

ALGEBRAIC REPRESENTATION OF VINCULUM AND GENERALIZED RULE FOR VINCULUM STRUCTURE

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ABSTRACT

The term ‘Vinculum’ plays an important role in Vedic Mathematics. It makes the digits over 5 to less than 5. So calculation becomes easier due to less probability of appearance of carry number. In the present study the concept of vinculum has been expressed by framing generalized algebraic structure with suitable logic.

Keywords: Vinculum, Algebraic Structure, Vedic mathematics, Bhāratī Kṛṣṇa Tīrthajī Mahārāja.

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1. INTRODUCTION

Mathematical ideas written in Veda were collected and arranged in a systematic manner by Jagadguru Śaṅkarācārya Śrī Bhāratī Kṛṣṇa Tīrthajī Mahārāja of Govardhana Maṭha, Puri, India (1884-1960).

Prof. Ginsburg says, “The Hindu notation was carried to Arabia about 770 A.D by a Hindu scholar named Kaṅka who was invited from Ujjain to the famous Court of Baghdad by the Abbaside Khalif Al-Mansur. Kaṅka taught Hindu astronomy and mathematics to the Arabian scholars; and, with his help, they translated into Arabic the Brahma-Sphuta-Siddhānta of Brahma Gupta. The recent discovery by the French savant M.F. Nau proves that the Hindu numerals were well-known and much appreciated in Syria about the middle of the seventh century A.D”

In Vedic Mathematics ‘Vinculum’ is an ingenious device to reduce single digits larger than 5. A digit larger than five creates problem for calculation by creating carry number. If all the digits are less than five then calculation becomes easier and probability of appearing the carry number is very less. With this idea the concept of vinculum was entered in Vedic mathematical calculation of numbers.

Kenneth R Williams (2005), in his book ‘Vedic Mathematics Teacher’s Manual’ has given the concept of vinculum by giving particular examples.

e.g., 49 is very close to 50 and the digit 9 of 49 is greater than 5, so it may be written that

$$49 = 50 - 1 = 5\overline{1}$$

Similarly

$$7\overline{2} = 70 - 2 = 68$$

Here, we shall try to make a generalized algebraic structure of vinculum by giving suitable logic.

2. ALGEBRAIC STRUCTURE OF VINCULUM OF TWO DIGIT NUMBER

Let, ab be a given number. Where, $a, b \in \mathbb{N} \cup \{0\}$ and \mathbb{N} = Set of natural numbers.

Here, $ab = 10 \times a + 1 \times b$

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Vinculum on a digit of a number represents that the digit becomes negative. i.e., $\bar{a} = -a$

$$\begin{aligned}\therefore \overline{ab} &= 10 \times a + 1 \times (-b) \\ &= 10 \times (a - 1) + 1 \times (10 - b) \\ &= mn, \text{ where } mn = 10 \times m + 1 \times n; \text{ here } m = a - 1, n = 10 - b\end{aligned}$$

Working Process: $\overline{ab} = (a - 1) / (10 - b)$
 $= mn$; here $m = a - 1, n = 10 - b$

Example 1:

$$\begin{aligned}\text{(i)} \quad 5\bar{1} &= (5 - 1) / (10 - 1) \\ &= 49 \\ \text{(ii)} \quad 7\bar{2} &= (7 - 1) / (10 - 2) \\ &= 68\end{aligned}$$

Conversely, if ab is a number and b is greater than 5 then it may be transferred in the vinculum form as follow -

$$\begin{aligned}ab &= 10 \times a + 1 \times b \\ &= 10 \times (a + 1) + 1 \times (b - 10) \\ &= p\bar{q}; \text{ where } p\bar{q} = 10 \times p + 1 \times \bar{q}\end{aligned}$$

Here, $p = a + 1, \bar{q} = -q$ and $q = 10 - b$

Working Process: $ab = (a + 1) / \overline{10 - b}$

Example 2:

$$\begin{aligned}\text{(i)} \quad 49 &= (4 + 1) / \overline{10 - 9} \\ &= 5\bar{1} \\ \text{(ii)} \quad 46 &= (4 + 1) / \overline{10 - 6} \\ &= 5\bar{4}\end{aligned}$$

3. ALGEBRAIC STRUCTURE OF VINCULUM OF THREE OR MORE DIGIT NUMBERS

We may remove vinculum from any place – Unit or Tens place of three digit numbers,

(i) Removal of vinculum from unit place

$$\begin{aligned}\therefore ab\bar{c} &= 100 \times a + 10 \times b + 1 \times (-c) \\ &= 100 \times a + 10 \times (b - 1) + 1 \times (10 - c)\end{aligned}$$

It is seen that digit in hundreds place remains unchanged but digits in tens place and Unit place are changed by their mutual calculation.

$$\begin{aligned}\therefore ab\bar{c} &= a / (b - 1) / (10 - c) \\ &= amn; \text{ where } m = b - 1, n = 10 - c\end{aligned}$$

Example 3: Remove vinculum from $34\bar{1}$

$$\begin{aligned}\text{Solution: } 34\bar{1} &= 3 / (4 - 1) / (10 - 1) \\ &= 339\end{aligned}$$

Here, digit in hundreds place remains unchanged.

(ii) Removal of vinculum from tens place

$$\begin{aligned}\therefore a\bar{b}c &= 100 \times a + 10 \times (-b) + 1 \times c \\ &= 100 \times (a - 1) + 10 \times (10 - b) + 1 \times c \\ &= (a - 1) / (10 - b) / c\end{aligned}$$

Here, digit in the Unit place remains unchanged but digits in hundreds and tens place are changed by their mutual calculation.

Example 4: Remove vinculum from $2\bar{3}4$

$$\begin{aligned}\text{Solution: } 2\bar{3}4 &= (2 - 1) / (10 - 3) / 4 \\ &= 174\end{aligned}$$

Here, it is seen the digit 4 in unit place is unchanged.

Conversely, If abc is a number then we may write this number in vinculum form by imposing vinculum is unit or tens place.

(i) Vinculum in Unit place

$$\begin{aligned}\therefore abc &= 100 \times a + 10 \times b + 1 \times c \\ &= 100 \times a + 10 \times (b + 1) + 1 \times (c - 10) \\ &= a / (b + 1) / \overline{10 - c} \\ &= \overline{am\overline{n}}; \text{ Where, } m = b + 1, n = 10 - c\end{aligned}$$

Here, hundreds place remains unchanged.

Example 5:

(a) Put vinculum in unit place for the number 328

Solution: $328 = 3 / (2 + 1) / \overline{10 - 8}$
 $= 33\overline{2}$, Hundreds place remains unchanged.

(b) Put vinculum in unit place for the number 539

Solution: $539 = 5 / (3 + 1) / \overline{10 - 9}$
 $= 54\overline{1}$

(ii) Vinculum in tens place

$$\begin{aligned}\therefore abc &= 100 \times a + 10 \times b + 1 \times c \\ &= 100 \times (a + 1) + 10 \times (b - 10) + 1 \times c \\ &= (a + 1) / \overline{10 - b} / c \\ &= \overline{pqc}; \text{ Where, } p = a + 1, q = 10 - b\end{aligned}$$

Here, unit place remains unchanged.

Example 6: Put vinculum in tens place for the number 593

$$\begin{aligned}\therefore 593 &= (5 + 1) / \overline{10 - 9} / 3 \\ &= 6\overline{1}3\end{aligned}$$

(iii) Theorem: $\overline{abc} = \overline{a\overline{b}c}$

$$\begin{aligned}\text{L.H.S} &= \overline{abc} \\ &= 100 \times a + \overline{10 \times b + 1 \times c} \\ &= 100 \times a - (10 \times b + 1 \times c) \\ &= 100 \times a - (10 \times b) - (1 \times c)\end{aligned}$$

$$\begin{aligned}\text{Again, R.H.S} &= \overline{a\overline{b}c} \\ &= 100 \times a + 10 \times (-b) + 1 \times (-c) \\ &= 100 \times a - (10 \times b) - (1 \times c)\end{aligned}$$

\therefore L.H.S = R.H.S, **Hence proved.**

(iv) Some solved problems

1. Remove vinculum signs from followings by using formula $\overline{a\overline{b}} = (a - 1) / (10 - b)$ and $\overline{a\overline{b}c} = \overline{a\overline{b}c}$

(i) $893\overline{1}$ (ii) $56\overline{2}4$ (iii) $8\overline{2}43$ (iv) $74\overline{2}3$ (v) $74\overline{2}3$ (vi) $74\overline{2}3$

Answer: (i) $893\overline{1} = 8 / 9 / (3 - 1) / (10 - 1)$
 $= 8929$
(ii) $56\overline{2}4 = 5 / (6 - 1) / (10 - 2) / 4$
 $= 5584$
(iii) $8\overline{2}43 = (8 - 1) / (10 - 2) / 4 / 3$
 $= 7843$
(iv) $74\overline{2}3 = 74\overline{2}3$, ($\therefore \overline{ab} = \overline{a\overline{b}}$)
 $= 7 / (4 - 1) / (10 - 2) / \overline{3}$
 $= 7 / 3 / 8 / \overline{3}$
 $= 7 / 3 / (8 - 1) / 10 - 3$
 $= 7377$
(v) $74\overline{2}3 = 74\overline{2}3$, ($\therefore \overline{ab} = \overline{a\overline{b}}$)
 $= (7 - 1) / (10 - 4) / \overline{2} / 3$
 $= 6 / 6 / \overline{2} / 3$
 $= 6 / (6 - 1) / (10 - 2) / 3$
 $= 6585$

$$\begin{aligned}
 \text{(vi) } \overline{7423} &= \overline{742\overline{3}}, (\because \overline{abc} = \overline{a\overline{b\overline{c}}}) \\
 &= (7-1) / (10-4) / \overline{2} / \overline{3} \\
 &= 6 / 6 / \overline{2} / \overline{3} \\
 &= 6 / (6-1) / (10-2) / \overline{3} \\
 &= 6 / 5 / 8 / \overline{3} \\
 &= 6 / 5 / (8-1) / (10-3) \\
 &= 6577
 \end{aligned}$$

2. Put the vinculum in (i) Unit place (ii) Tens place (iii) Hundreds place (iv) Both Unit and Tens place (v) Both Tens and Hundreds place (vi) Unit, Tens and Hundreds place of the number 2676 by using formula

$$ab = (a+1) / \overline{10-b}.$$

$$\begin{aligned}
 \text{(i) } 2676 &= 2 / 6 / (7+1) / \overline{10-6} \\
 &= 268\overline{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } 2676 &= 2 / (6+1) / \overline{10-7} / 6 \\
 &= 27\overline{3}6
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) } 2676 &= (2+1) / \overline{10-6} / 7 / 6 \\
 &= 3\overline{4}76
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } 2676 &= 2 / (6+1) / \overline{10-7} / 6 \\
 &= 2 / 7 / \overline{3} / 6 \\
 &= 2 / 7 / (\overline{3}+1) / \overline{10-6} \\
 &= 2 / 7 / \overline{2} / \overline{4} \\
 &= 27\overline{2} \overline{4} \\
 &= 2724
 \end{aligned}$$

$$\begin{aligned}
 \text{(v) } 2676 &= (2+1) / \overline{10-6} / 7 / 6 \\
 &= 3 / \overline{4} / 7 / 6 \\
 &= 3 / (\overline{4}+1) / \overline{10-7} / 6 \\
 &= 3 / \overline{3} / \overline{3} / 6 \\
 &= 3\overline{3} \overline{3}6 \\
 &= 33\overline{3}6
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi) } 2676 &= (2+1) / \overline{10-6} / 7 / 6 \\
 &= 3 / \overline{4} / 7 / 6 \\
 &= 3 / (\overline{4}+1) / \overline{10-7} / 6 \\
 &= 3 / \overline{3} / \overline{3} / 6 \\
 &= 3 / \overline{3} / (\overline{3}+1) / \overline{10-6} \\
 &= 3 / \overline{3} / \overline{2} / \overline{4} \\
 &= 3\overline{3} \overline{2} \overline{4} \\
 &= 3324
 \end{aligned}$$

4. CONCLUSION

We have framed algebraic structure of vinculum number and generalized rule for removing and imposing vinculum. Iteration method of structure has been given to make computerised programme. As idea of vinculum makes the digits over 5 to less than 5, So, in any number only some of the digits 0, 1, 2, 3, 4, 5 are found. Hence calculation becomes easier.

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