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NAGENDRAM Γ-SEMI SUB NEAR-FIELD SPACES OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD

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ABSTRACT

In this paper, In depth study makes me to introduce the Nagendram Gamma-semi sub near-field spaces in Γ-near-field space over a near-field, Dr. N V Nagendram together investigate the related properties of Left Invariant vector Γ-semi sub near-field spaces of a Γ-near-field space, Nagendram Γ-semi sub near-field space Homomorphisms of a Γ-near-field space over near-field and Topological Nagendram Γ-semi sub near-field spaces of a Γ-near-field space over near-field.

Keywords: *Γ-near-field space; Γ-Semi normal sub near-field space of Γ-near-field space; Semi near-field space of Γ-near-field space.*

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SECTION 1: NAGENDRAM Γ-SEMI SUB NEAR-FIELD SPACES OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD

1.1. Nagendram Γ-semi sub near-field spaces**.**

Definition 1.1.1: A Nagendram Γ-semi sub near-field space N is a C^* manifold with a Γ-semi sub near-field space structure so that the near-field space operations are smooth. More precisely, the maps m : N X N → N (multiplication)
inv : N → N (inversion) are C^∞ maps of manifolds.

Example 1.1.2: Take N = R with map m(a,b) = a + b, inv(a) = – a for all a, b ∈ R. then N is an abelian Nagendram Γ-semi sub near-field space.

Example 1.1.3: Let V be a finite dimensional vector Nagendram Γ-semi sub near-field space over R. Then, V has a canonical manifold structure, and is a Γ-near-field space over near-field under vector addition. It can be shown that vector addition and negation are smooth, so V is a Nagendram Γ-semi sub near-field space.

Example 1.1.4: Let M_n (N) denote the set of all Γ-near-field spaces over near-field n x n matrices over R. define NN(n, R) = {A ∈ M_n(N) / |A| ≠ 0} then NN(b, R) is a Γ-near-field space over near-field under the operations m(A,B) = AB and inv(A) = $A^{-1} = \dfrac{adj\ A}{|A|}$ where adj. A denotes the Adjoint of A. As these operations are smooth on GL(n, R) considered as a sub manifold of R^{n^2}, NN(n, R) is a Nagendram Γ-semi sub near-field space called the real general linear Nagendram Γ-semi sub near-field space. Completely analogously, we have the Nagendram Γ-semi sub near-field space NN(n, C) = {A = M_n(C) / |A| ≠ 0} the complete general linear Nagendram Γ-semi sub near-field space.

Example 1.1.5: the orthogonal Γ-semi sub near-field space O (n) = {A ∈ $M_n(R)/AA^T = I$} is a Nagendram Γ-semi sub near-field space as a Γ-semi sub near-field space and sub manifold of NN(n, R).

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We shall now state the following closed graph theorem without proof

Theorem 1.1.6: Let G be the Nagendram Γ-semi sub near-field space and H < G a closed graph Γ-semi sub near-field space of G. Then, H is a Nagendram Γ-semi sub near-field space in the induce topology as an embedded sub manifold of G.

Corollary 1.1.7: If G and G′ are Nagendram Γ-semi sub near-field spaces over a Γ-near-field space over near-field and $\phi : G \to G'$ is a continuous homomorphism, then ϕ is smooth.

Example 1.1.8: The following Γ-near-field spaces are Nagendram Γ-semi sub near-field spaces :
(a) The real special linear Γ-near-field space SL(n, R) = {A ∈ NN(n, R) / |A| = 1}
(b) The complex special linear Γ-near-field space SL(n, C) = {A ∈ NN(n, C) / |A| = 1}
(c) The special orthogonal Γ-near-field space SO(n, R) = S L(n, R) ∩ O (n).
(d) The unitary Γ-near-field space U (n) = {A ∈ NN(n, C) / $AA^* = 1$} where A^* denotes the Hermitian transpose of A.
(e) The special unitary Γ-near-field space SU(n) = U(n) ∩ SL (n, C).

Example 1.1.9: we now define the Euclidean Γ-near-field space of rigid motions. *Euc* (n). Let End (V, W) denote the vector Γ-near-field space of all linear endomorphisms from vector Γ-near-field space V to itself. A near-field space, we have *Euc* (n) = $\{T \in End(R^n) / \|Tx - Ty\| = \|x - y\| \ \forall\ x, y \in R^n\}$ where $\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2}$. Now one can check that if T ∈ Euc (n) and T(0) = 0. Then, T ∈ O (n). Then we an write x ↦ Tx – T(0) ∈ O (n) and so T(x) = (T(x) – T(0)) + T(0). This shows that T ∈ R^n x O (n). we can think of *Euc (n)* as a slightly different Γ-near-field space. Write Euc (n) = $\left\{\begin{bmatrix} A & v \\ 0 & 1 \end{bmatrix} / A \in O(n), v \in R^n\right\}$. If we identify R^n with the set of all vector Γ-near-field spaces of the form $\begin{bmatrix} \omega \\ 1 \end{bmatrix}$ with $\omega \in R^n$, then we have $\begin{bmatrix} A & v \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \omega \\ 1 \end{bmatrix} = \begin{bmatrix} A\omega + v \\ 1 \end{bmatrix}$.

Example 1.1.10: Is Euc(n) ≅ R^n x O (n) as Γ-near-field spaces ?

SECTION 2: NAGENDRAM Γ-SEMI SUB NEAR-FIELD SPACE ALGEBRAS OF Γ-SEMI SUB NEAR-FIELD SPACE OVER A NEAR-FIELD.

Definition 2.2.1: A real Nagendram Γ-semi sub near-field space Algebra L is a vector Nagendram Γ-semi sub near-field space of Γ-semi sub near-field space over a near-field N with a linear map called the Nagendram Bracket as below

$$[\cdot,\cdot]: L \ X \ L \to L \text{ and } (X, Y) \mapsto [X, Y]$$

such that for all X, Y, Z ∈ L.

1. [X, Y] = - [Y, X]
2. [X, [Y,Z]] = [[X,Y],Z] + [Y, [X, Z]]

Note 2.2.3: If we write ad(X)Y = [X, Y] then 2) reads ad(X) is a derivation of (L, [,]).

Example 2.2.4: Let L = $M_n(N)$. Then, L is a Nagendram Γ-semi sub near-field space Algebra with the commutator i.e. [X, Y] = XY – YX.

Note 2.2.5: Is obviously one can prove that $M_n(N)$ is a Nagendram Γ-semi sub near-field space Algebra with the commutator bracket.

SECTION 3: LEFT INVARIANT VECTOR Γ-SEMI SUB NEAR-FIELD SPACES OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD

3.1 Left Invariant vector Γ-semi sub near-field spaces of a Γ-near-field space

Definition 3.1.1: Let N be a Nagendram Γ-semi sub near-field space and M a smooth manifold. An action of G on M is a smooth map G X M → M satisfying the two following axioms.

1. I_G . x = x for each x ∈ M
2. g. (g′.x) = (gg′). x for each g, g′ ∈ G, x ∈ M.

Example 3.1.2: Any Nagendram Γ-semi sub near-field space N acts on itself by left multiplication. If a $\in$ N is fixed, we denote this action by $L_a(g) = ag$ for any $g \in$ N. N also acts on itself by right multiplication we denote this by R_a.

Note 3.1.3: L_a or respectively R_a is a diffeomorphism for each a $\in$ N since we have a smooth inverse given by a map $L_a^{-1}(g) = a^{-1} g = L_{a-1}(g)$ for any $g \in$ N.

Note 3.1.4: If N is a Nagendram Γ-semi sub near-field space acts on a manifold M and we write gM : m $\mapsto$ g . m, then we have a map ρ : N → Dif f (M) where m $\mapsto$ gM for each $g \in$ N. Now, $\rho(I_N) = id_M$ and $\rho(g_1 g_2) = \rho(g_1)\rho(g_2)$ so that ρ is a near-field space homomorphism from N to the near-field space of diffeomorphism of M.

Example 3.1.5: Define a map L: N → Dif f(N) ; $g \mapsto L_g$ by $L_g(g') = gg'$. then L_g is a homomorphism for each fixed $g \in$ N and represents the usual left action of N on itself.

Definition 3.1.6: A vector Nagendram Γ-semi sub near-field space of Γ-semi sub near-field space over a near-field X on a Nagendram Γ-semi sub near-field space N is left invariant if $(dL_g)(X(x)) = X(L_g(x)) = X(gx)$ for each x, $g \in$ N.

We will now use left invariant vector Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field to show that the tangent Γ-semi sub near-field space of N at the identity, denoted by T_1N is a Nagendram Γ-semi sub near-field space Algebra.

Proposition 3.1.7: Let N be a Nagendram Γ-semi sub near-field space of Γ-semi sub near-field space over a near-field. Then, the vector Nagendram Γ-semi sub near-field space of Γ-semi sub near-field space over a near-field of all left invariant vector Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field on N is isomorphic (as a vector Nagendram Γ-semi sub near-field space) to T_1N.

Proof: Since X is left invariant the following diagram commutes

$$\begin{tikzcd} TN \arrow[r, "dL_g"] & TN \\ N \arrow[u, leftarrow, "X"'] \arrow[r, "L_g"'] & N \arrow[u, leftarrow, "X"] \end{tikzcd}$$

So that $X(a) = (dL_a)_1(X(1))$ for all a $\in$ N. Denote by $\Gamma(TN)^N$ the set of all left invariant vector Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field on N. Define a map $\varnothing : \Gamma(TN)^N \to T_1N$ by $\varnothing$: X $\mapsto$ X(1). Then, $\varnothing$ is linear and injective since if X, Y $\in \Gamma(TN)^N$ and $\varnothing(X) = \varnothing(Y)$, $X(g) = dL_g(X(1)) = dL_g(Y(1)) = Y(g)$ for each $g \in$ N.

Now, $\varnothing$ is also surjective. For $\nu \in T_1N$, define $X_\nu \in \Gamma(TN)^N$ by $X_\nu(a) = (dL_a)_1(\nu)$ for a $\in$ N. we claim that X_ν is a left invariant vector Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field. Now, X_ν : N → TN is a C^∞ map of manifold since if $f \in C^\infty N$, then for a $\in$ N $(X_\nu(f))(a) = (dL_a(\nu))f = \nu(f \circ L_a)$.

Now, if x $\in$ N we have $(f \circ L_a)(x) = (f \circ m)(a, x)$ which is a smooth map of a, x where x is the multiplication map on N. Thus, $\nu(f \circ L_a)$ is smooth and hence so is X_ν.

We now show X_ν is left invariant. For a, $g \in$ N, we have
$(dL_g)(X_\nu(a)) = dL_g((dL_a)_1(\nu)) = d(L_g \circ L_a)(\nu) = d(L_{ga})(\nu) = X_\nu(ga) = X_\nu(L_g(a))$.

So that X_ν is left invariant. Therefore $\varnothing$ is onto and $\Gamma(TN)^N \cong T_1N$. This completes the proof of the proposition.

Proposition 3.1.8: The Nagendram bracket of two left vector Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field is a left invariant vector Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field.

Thus we can regard T_1N as a Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field algebra and make the following definition.

Definition 3.1.9: Let N be a Nagendram Γ-semi sub near-field space of Γ-semi sub near-field space over a near-field. The Nagendram Γ-semi sub near-field space algebra *g* of N is T_1N with the Nagendram bracket induced by its identification with $\Gamma(TN)^N$.

Example 3.1.10: Let N = $(N^n, +)$ what is *g*? Notice that for this Nagendram Γ-semi sub near-field space. $L_a(x) = a + x$, so that $(dL_a)_0 = id_{T0R}{}^n$. So, $(dL_a)_0$ (v) = v for all v ∈ T_0N^n and thus $g = T_0N^n \cong N^n$. So the Nagendram Γ-semi sub near-field space algebra of Γ-semi sub near-field space over a near-field contains all constant vector Nagendram Γ-semi sub near-field spaces, and the Nagendram bracket is identically 0.

Example 3.1.11: Consider the Nagendram Γ-semi sub near-field space algebra of Γ-semi sub near-field space over a near-field NN(n, N). we have T_1NN(n, N), the Nagendram bracket is the commutator i.e. [A, B] = AB – BA

To prove this, we compute X_A, the left invariant vector Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field associated with the matrix A ∈ T_1NN(n, N). Now, on $M_n(N)$, we have global coordinate maps given by x_{ij} (A) = A_{ij}, the ij th entry of the matrix B. So, for *g* ∈ NN(n, N), $(X_A(x_{ij}))(g) = X_A(I)(x_{ij} \circ L_g)$. Also, if h ∈ NN(n, N).

$(x_{ij} \circ L_g)(h) = x_{ij}(gh) = \sum_k g_{ik} h_{kj} = \sum_k g_{ik} x_{kj}(h)$ which implies that $x_{ij} \circ L_g = \sum_k g_{ik} x_{kj}(h)$

Now, if $f \in C^\infty$ (NN(n N)), $X_A(I) f = \frac{d}{dt}\Big|_{t=0} f(I + tA)$ so that $X_A(I)_{xij} = \frac{d}{dt}\Big|_{t=0} f(I + tA) = A_{ij}$.

Putting these remarks together, we see that $X_A(x_{ij} \circ L_g) = \sum_k g_{ik} A_{kj} = \sum_k x_{ik}(g) A_{kj}$.

We are now in a position to calculate the Nagendram bracket of the left invariant vector Nagendram Γ-semi sub near-field spaces of Γ-semi sub near-field space over a near-field associated with elements of $M_n(N)$.

$$([X_A, X_B)(I)]_{ij} = [X_A, X_B](I)x_{ij} = X_A X_B(x_{ij}) - X_B X_A(x_{ij}) = (X_A(\sum_k B_{kj} x_{ik}) - X_B(\sum_k A_{kj} x_{ik}))(I)$$

$$= (\sum_{k,\ j} B_{kj} x_{il} A_{lk} - A_{kj} x_{il} B_{lk})(I) = \sum_{k,\ l} B_{kj} \delta_{il} A_{lk} - A_{kj} \delta_{il} B_{lk}$$

$$= \sum_k A_{ik}\ B_{kj} - \sum_k B_{ik}\ A_{kj} = (AB - BA)_{ij}$$

Therefore, [A, B] = AB – BA.

SECTION 4: NAGENDRAM Γ-SEMI SUB NEAR-FIELD SPACE HOMOMORPHISMS OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD

4.1 Nagendram Γ-semi sub near-field space Homomorphisms of a Γ-near-field space over near-field

Definition 4.1.1: Let P and Q be Nagendram Γ-semi sub near-field spaces of a Γ-near-field space over near-field. A map ρ : P → Q is a Nagendram Γ-semi sub near-field space homomorphism if (i) ρ is a C^∞ map of manifolds and (ii) ρ is a Γ-semi sub near-field space homomorphism of a Γ-near-field space over near-field.

Furthermore, we say ρ is a Nagendram Γ-semi sub near-field space homomorphism if it is a Nagendram Γ-semi sub near-field space isomorphism and a diffeomorphism.

If *s* and *t* are Nagendram Γ-semi sub near-field space algebras, a Nagendram Γ-semi sub near-field space algebra homomorphism ψ : *s* → *t* is a map such that (i) ψ is linear and (ii) ψ([X, Y]) = [ψ(X), ψ(Y)] for all X, Y ∈ *s*.

Now Suppose W is n-dimensional vector Nagendram Γ-semi sub near-field spaces of a Γ-near-field space over near-field N. we define NN(W) = {A/ W → W | A is a linear isomorphism }

Since W ≅ N^n , NN(W) ≅ NN(n, N). Note that NN(W) ⊆ Hom(W, W) as an open Γ-semi sub near-field space over near-field.

Definition 4.1.2: A (real) representation of a Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field N is a Nagendram Γ-semi sub near-field space homomorphism of a Γ-near-field space over near-field ρ : N → NN(W).
We may similarly define NN(W) for a complex vector Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field W and thus notation of a complex representation.

There are basic two problems in Nagendram Γ-semi sub near-field space theory:

a. classify all Nagendram Γ-semi sub near-field spaces and Nagendram Γ-semi sub near-field space algebras of a Γ-near-field space over near-field,
b. Classify all representations of Nagendram Γ-semi sub near-field spaces.

One step of understanding in this direction is the association between Nagendram Γ-semi sub near-field space homomorphisms and homomorphisms of Nagendram Γ-semi sub near-field space algebras of a Γ-near-field space over near-field.

Theorem 4.1.3: Suppose, $\rho: N \to P$ is a Nagendram Γ-semi sub near-field space homomorphism. Write $d\rho_1 = \delta\rho$. Then, $\delta\rho : T_1N \to T_1P$ is a Nagendram Γ-semi sub near-field space algebras homomorphism.

Proof: It is enough to show that any two left invariant vector Nagendram Γ-semi sub near-field spaces on N and P are ρ-related. So let $X \in \Gamma(SN)^N$ and $X \in \Gamma(SP)^P$.

Then for each a, $g \in N$, we have $(\rho \circ L_a)(g) = \rho(ag) = \rho(a)\,\rho(g) = (L_{\rho(a)} \circ \rho)(g)$.

So that $\rho \circ L_a = L_{\rho(a)} \circ \rho$.

Now, $d\rho_a(X(a)) = d\rho_a(L_a(X(1))) = d(\rho \circ L_a)(X(1)) = d(L_{\rho(a)} \circ \rho)(X(1))$

$= dL_{\rho(a)}(d\rho(X(1))) = dL_{\rho(a)}(\delta\rho(X(1)))= dL_{\rho(a)}(\overline{X}(1)) = \overline{X}(\rho(a))$ since $\overline{X}$ is left invariant and thus X and $\overline{X}$ are ρ-related. This completes the proof of the theorem.

4.2 Nagendram Γ-semi sub near-field spaces and Nagendram Γ-semi sub near-field space algebras of a Γ-near-field space over near-field.

Definition 4.2.1: Let N be a Nagendram Γ-semi sub near-field space. A Γ-semi sub near-field space P of N is a Nagendram Γ-semi sub near-field space if

a. P is an abstract Γ-semi sub near-field space of a Γ-near-field space over near-field N
b. P is a Nagendram Γ-semi sub near-field space and
c. The inclusion $i : P \to N$is an immersion.

A linear Γ-semi sub near-field space *h* of a Nagendram Γ-semi sub near-field space algebras *g* is a Nagendram Γ-semi sub near-field space sub algebras if *h* is closed under the Nagendram Γ-semi sub near-field space bracket in *g*.

Example 4.2.2: Let *g* be a Nagendram Γ-semi sub near-field space algebras and $v \in g$ a non-zero vector. Then N*v*, the span of *v*, is a Nagendram Γ-semi sub near-field space sub algebras of *g*.

Example 4.2.3: Let $P \to N$ be a Nagendram Γ-semi sub near-field space. By theorem 8.1.3 $T_1P \to T_1N$ is a Nagendram Γ-semi sub near-field space sub algebras.

Example 4.2.4: Consider the Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field NN(2, N). The Γ-semi sub near-field space SO(2) = $\left\{ \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} / \ t \in N \right\}$ is a Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field. We will compute the Nagendram Γ-semi sub near-field space algebras of SO(2). It will be sufficient to find a non-zero vector in $T_1SO(2)$ since this Nagendram Γ-semi sub near-field space algebras is of co-dimension 1 in $T_1NN(n, N)$. Now, $\frac{d}{dt}\Big|_{t=0} \begin{bmatrix} \cos t & Sin\,t \\ -Sin\,t & Cos\,t \end{bmatrix} = \begin{bmatrix} -Sin\,t & Cos\,t \\ -Cos\,t & -Sin\,t \end{bmatrix}\Big|_{t=0} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Thus, the Nagendram Γ-semi sub near-field space algebras, so(2) of SO(2) is so(2) = $\left\{ \begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix} / \ x \in N \right\}$.

Example 4.2.5: Consider the Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field O(n) of NN(n, N). we compute o(n). The Nagendram Γ-semi sub near-field space algebras of O(n). Let A(t) be a path in O(n) with A(0) = 1. Then since $BB^T = I$ for all $B \in O(n)$ we have $A(t)\,A(t)^T = I$ for every t.

Thus $0 = \frac{d}{dt}\Big|_{t=0} I = \left(\frac{d}{dt}\Big|_{t=0} A(t)\right)A(0)^T + A(0)\left(\frac{d}{dt}\Big|_{t=0} A(t)^T\right) = A'(0) + A'(0)^T$. Thus we have $o(n) \subseteq S$ where $S = \{X \in M_n(N) / X + X^T = 0\}$. To prove equality, we proceed by dimension count. Now, dim S = $\frac{n^2 - n}{2}$ the easiest way to see this is to write down the form of a general element of S and determine where to place 1's.

View I $\in$ Sym^2 (N^n), the set of all symmetric n x n real matrices. Then, I is a regular value of the map A $\mapsto$ AA^T where A $\in$ NN(n, N). Thus dim o(n) = dim NN(n, N) = $n^2 - \left(\frac{n^2-n}{2}+n\right) = \frac{n^2-n}{2}$.

We conclude that this section by discussing the induced maps δm and δinv on the Nagendram Γ-semi sub near-field space algebras of a Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field N.

Proposition 4.2.6: Let N be a Nagendram Γ-semi sub near-field space of a Γ-near-field space over near-field. Then, for all X, Y $\in$ *g*, (a). δm(X, Y) = X + Y (b). δinv(X) = - X.

Proof: (a). First note that since $\delta m = (dm)_1$ is linear, it is enough to prove that $(dm)_1(X, 0) = X$. So, let γ : (a, b) $\to$ N be a curve with $\gamma(0) = 1$ and $\frac{d}{dt}\gamma(t)$ = X. Then, δm (X, 0) = $(dm)_1(X, 0) = \frac{d}{dt}\big|_{t=0} m(\gamma(t),1) = \frac{d}{dt}\big|_{t=0}\gamma(t)$ = X.
(b). Now, $m(\gamma(t), inv(\gamma(t))$ = 1 for each t $\in$ (a, b). Consider, F : N $\to$ N defined by F(*g*) = gg^{-1}. Denote by Δ the diagonal map Δ(*g*) = (*g*, *g*) for each N $\in$ N. Then, F = *m* o (id_N x *inv* $_)$ o Δ. Thus, 0 = $(dF)_1$ (X) = $((dm)_1$ o (d $id_N)_1$ x $(dinv)_1$ o $(d\Delta)_1)$(X) = X + $(d\ inv)_1$(X) and so δ*inv*(X) = – X. This completes the proof of the proposition.

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