CONSTRUCTION OF SECOND ORDER ROTATABLE DESIGNS USING BALANCED TERNARY DESIGNS

KANNA E1, AMEEN SAHEB SK*2 AND BHATRA CHARYULU N. CH.1

1Department of Statistics, University College of Science, Osmania University, Hyderabad-7. India.
2School of Mathematics and Statistics, University of Hyderabad, Hyderabad-500046, India.

(Received On: 11-04-18; Revised & Accepted On: 18-05-18)

ABSTRACT

The variance of estimated response of Second Order Response Surface Design Model satisfying the property that, at any given point in a design, it is a function of the distance from that point to the origin, specifically; it is a spherical or nearly spherical variance function, are called rotatable designs. This paper provides new series for the construction of Second Order Rotatable Design using Balanced Ternary Design.

Keywords: Balanced Incomplete Block Design, Balanced Ternary Design, Second Order Rotatable Designs,

1. INTRODUCTION

Let $X_1, X_2, \ldots, X_v$ are ‘v’ factors, each has ‘s’ levels for experimentation. Let $D$ denote the design matrix of combination of the factor levels, given by

$$D = ( (x_{u1}, x_{u2}, \ldots, x_{uv}) )$$

where $x_{ui}$ be the level of the $i^{th}$ factor in the $u^{th}$ factors combination $(i=1, 2, \ldots v; u =1, 2 \ldots N)$. Let $Y_u$ denote the response at the $u^{th}$ combination. The factor-response relationship is given by

$$E(Y_u) = f(x_{u1}, x_{u2}, \ldots, x_{uv})$$

is called the ‘Response Surface’. The design ‘D’ used for fitting the response surface models are termed as ‘Response Surface Design’. The functional form of the response surface to be fitted to the design is polynomial of degree $k$, may be linear, second order, third order etc. The second order response surface design model at the $u^{th}$ design point is

$$Y_u = \beta_0 + \sum_{i=1}^{v} \beta_i x_{ui} + \sum_{i=1}^{v} \beta_{ii} x_{ui}^2 + \sum_{i<j}^{v} \beta_{ij} x_{ui} x_{uj} + \varepsilon_u$$

where, $Y_u$ is the response at the $u^{th}$ design point,

$$\hat{\beta} = (\beta_0, \beta_1, \beta_2, \ldots, \beta_v, \beta_{v1}, \beta_{v2}, \ldots, \beta_{vv})'$$

be the vector of parameters

$$x_u = (1, x_{u1}, x_{u2}, \ldots x_{uv}, x_{u1}^2, x_{u2}^2, \ldots x_{uv}^2, x_{u1}x_{u2}, \ldots, x_{uv1}x_{uv})$$

be the $u^{th}$ row of the design matrix $X$,

$$\varepsilon_u$$

is the random error corresponding to $Y_u$. Assume the random errors are independent follows $N(0, \sigma^2)$.

The model (1.3) can be expressed in the matrix form as

$$Y = X \hat{\beta} + \varepsilon$$

where, $Y_{Nx1}$ is the vector of responses, $\hat{\beta}$ is the vector of parameters and $\varepsilon_{N \times 1}$ is the vector of random errors. $X_{N \times k}$ is the design matrix,

$$X = \begin{bmatrix}
1 & x_{11} & x_{12} & \ldots & x_{1v} & x_{11}^2 & x_{12}^2 & \ldots & x_{1v}^2 & x_{11}x_{12} & x_{11}x_{13} & \ldots & x_{1v}x_{1v} \\
1 & x_{21} & x_{22} & \ldots & x_{2v} & x_{21}^2 & x_{22}^2 & \ldots & x_{2v}^2 & x_{21}x_{22} & x_{21}x_{23} & \ldots & x_{2v}x_{2v} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & x_{N1} & x_{N2} & \ldots & x_{Nv} & x_{N1}^2 & x_{N2}^2 & \ldots & x_{Nv}^2 & x_{N1}x_{N2} & x_{N1}x_{N3} & \ldots & x_{Nv}x_{Nv}
\end{bmatrix}$$

Corresponding Author: Ameen Saheb Sk*2

2School of Mathematics and Statistics, University of Hyderabad, Hyderabad-500046, India.
The least square estimate of the parameters and responses are $\hat{\beta} = (X'X)^{-1}X'Y$ and $\hat{Y} = (X'X)^{-1}X'Y$ and the variance of estimated response is $V(\hat{Y}) = (X'X)^{-1} \sigma^2$, where the moment matrix $(X'X)$ is

$$X'X = \begin{bmatrix}
N & \sum_{i=1}^{N} X_{i1} & \ldots & \sum_{i=1}^{N} X_{iN} & \sum_{i=1}^{N} X_{i1} X_{i2} & \ldots & \sum_{i=1}^{N} X_{i1} X_{iN} \\
\sum_{j=1}^{N} X_{j1} & \sum_{i=1}^{N} X_{i1}^2 & \ldots & \sum_{i=1}^{N} X_{i1} X_{iN} & \sum_{i=1}^{N} X_{i1}^2 & \ldots & \sum_{i=1}^{N} X_{i1}^2 X_{iN} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\sum_{j=1}^{N} X_{jN} & \sum_{i=1}^{N} X_{iN}^2 & \ldots & \sum_{i=1}^{N} X_{i1} X_{iN} & \sum_{i=1}^{N} X_{iN}^2 & \ldots & \sum_{i=1}^{N} X_{iN}^2 X_{iN} \\
\sum_{i=1}^{N} X_{i1} X_{N+1} & \sum_{i=1}^{N} X_{i1} X_{N+1} & \ldots & \sum_{i=1}^{N} X_{i1} X_{iN} & \sum_{i=1}^{N} X_{i1} X_{iN} & \ldots & \sum_{i=1}^{N} X_{i1} X_{N} \\
\sum_{i=1}^{N} X_{iN+1} & \sum_{i=1}^{N} X_{iN+1}^2 & \ldots & \sum_{i=1}^{N} X_{i1} X_{iN} & \sum_{i=1}^{N} X_{iN+1}^2 & \ldots & \sum_{i=1}^{N} X_{iN+1}^2 X_{iN} \\
\sum_{i=1}^{N} X_{N+1}^2 & \sum_{i=1}^{N} X_{N+1}^2 & \ldots & \sum_{i=1}^{N} X_{i1} X_{iN} & \sum_{i=1}^{N} X_{N+1}^2 & \ldots & \sum_{i=1}^{N} X_{N+1}^2 \sum_{i=1}^{N} X_{iN+1}^2 X_{iN} \\
\end{bmatrix}
$$

The $V(\hat{Y})$ is not in a simplified form as the elements in moment matrix are in higher order, hence imposing the restrictions on the moment matrix that all odd power summations are zero, towards reaching to near orthogonality, i.e.

$\sum_{i=1}^{N} x_{ui} x_{uj} x_{uk} x_{ul} = 0$, where all the summations are over the design points $u = 1, 2, …, N$ and for distinct $i, j, k, l = 1, 2, v$. Let $\Sigma x_{ui} x_{ui} = N \lambda_2$, $\Sigma x_{u1} x_{u1} = N \lambda_4$, then the moment matrix will be in the form

$$N^2 X'X = \begin{bmatrix}
1 & 0 & \lambda_2 J & 0 \\
0 & \lambda_2 J & 0 & 0 \\
\lambda_2 J & 0 & [(c-1)I + J] \tilde{\beta}_4 & 0 \\
0 & 0 & 0 & \lambda_4 I
\end{bmatrix}
$$

and $N(X'X)^{-1} = \begin{bmatrix} 
\lambda_4 (c+v+1) \Delta^{-1} & 0 & (c+v-1)(c-1) \Delta^{-1} & 0 \\
0 & \lambda_4 & 0 & 0 \\
-2\lambda_4 \Delta^{-1} & 0 & Z_{v,v} & 0 \\
0 & 0 & 0 & \lambda_4 \Delta^{-1}
\end{bmatrix}$

where, $\Delta = [\lambda_4(c+v-1) - \lambda_4^2] > 0$; $Z_{v,v} = \frac{(c+v-1)I_v - J_{v,v}}{\lambda_4(c-1)(c+v-1)} + \frac{\lambda_4^2(c+v-1)(c-1)}{\lambda_4(c-1)(c+v-1) - k \lambda_4^2} J_{v,v}

Let $\sum_{i=1}^{v} x_{ui}^2 = \rho^2$; then $\sum_{i=1}^{v} x_{ui} = \rho^2 - 2 \sum_{i=1}^{v} x_{ui} x_{uj}$. The variance of estimated response at the $u^{th}$ design point is

$$V(\hat{Y}_u) = V(\hat{\beta}_0) + \rho^2 V(\hat{\beta}_1) + \left[ \rho^2 - 2 \sum_{i<j} x_{ui} x_{uj} \right] V(\hat{\beta}_2) + \sum_{i<j} x_{ui} x_{uj}^2 V(\hat{\beta}_3) + 2 \text{Cov}(\hat{\beta}_0, \hat{\beta}_2) \rho^2

+ 2 \text{Cov}(\hat{\beta}_2, \hat{\beta}_3) \sum_{i<j} x_{ui} x_{uj}^2$$

(1.5)

where,

$$V(\hat{\beta}_0) = \frac{\lambda_4 (c+v+1)}{N \Delta} \sigma^2$$

$$V(\hat{\beta}_1) = \frac{1}{N \lambda_2} \sigma^2$$

$$V(\hat{\beta}_2) = \frac{1}{N \lambda_4} \sigma^2$$

$$\text{Cov}(\hat{\beta}_0, \hat{\beta}_2) = \left[ -\lambda_2 \right] \sigma^2$$

$$V(\hat{\beta}_3) = \left[ (\lambda_4(c+v+1)(c-1) \Delta^{-1} \right] \sigma^2$$

$$\text{Cov}(\hat{\beta}_2, \hat{\beta}_3) = \left[ \left( \lambda_4^2 - \lambda_4 \right) / (c-1) \right] \Delta \sigma^2$$

© 2018, IJMA. All Rights Reserved

165
V(Ŷ) = V(β̂₀) + [V(β̂₁) + 2Cov(β̂₀, β̂₁)]p² + [V(β̂₀) - 2V(β̂₁) + 2Cov(β̂₀, β̂₁)] \sum_{i<j} x_{ui} x_{uj} \\
⇒ V(Ŷ) = \frac{σ^2}{NΔ} \left[ \frac{Δ - λ_2^2}{λ_4(c-1)} \right] p^4 + \left[ \frac{Δ - 2λ_2^2}{λ_2^2} \right] p^2 - \left[ Δ + vλ_2^2 \right] \right] + \left[ \frac{(c-3)}{(c-1)Nλ_4} σ^2 \right] \sum_{i<j} x_{ui} x_{uj}^2 

(1.6)

The condition \([λ_2(v+2) - vλ_2^2] > 0\) is a non-singularity condition necessary for the \(v+2\) coefficients in the response function to be estimated. The condition can always be satisfied by mere addition of central points. The variance of the estimated response at any design point in the design is a function of \(p^2\), i.e. the distance from design point to the origin. Such a design is called as Second Order Rotatable Design. From (1.6), when \(c = 3\), the variance of estimated response can be expressed in the form of a function of \(p^2\) as

\[ V(Ŷ) = \frac{σ^2}{NΔ} \left[ \frac{Δ - λ_2^2}{λ_4(c-1)} \right] p^4 + \left[ \frac{Δ - 2λ_2^2}{λ_2^2} \right] p^2 - \left[ Δ + vλ_2^2 \right] \right] + \left[ \frac{(c-3)}{(c-1)Nλ_4} σ^2 \right] \sum_{i<j} x_{ui} x_{uj}^2. \]

(1.7)

where, \(A = \frac{σ^2}{NΔ} \left[ \frac{Δ - λ_2^2}{λ_4(c-1)} \right]; B = \frac{σ^2}{NΔ} \left[ \frac{Δ - 2λ_2^2}{λ_2^2} \right]; C = \frac{σ^2}{NΔ} \left[ Δ + vλ_2^2 \right].\)

This paper provides new series for the construction of Second Order Rotatable Designs using Balanced Ternary Designs.

2. CONSTRUCTION OF NEW CLASS OF SORD

This section provides a new series for the construction Second Order Rotatable Design using Balanced Ternary Designs. The constructions are illustrated with suitable examples and presented.

Series 2.1: Second Order Rotatable Designs can be constructed using Balanced Ternary Designs provided by Tyagi and Rizwi (1979) are presented below

Step-1: Let \(N_1\) be the incidence matrix of a balanced incomplete block design with parameters \(v, b, r, λ\) (assume \(9λ^2 ≥ 4rλ(v-1)\) and let \(N_2 = I_v\), where \(I_v\) is the identity matrix of order \(v\) and \(p\) is a positive integer. The balanced ternary designs are derived by adding the elements of \(j^{th}\) row of \(N_2\) to those rows of \(N_1\), which contain unity in the \(j^{th}\) column, constitute a balanced ternary design with parameters \(V = v, B = vr, R = (k+1)r, K = k+1\) and \(π = λ(k+2)\).

Step-2: Replace the elements 2 with \(α\) and 1 with \(β\), then associate each block with an appropriate fraction of factorials (say \(2^{k_i}\)) with levels \(±1\) such that no lower order interaction effects are confounded.

Step-3: Add \(n_0(n_0 > 0)\) central design points \((0, 0, …, 0)\) to the resulting design, then total number design points in the design are: \(N = 2^{k_i} vr + n_0\).

Step-4: The levels ‘\(α\)’ and ‘\(β\)’ can be obtained such that \(t = α^2 / β^2\). Real roots for \(t\) can be obtained as \(t = \frac{32 ± \sqrt{92^2 - 4rλ(v-1)}}{4rλ(v-1)}\). Choose the value for \(β\) then obtain the value of \(α\) as \(α^2 = tβ^2\). The resulting design provides a \(v\)-dimensional second order rotatable design with \(5\) levels \((±α, ±β, 0)\).

Theorem 2.1: A Second Order Rotatable Design exists with five levels \((±α, ±β, 0)\) using a balanced ternary design with parameters \(V = v, B = vr, R = (k+1)r, K = k+1\) and \(π = λ(k+2)\) satisfying the condition \(9λ^2 ≥ 4rλ(v-1)\).

Proof: Let \(N_{BTB}\) be the incidence matrix of a balanced ternary Design with parameters \(V = v, B = vr, R = (k+1)r, K = k+1\) and \(π = λ(k+2)\). Each column of \(N_{BTB}\) contains \(‘r’\) times \(2, λ(v-1)\) times \(1\) and \((r-λ)\) times \(0\). Every pair of columns consists the pairs \((1, 2)\) or \((1, 2)\)'s \(2λ\) times and \((2, 2)\) and \((1, 1)\) pairs zero times. Replace the elements 1 with \(β\) and 2 with \(α\). Associate each block with an appropriate fraction of factorials (say \(2^{k_i}\)) for \(v\) factors, with levels \(±1\). After augmenting \(n_0\) central points, the resulting design has \(N = 2^{k_i} (v(r) + n_0\) design points. Then from the rotatable condition \(\sum_{u=1}^{N} X_{ui}^2 = 3, \sum_{u=1}^{N} X_{ui} X_{uj}\), we can obtain

\[2^{k_i} (rα^4 + λ(v-1)β^4) = 3. 2^{k_i} (2λα^2β^2) \]

⇒ \(rα^4 + λ(v-1)β^4 - 6λα^2β^2 = 0\)

(2.1)

Let \(t = α^2 / β^2\), then (2.2) can be expressed in the quadratic form as

\(rt - 3λt + λ(v-1) = 0\)

(2.3)

The roots are real if \(9λ^2 ≥ rλ(v-1)\). Choose any real value for ‘\(β\)’, then the real value for ‘\(α\)’ can be obtained as \(α^2 = tβ^2\). The resulting design provides a \(v\)-dimensional Second Order Rotatable Design in five levels.
Example 2.1: Let \( N_1 \) be the incidence matrix of a Balanced Incomplete Block Design with parameters \( v = 4, b = 6, r = 3, k = 2 \) and \( \lambda = 1 \), and \( N_2 = 1 \). The resulting Balanced Ternary Design is with parameters \( V = 4, B = 12, R = 9, K = 3 \) and \( \pi = 4 \). The Second Order Rotatable Design with four factors is presented in Table 2.1.

<table>
<thead>
<tr>
<th>( N_1 )</th>
<th>( N_2 )</th>
<th>( N )</th>
<th>SORD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 0 0</td>
<td>1 0 0 0</td>
<td>2 1 0 0</td>
<td>( \pm \alpha \pm \beta \ 0 \ 0 )</td>
</tr>
<tr>
<td>1 0 1 0</td>
<td>0 1 0 0</td>
<td>2 0 1 0</td>
<td>( \pm \alpha 0 \pm \beta \ 0 )</td>
</tr>
<tr>
<td>1 0 0 1</td>
<td>0 0 1 0</td>
<td>2 0 0 1</td>
<td>( \pm \alpha 0 0 \pm \beta )</td>
</tr>
<tr>
<td>0 1 1 0</td>
<td>0 0 0 1</td>
<td>1 2 0 0</td>
<td>( \pm \beta \pm \alpha 0 )</td>
</tr>
<tr>
<td>0 0 1 0</td>
<td>0 2 1 0</td>
<td>0 ( \pm \alpha \pm \beta \ 0 )</td>
<td></td>
</tr>
<tr>
<td>0 1 0 1</td>
<td>0 2 0 1</td>
<td>0 ( \pm \alpha 0 \pm \beta )</td>
<td></td>
</tr>
<tr>
<td>0 0 1 0</td>
<td>0 1 2 0</td>
<td>0 ( \pm \beta \pm \alpha )</td>
<td></td>
</tr>
<tr>
<td>0 2 1 0</td>
<td>1 0 2 0</td>
<td>0 ( \pm \beta \pm \alpha )</td>
<td></td>
</tr>
<tr>
<td>0 1 0 2</td>
<td>1 0 0 2</td>
<td>0 ( \pm \beta 0 \pm \alpha )</td>
<td></td>
</tr>
<tr>
<td>0 0 1 2</td>
<td>0 ( \pm \beta \ 0 \pm \alpha )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The real solution for \( t \) is 1. Select value for \( \beta \) arbitrarily and accordingly \( \alpha \) can be evaluated.

Series 2.2: A series of Second Order Rotatable Designs can be constructed using Balanced Ternary Designs of Tyagi and Rizwi (1979) is like as follows.

Step-1: Let \( N_1 \) be the incidence matrix of a Balanced Incomplete Block Design with parameters \( v, b, r, k, \) and \( \lambda \) and let \( N_2 = 2I_v \), where \( I_v \) is the identity matrix of order \( v \). The Balanced Ternary Designs are derived by adding the elements of the row of \( N_2 \) those rows of \( N_1 \) which contain zero in the \( j \)th column, then \( v(b-r) \) blocks so formed constitute a Balanced Ternary Design with parameters \( \pi = (v-k)(v-k-1) \), \( \beta = (r-\lambda)(v-1) \). Each column of \( N_1 \) contains \( 2 \) and \( 1 \) in \( b-r \) and \( r \) out of \( b-r \). Then \( \lambda (v-k)(v-k-1) \) times. Every pair of columns contains the pairs \( (1,2) \) and \( (2,1) \) occurs \( r-\lambda \) and \( r-\lambda \) times. Replace the elements 1 with \( \beta \) and 2 with \( \alpha \). Associate each block with an appropriate fraction of factorials (say \( 2^{k_i} \)) for \( \alpha \) factors, with levels \( z \) and \( \pi \) in the resulting design provides a \( \pi \)-dimensional Second Order Rotatable Design with five levels \( (\pm \alpha, \pm \beta, 0) \).

Theorem 2.2: A Second Order Rotatable Design exists with five levels \( (\pm \alpha, \pm \beta, 0) \) using a Balanced Ternary Design with parameters \( V = v, B = v(b-r), R = (k+2)(b-r), K = k+2 \) and \( \pi = (r-\lambda)(4+k-1) \), where \( \lambda = (v-k)(v-k-1) \). Each column of \( N_BxV \) contains \( 2 \)’s and \( 1 \)’s \( b-r \) and \( r-\lambda \) times. Every pair of columns contains the pairs \((1,2)\) and \((2,1)\) occurs \( r-\lambda \) and \( r-\lambda \) times. Replace the elements 1 with \( \beta \) and 2 with \( \alpha \). Associate each block with an appropriate fraction of factorials (say \( 2^{k_i} \)) for \( \beta \) factors, with levels \( z \). After augmenting \( n_0 \) central points, the resulting design has \( N = 2^{k_i}v(b-r)+n_0 \) design points. Then from the rotatable condition

\[
\sum_{u=1}^{N} x_{ui}^4 = 3 \sum_{u=1}^{N} x_{ui}^2 x_{uj}^2 \quad \text{we can obtain}
\]

\[
2^{k_i} \{ (b-r) \alpha^4 + (r-\lambda)(v-1) \beta^4 \} + 3 \cdot 2^{k_i} \{ \lambda(v-k) \beta^4 + (b-2\lambda)\alpha^2 \beta^2 \} = 0
\]

From the rotatable condition, we obtain

\[
4 \alpha^2 - 3(b-2\lambda) \beta^2 = 0
\]
Let $t = \alpha^2/\beta^2$, then (2.5) can be expressed in the quadratic form as

$$ (b-r) t^2 - 3(b-2\lambda) t + [(v-1)(r-\lambda) - \lambda(v-k)] = 0 \tag{2.6} $$

The roots of the quadratic equation are $t = \frac{(b-r)+\sqrt{9(b-2\lambda)^2 - 4(b-r)[(v-1)(r-\lambda) - \lambda(v-k)]}}{2(b-r)}$.

The roots are real if the discriminant is nonnegative. Choose any real value for ‘$\beta$’ then the real value for ‘$\alpha$’ can be obtained as $\alpha^2 = t\beta^2$. The resulting design ‘D’ provides a $v$-dimensional Second Order Rotatable Design in five levels.

**Example 2.2:** Let $N_1$ be the incidence matrix of a Balanced Incomplete Block Design with parameters $v = 4$, $b = 6$, $r = 3$, $k = 2$ and $\lambda = 1$, and $N_2 = 2I_v$. The resulting Balanced Ternary Design is with parameters $V = 4$, $B = 12$, $R = 12$, $K = 4$ and $\pi = 10$. The Second Order Rotatable Design with four factors is presented in Table 2.2.

| Table 2.2: Construction of Second Order Rotatable Design (SORD) |
|-------------------|-----------------|------------------|
| $N_1$             | $N_2$           | $N$              | SORD             |
| [1 1 0 0]         | [2 0 0 0]       | [2 1 1 0]        | $\pm \alpha \pm \beta \pm \beta$ 0 |
| 1 0 1 0           | 0 2 0 0         | 2 1 0 1          | $\pm \alpha \pm \beta$ 0 $\pm \beta$ |
| 1 0 0 1           | 0 0 2 0         | 2 0 1 1          | $\pm \alpha 0 \pm \beta \pm \beta$ |
| 0 1 1 0           | 0 0 0 2         | 1 2 1 0          | $\pm \beta \pm \alpha \pm \beta$ 0 |
| 0 1 0 1           |                  | 1 2 0 1          | $\pm \beta \pm \alpha 0 \pm \beta$ |
| 0 0 1 1           |                  | 0 2 1 1          | $0 \pm \alpha \pm \beta \pm \beta$ |
|                   |                  | 1 1 2 0          | $\pm \beta \pm \alpha \pm \beta$ 0 |
|                   |                  | 1 0 2 1          | $\pm \beta 0 \pm \alpha \pm \beta$ |
|                   |                  | 0 1 2 1          | $0 \pm \beta \pm \alpha \pm \beta$ |
|                   |                  | 1 1 0 2          | $\pm \beta \pm \beta 0 \pm \alpha$ |
|                   |                  | 1 0 1 2          | $\pm \beta 0 \pm \beta \pm \alpha$ |
|                   |                  | 0 1 1 2          | $0 \pm \beta \pm \beta \pm \alpha$ |

The real solution for ‘$t$’ is 3.63. Select value for $\beta$ arbitrarily and accordingly $\alpha$ can be evaluated.

**ACKNOWLEDGEMENT**

The authors are thankful to the referees for improving the version of manuscript. The first author is also thankful to the UGC for providing financial assistance in the form of Fellowship under BSR-RFSMS.

**REFERENCES**


Source of support: UGC, India., Conflict of interest: None Declared.

© 2018, IJMA. All Rights Reserved