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MHD FLOW OF VISCO-ELASTIC OLDROYD LIQUID WITH TRANSIENT PRESSURE GRADIENT THROUGH POROUS MEDIUM IN A LONG RECTANGULAR CHANNEL

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ABSTRACT

The aim of the present paper is to investigate the magnetohydrodynamic unsteady flow of a conducting visco-elastic Oldroyd liquid through porous medium in a long rectangular channel under the influence of transient pressure gradient. The numerical computations of the velocity profiles have been presented. The results for visco-elastic Maxwell liquid, visco-elastic Kuvshinisky and purely viscous liquid have been deduced from Oldroyd visco-elastic liquid by taking limits (i) $\mu_1 \to 0$ (ii) $\lambda_1 \to 0$ and (iii) $\mu_1 \to 0$, $\lambda_1 \to 0$ respectively. We have discussed the particular cases for (I) $M \to 0$ i.e. magnetic field is withdrawn and (II) $k \to \infty$ i.e. porous media is withdrawn. If magnetic field and porous media both are withdrawn then all expressions of Kundu and Sengupta (2001) are obtained.

Keywords: *MHD Flow, Visco-elastic Fluid, Transient Pressure Gradient, Porous media.*
Classification: MSC 2000: *76A05, 76S05, 76W05*

INTRODUCTION:

The Fluids which exhibit the elasticity property of solids and viscous property of liquids are called visco-elastic fluids or non-Newtonian fluids. In hydromagnetic flow we study of the flow of electrically conducting fluid in presence of Maxwell electromagnetic field. The flow of the conducting fluid is effectively changed by the presence of the magnetic field and the magnetic field is also perturbed due to the motion of the conducting fluid. This phenomena is therefore interlocking in character and the discipline of this branch of science is called Magnetohydrodynamics (MHD). It is equally rich and admits wider applications in Engineering, Technology, Cusmology, Astrophysics and other applied sciences. It has tremendously developed in last forty years and some of the monographs in this field are due to Ferraro and Plumton (1961); Pai (1962); Shercliff (1965); Sutton and Shermann (1965); Jefferey (1966) and Cowling (1976).

Flow behaviour of visco-elastic fluids through channels of different cross-section have been studied by a number of authors Ghosh and Sengupta (1996); Kundu and Sengupta (2001); Abdul Hadi and Sharma (2002) and Mishra and Panda (2005). Many research workers have paid their attention towards the application of visco-elastic fluid flow through various types of channel under the influence of magnetic field such as Sengupta and Basak (2002); Kundu and Sengupta (2003); Sengupta and Paul (2004); Rehman and Alam Sarkar (2004); Krishna and Rao (2005); Ghosh and Ghosh (2006) and Radhakrishnamacharya and Rao (2007).

The study of physics of flow through porous medium has become the basis of many scientific and engineering applications. This type of flow is of great importance in the petroleum engineering concerned with the movement of oil, gas and water through reservoir of oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineer in the filtration process. Many research workers have paid their attention towards the application of visco-elastic fluid flow of different category through porous medium in channels of various cross-section such as Sudhakar and Venkataramana (1988); Kumar and Singh (1990); Singh, Shankar and Singh (1995); Gupta and Gupta (1996); Hassanien (2002); Agarwal and Agarwal (2006); Saroa (2006); Sharma and Pareek (2006); Ahmed (2007); Sarangi and Sharma (2007); Thakur and Kumar (2008); Singh, Kumar and Sharma (2008);

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Singh, Mishra and Sharma (2008); Das and Haque (2008) and Kumar, Sharma and Singh (2008) etc.

Present paper is concerned with MHD unsteady flow of a conducting visco-elastic Oldroyd liquid through porous medium in a long rectangular channel under the influence of transient pressure gradient. The velocity of liquid has been presented in elegant form. The results for visco-elastic Maxwell liquid, visco-elastic Kuvshinisky and purely viscous liquid have been deduced. If magnetic field and porous medium are withdrawn then all expressions of Kundu and Sengupta (2001) are obtained.

BASIC THEORY:

For slow motion, the Rheological equations for Oldroyd visco-elastic liquid are:

$$\tau_{ij} = -p\delta_{ij} + \tau_{ij}^{'} \tag{1}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\tau_{ij}^{'} = 2\mu\left(1+\mu_1 \frac{\partial}{\partial t}\right)e_{ij} \tag{2}$$

$$e_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right) \tag{3}$$

$$v_{i,i} = 0 \tag{4}$$

where $\tau_{ij} =$ The stress tensor

$\tau_{ij}^{'} =$ The deviatoric stress tensor

$e_{ij} =$ The rate of strain tensor

$p =$ The pressure

$\lambda_1 =$ The stress relaxation time parameter

$\mu_1 =$ The strain retardation time

$\delta_{ij} =$ The metric tensor

$\mu =$ The coefficient of viscosity

$v_{i,j} =$ The velocity components

FORMATION OF THE PROBLEM:

Using a rectangular Cartesian coordinate system (x, y, z) such that the z- axis is along the axis of the channel and the walls of the channel are taken to be planes $x = \pm a$ and $y = \pm b$. Let us consider the flow of visco-elastic Oldroyd liquid along the axis of rectangular channel i.e. along z-axis only, therefore $0, 0, w(x, y, z)$ are the velocity components along x, y, z directions respectively. A transient pressure gradient $-Pe^{-\omega t}$ varying with time *t* is applied to the liquid.

Following the stress-strain relations (1)-(4), the equation for unsteady motion through porous medium in a rectangular channel under the influence of an uniform magnetic field B_0 applied perpendicular to flow of the conducting visco-elastic Oldroyd liquid when induced magnetic field be neglected is given by

$$\begin{aligned}\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} &= -\frac{1}{\rho}\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} \\ &+ \upsilon\left(1+\mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\ &- \left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\sigma B_0^2}{\rho} w - \frac{\upsilon}{k}\left(1+\lambda_1 \frac{\partial}{\partial t}\right)w\end{aligned} \tag{5}$$

Where B_0 is an uniform magnetic field, σ the electrical conductivity, υ the kinematic coefficient of viscosity, ρ the density, μ the coefficient of viscosity and *k* is the permeability. Introducing the following non-dimensional quantities:

$$w^* = \frac{a}{\upsilon} w,\ p^* = p\frac{a^2}{\rho \upsilon^2},\ t^* = \frac{\upsilon}{a^2} t,\ \left(x^*, y^*, z^*\right) = \frac{1}{a}(x, y, z),$$

$$\omega^* = \omega \frac{a^2}{\upsilon},\ \lambda^* = \lambda \frac{\upsilon}{a^2}, \mu_1^* = \frac{\upsilon}{a^2}\mu_1,\ k^{'} = \frac{1}{a^2} k$$

in equation (1), we get (after dropping the stars)

$$\begin{aligned}\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} &= -\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \left(1+\mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\ &-\left(M^2 + \frac{1}{k}\right)\left(1+\lambda_1 \frac{\partial}{\partial t}\right)w\end{aligned} \tag{6}$$

where $M = aB_0\sqrt{\dfrac{\sigma}{\mu}}$ (Hartmann number)

We have considered those types of situations of the flow which is transient in nature with respect to time and periodic in nature with respect to y. From the nature of the boundary conditions it is reasonable to choose the solution of (6) as

$$w = W(x)\cos my\, e^{-\omega t} \tag{7}$$

The boundary conditions are:

(i) w=0 when $x = \pm 1,\quad -\frac{b}{a} \le y \le \frac{b}{a}$ (8)

(ii) w=0 when $y = \pm \frac{b}{a},\quad -1 \le x \le 1$ (9)

Boundary condition (9) will be satisfied if

$$\cos m\frac{b}{a} = 0$$

$$\text{or}\quad m\frac{b}{a} = (2n+1)\frac{\pi}{2}$$

$$\text{or}\quad m = (2n+1)\frac{\pi a}{2b},\quad n = 0, 1, 2, 3, \ldots \tag{10}$$

We construct the solution as the sum of all possible solutions for each value of n:

$$w=\sum_{n=0}^{\infty} W(x)\cos m\,y.e^{-\omega t} \tag{11}$$

By putting $\frac{\partial p}{\partial z}=-Pe^{-\omega t}$, $(\omega>0)$ in (6) and using (11), we get

$$\sum_{n=0}^{\infty}\left\{\frac{d^2W(x)}{dx^2}-m^2W(x)\right\}\cos my+\sum_{n=0}^{\infty}\frac{\omega(1-\lambda_1\omega)}{(1-\mu_1\omega)}W(x)\cos my-\sum_{n=0}^{\infty}\frac{\left(M^2+\frac{1}{k}\right)(1-\lambda_1\omega)}{(1-\mu_1\omega)}$$
$$\times W(x)\cos my+\frac{4P}{\pi}\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos my=0$$

$$\sum_{n=0}^{\infty}\left[\left\{\frac{d^2W(x)}{dx^2}-\left(\begin{array}{l}m^2-\frac{\omega(1-\lambda_1\omega)}{(1-\mu_1\omega)}\\+\left(M^2+\frac{1}{k}\right)\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}\end{array}\right)W(x)+\frac{4P}{\pi}\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}\frac{(-1)^n}{(2n+1)}\right\}\cos my\right]=0 \tag{12}$$

Now from (12) equating the coefficient of *cos my* equal to zero, the value of W(*x*) can be determined from

$$\frac{d^2W(x)}{dx^2}-\left\{m^2-\frac{\omega(1-\lambda_1\omega)}{(1-\mu_1\omega)}+\left(M^2+\frac{1}{k}\right)\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}\right\}W(x)$$
$$+\frac{(-1)^n}{(2n+1)}.\frac{4P}{\pi}\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}=0$$

or
$$\frac{d^2W(x)}{dx^2}-\frac{K^2}{a^2}W(x)+A_n=0 \tag{13}$$

where

$$K^2=\left\{m^2-\frac{\omega(1-\lambda_1\omega)}{(1-\mu_1\omega)}+\left(M^2+\frac{1}{k}\right)\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}\right\}a^2$$
$$and\;\; A_n=\frac{(-1)^n}{(2n+1)}.\frac{4P}{\pi}\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)} \tag{14}$$

Solving (13) subject to boundary condition (8), we get

$$W(x)=\frac{A_n}{K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\}$$

or

$$W(x)=\frac{(-1)^n}{(2n+1)}.\frac{4P}{\pi}\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\} \tag{15}$$

Putting the value of W(*x*) in (11), we get the velocity of visco-elastic Oldroyd liquid under the influence of uniform transverse magnetic field.

$$w=\sum_{n=0}^{\infty}\left[\begin{array}{l}\frac{(-1)^n}{(2n+1)}.\frac{4P}{\pi}\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\}\\ \cos\frac{(2n+1)\pi a}{2b}y\end{array}\right]e^{-\omega t} \tag{16}$$

DEDUCTIONS:

(i) Taking limit $\mu_1\to 0$, the visco-elastic liquid becomes Maxwell liquid, the velocity is given by the expression:

$$w=\sum_{n=0}^{\infty}\left[\begin{array}{l}\frac{(-1)^n}{(2n+1)}.\frac{4P}{\pi}\frac{(1-\lambda_1\omega)}{K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\}\\ \cos\frac{(2n+1)\pi a}{2b}y\end{array}\right]e^{-\omega t} \tag{17}$$

where $K^2=\left\{(m^2+M^2+\frac{1}{k}-\omega)(1-\lambda_1\omega)\right\}a^2$

(ii) Taking limit $\lambda_1\to 0$, the visco-elastic liquid becomes Kuvshinisky liquid, the velocity is given by the expression:

$$w=\sum_{n=0}^{\infty}\left[\begin{array}{l}\frac{(-1)^n 4P}{(2n+1)\pi(1-\mu_1\omega)K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\}\\ \cos\frac{(2n+1)\pi a}{2b}y\end{array}\right]e^{-\omega t} \tag{18}$$

where $K^2=\left\{m^2+\frac{\left(M^2+\frac{1}{k}-\omega\right)}{(1-\mu_1\omega)}\right\}a^2$

(iii) Taking limits $\mu_1 \to 0$ and $\lambda_1 \to 0$, the visco-elastic liquid becomes purely viscous liquid, the velocity is given by the expression:

$$w = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4P}{(2n+1)\pi K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\}\cos\frac{(2n+1)\pi a}{2b}y\right]e^{-\omega t} \tag{19}$$

where $K^2 = \left\{m^2 + M^2 + \frac{1}{k} - \omega\right\}a^2$

PARTICULAR CASES:

Case I: Taking limit $M \to 0$ i.e. if uniform magnetic field is withdrawn, the velocity of visco-elastic Oldroyd liquid through porous medium is given by the expression:

$$w = \sum_{n=0}^{\infty}\left[\begin{array}{l}\frac{(-1)^n 4P(1-\lambda_1\omega)}{(2n+1)\pi(1-\mu_1\omega)K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\} \\ \cos\frac{(2n+1)\pi a}{2b}y\end{array}\right]e^{-\omega t} \tag{20}$$

where $K^2 = \left\{m^2 - \frac{\omega(1-\lambda_1\omega)}{(1-\mu_1\omega)} - \frac{1}{k}\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}\right\}a^2$

CaseII: Taking limit $k \to \infty$ i.e. porous medium is withdrawn; velocity of visco-elastic Oldroyd liquid under the influence of magnetic field is given by

$$w = \sum_{n=0}^{\infty}\left[\begin{array}{l}\frac{(-1)^n 4P(1-\lambda_1\omega)}{(2n+1)\pi(1-\mu_1\omega)K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\} \\ \cos\frac{(2n+1)\pi a}{2b}y\end{array}\right]e^{-\omega t} \tag{21}$$

where $K^2 = \left\{m^2 - \frac{\omega(1-\lambda_1\omega)}{(1-\mu_1\omega)} + M^2\frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}\right\}a^2$

Case III: Taking limit $M \to 0$ and $k \to \infty$ i.e. uniform magnetic field and porous media both are withdrawn; the velocity of visco-elastic Oldroyd liquid is given by

$$w = \sum_{n=0}^{\infty}\left[\begin{array}{l}\frac{(-1)^n 4P(1-\lambda_1\omega)}{(2n+1)\pi(1-\mu_1\omega)K^2}\left\{1-\frac{\cos h\frac{K}{a}x}{\cos h\frac{K}{a}}\right\} \\ \cos\frac{(2n+1)\pi a}{2b}y\end{array}\right]e^{-\omega t} \tag{22}$$

where $K^2 = \left\{m^2 - \frac{\omega(1-\lambda_1\omega)}{(1-\mu_1\omega)}\right\}a^2$

All the expressions of Kundu and Sengupta (2001) are obtained.

NUMERICAL CALCULATION AND DISCUSSION:

To understand the physical situation of the problem some numerical calculations are carried out for the non dimensional velocity of conducting visco-elastic Oldroyd liquid with transient pressure gradient through a long rectangular channel under the influence of uniform magnetic field given by the equation (16) and the graphs are also traced out for the particular cases. The magnetic field and porous media affects the velocity of the liquid. We have taken the following values.

$$\lambda_1 = 0.0023,\ \mu_1 = 0.0005,\ x = 0.75,\ y = 0.45,\ n = 1,$$
$$m = 9.42478, a = 0.50,\ b = 0.25,\ \omega = 0.50,\ P = 10$$
$$M = 1, 8, 15,\ k = 0.02, 0.05, 0.10$$

$$t = 0, .05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, \ldots$$

From the graphs it is clear that the velocity of the liquid starts with a maximum value initially and therefore it gradually decreases with the increasing the value of time and ultimately velocity tends to zero as the time
t tends to infinity.

Table 1

t	w(k=0.02)	w(k=0.05)	w(k=0.10)
0	0.454	0.511	0.82
0.5	0.354	0.398	0.638
1	0.276	0.31	0.497
1.5	0.215	0.241	0.387
2	0.167	0.188	0.302
2.5	0.130	0.146	0.235
3	0.101	0.114	0.183
3.5	0.0789	0.0887	0.142
4	0.0615	0.0691	0.111

Table2

t	w (M =15	w(M=8)	w (M =1)
0	0.126	0.250	0.405
0.5	0.0982	0.195	0.316
1	0.0765	0.152	0.246
1.5	0.0596	0.118	0.191
2	0.0464	0.0921	0.149
2.5	0.0361	0.0718	0.116
3	0.0281	0.0559	0.0904
3.5	0.0219	0.0435	0.0704
4	0.0171	0.0339	0.0548

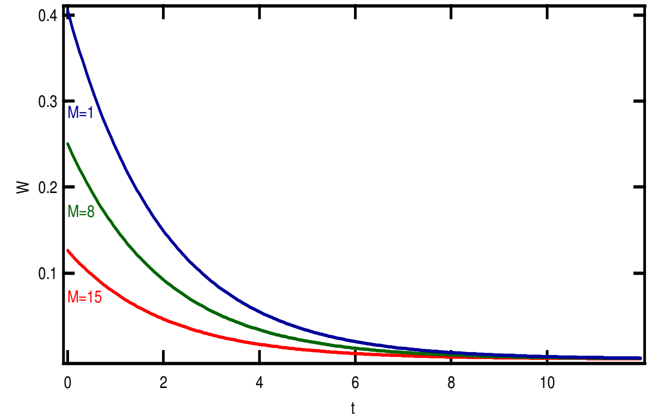

Fig.2: Velocity profile for equation 21

Table 3

t	w(omega=0.8)	w(omega=0.6)	w(omega=0.4)
0	0.410	0.410	0.409
0.5	0.275	0.303	0.335
1	0.184	0.225	0.274
1.5	0.124	0.167	0.224
2	0.0828	0.123	0.184
2.5	0.0555	0.0914	0.15
3	0.0372	0.0677	0.123
3.5	0.0249	0.0502	0.101
4	0.0167	0.0372	0.0826

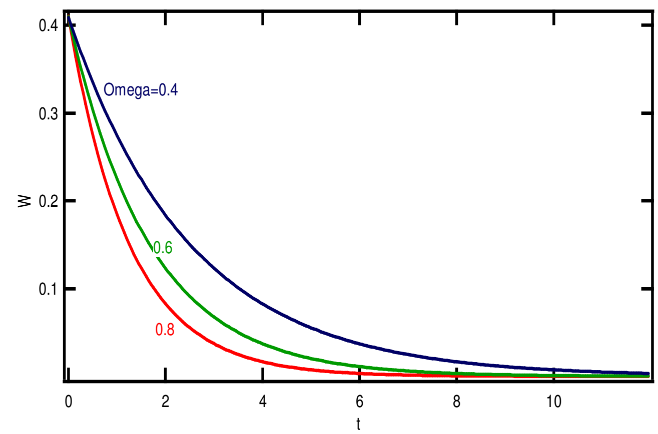

Fig.3: Velocity profile for equation 22

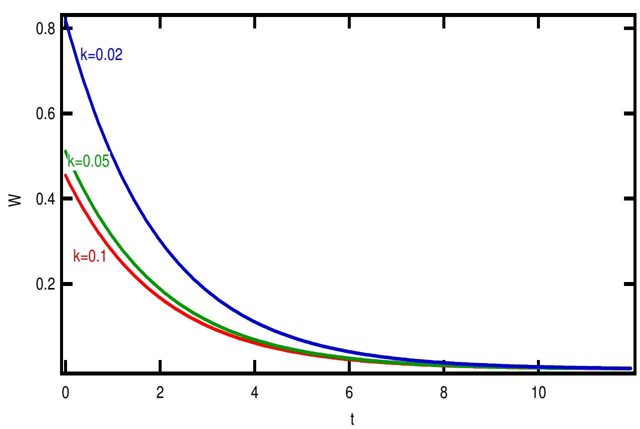

Fig.1: Velocity profile for equation 20

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