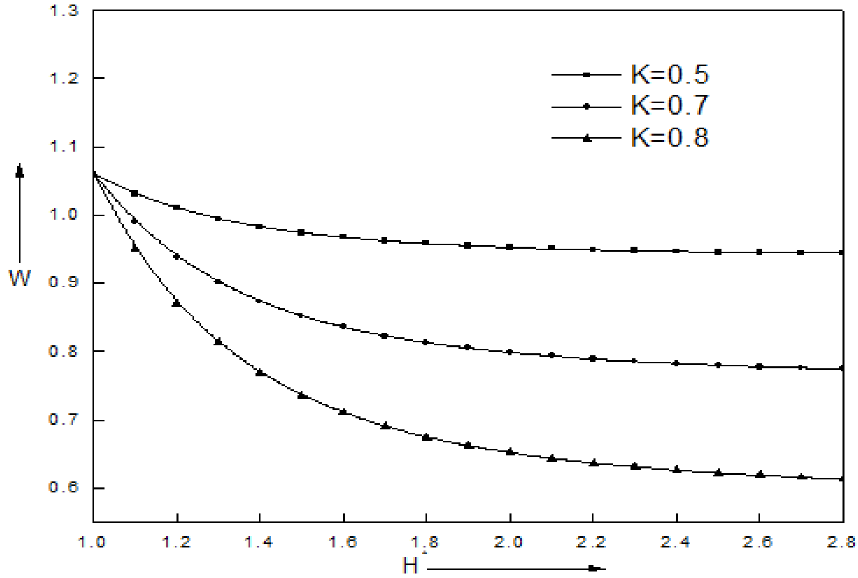


Fig 5. Variation of non-dimensional W with H* for different values of K with ψ = 0.01, N = 0.4, l* = 0.15.

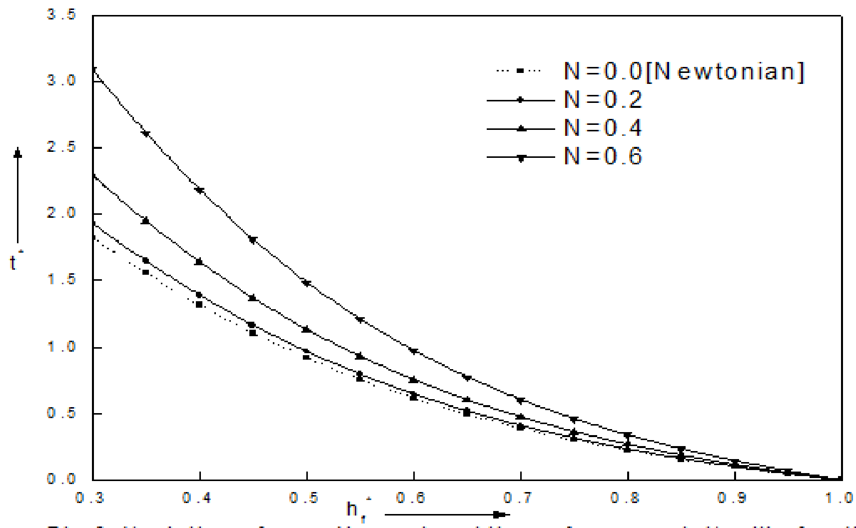


Fig 6. Variation of non-dimensional time of approach t* with for different values of N with ψ = 0.01, l* = 0.15, K = 0.6.

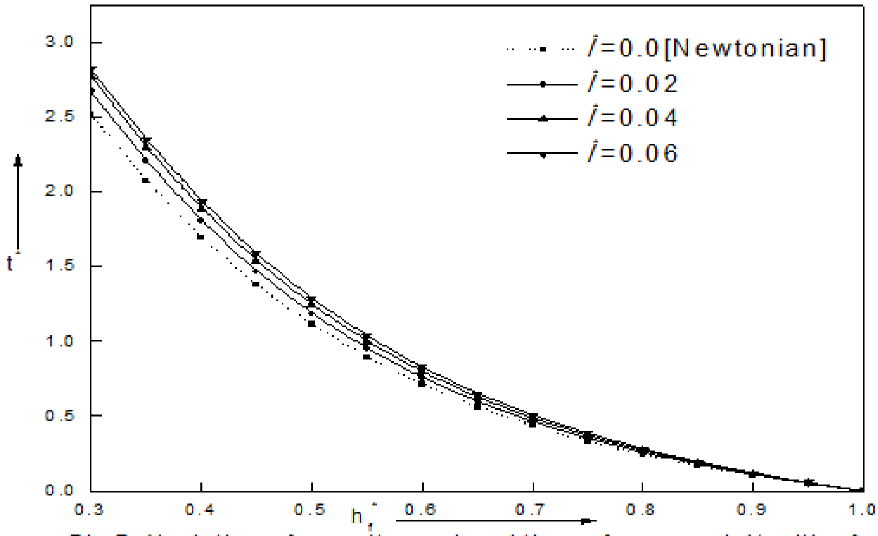


Fig 7. Variation of non-dimensional time of approach t* with for different values of l* with K = 0.6, ψ = 0.01, N = 0.5.

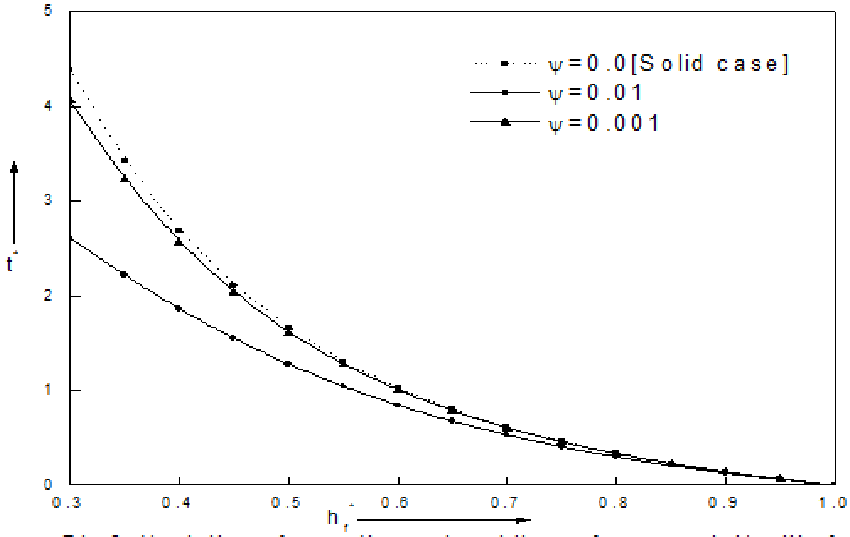


Fig 8. Variation of non-dimensional time of approach t* with for different values of ψ with K = 0.6, N = 0.5, l* = 0.5.

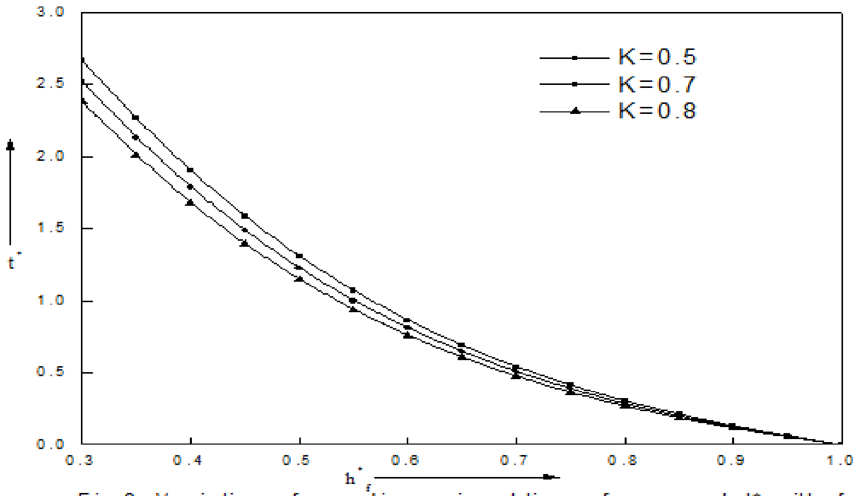


Fig 9. Variation of non-dimensional time of approach t* with for different values of K with ψ = 0.01, N = 0.5, l * = 0.15.

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