International Journal of Mathematical Archive-9(6), 2018, 164-168
Available online through www.ijma.info ISSN 2229 – 5046

CONSTRUCTION OF SECOND ORDER ROTATABLE DESIGNS USING BALANCED TERNARY DESIGNS

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(Received On: 11-04-18; Revised & Accepted On: 18-05-18)

ABSTRACT

The variance of estimated response of Second Order Response Surface Design Model satisfying the property that, at any given point in a design, it is a function of the distance from that point to the origin, specifically; it is a spherical or nearly spherical variance function, are called rotatable designs. This paper provides new series for the construction of Second Order Rotatable Design using Balanced Ternary Design.

***Keywords**: Balanced Incomplete Block Design, Balanced Ternary Design, Second Order Rotatable Designs,*

1. INTRODUCTION

Let X_1, X_2, … , X_v are 'v' factors, each has 's' levels for experimentation. Let D denote the design matrix of combination of the factor levels, given by

$$D = ((x_{u1}, x_{u2}, \dots , x_{uv})) \tag{1.1}$$

where x_{ui} be the level of the i^{th} factor in the u^{th} factors combination (i=1, 2, ... v; u =1, 2 … N). Let Y_u denote the response at the u^{th} combination. The factor-response relationship is given by

$$E(Y_u) = f(x_{u1}, x_{u2}, \dots , x_{uv}) \tag{1.2}$$

is called the 'Response Surface'. The design 'D' used for fitting the response surface models are termed as 'Response Surface Design'. The functional form of the response surface to be fitted to the design is polynomial of degree k, may be linear, second order, third order etc. The second order response surface design model at the u^{th} design point is

$$Y_u = \beta_0 + \sum_{i=1}^{v} \beta_i x_{ui} + \sum_{i=1}^{v} \beta_{ii} x_{ui}^2 + \sum_{i<j}^{v} \beta_{ij} x_{ui} x_{uj} + \boldsymbol{\varepsilon} \qquad u = 1, 2, \dots N \tag{1.3}$$

where, Y_u is the response at the u^{th} design point,

$\underline{\beta} = (\beta_0, \beta_1, \beta_2, \dots \beta_v, \beta_{11}, \beta_{22}, \dots \beta_{vv}, \beta_{12}, \dots \beta_{v-1v})'$ be the vector of parameters

$x_u = (1, x_{u1}, x_{u2}, \dots x_{uv}, x^2_{u1}, x^2_{u2}, \dots x^2_{uv}, x_{ul}x_{u2}, \dots x_{uv-1}x_{uv})$ be the u^{th} row of the design matrix X,

$\boldsymbol{\varepsilon_u}$ is the random error corresponding to Y_u. Assume the random errors are independent follows $N(0, \sigma^2)$.

The model (1.3) can be expressed in the matrix form as

$$\underline{Y} = X \underline{\beta} + \underline{\varepsilon} \tag{1.4}$$

where, $Y_{Nx1}= [Y_1\ Y_2 \dots Y_N]'$ is the vector of responses, $\underline{\beta}_{k\times1}$ is the vector of parameters and $\underline{\boldsymbol{\varepsilon}}_{N\times1} = [\varepsilon_1, \varepsilon_2, \dots , \varepsilon_N]'$ is the vector of random errors. $X_{N\times k}$ is the design matrix,

$$X = \begin{bmatrix} 1 & X_{11} & X_{12} & \dots & X_{1v} & X_{11}^2 & X_{12}^2 & \dots & X_{1v}^2 & X_{11}X_{12} & X_{11}X_{13} & \dots & X_{1v-1}X_{1v} \\ 1 & X_{21} & X_{22} & \dots & X_{2v} & X_{21}^2 & X_{22}^2 & \dots & X_{2v}^2 & X_{21}X_{22} & X_{21}X_{23} & \dots & X_{2v-1}X_{2v} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & X_{N1} & X_{N2} & \dots & X_{Nv} & X_{N1}^2 & X_{N2}^2 & \dots & X_{Nv}^2 & X_{N1}X_{N2} & X_{N1}X_{N3} & \dots & X_{Nv-1}X_{Nv} \end{bmatrix}$$

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The least square estimate of the parameters and responses are $\hat{\beta} = (X'X)^{-1}X'Y$ and $\hat{Y} = (X'X)^{-1}X'Y$ and the variance of estimated response is $V(\hat{Y}) = (X'X)^{-1}\sigma^2$, where the moment matrix $(X'X)$ is

$$X'X = \begin{bmatrix}
N & \sum_{i=1}^{N} x_{il} & \cdots & \sum_{i=1}^{N} x_{iv} & \sum_{i=1}^{N} x_{i1}^2 & \cdots & \sum_{i=1}^{N} x_{iv}^2 & \sum_{i=1}^{N} x_{i1}x_{i2} & \cdots & \sum_{i=1}^{N} x_{iv-1}x_{iv} \\
\sum_{i=1}^{N} x_{il} & \sum_{i=1}^{N} x_{i1}^2 & \cdots & \sum_{i=1}^{N} x_{i1}x_{iv} & \sum_{i=1}^{N} x_{i1}^3 & \cdots & \sum_{i=1}^{N} x_{i1}x_{iv}^2 & \sum_{i=1}^{N} x_{i1}^2x_{i2} & \cdots & \sum_{i=1}^{N} x_{i1}x_{iv-1}x_{iv} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\sum_{i=1}^{N} x_{iv} & \sum_{i=1}^{N} x_{i1}x_{iv} & \cdots & \sum_{i=1}^{N} x_{iv}^2 & \sum_{i=1}^{N} x_{i1}^2x_{iv} & \cdots & \sum_{i=1}^{N} x_{iv}^3 & \sum_{i=1}^{N} x_{i1}x_{i2}x_{iv} & \cdots & \sum_{i=1}^{N} x_{iv-1}x_{iv}^2 \\
\sum_{i=1}^{N} x_{i1}^2 & \sum_{i=1}^{N} x_{i1}^3 & \cdots & \sum_{i=1}^{N} x_{i1}^2x_{iv} & \sum_{i=1}^{N} x_{i1}^4 & \cdots & \sum_{i=1}^{N} x_{i1}^2x_{iv}^2 & \sum_{i=1}^{N} x_{i1}^3x_{i2} & \cdots & \sum_{i=1}^{N} x_{i1}^2x_{iv-1}x_{iv} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\sum_{i=1}^{N} x_{iv}^2 & \sum_{i=1}^{N} x_{i1}x_{iv}^2 & \cdots & \sum_{i=1}^{N} x_{iv}^3 & \sum_{i=1}^{N} x_{i1}^2x_{iv}^2 & \cdots & \sum_{i=1}^{N} x_{iv}^4 & \sum_{i=1}^{N} x_{i1}x_{i2}x_{iv}^2 & \cdots & \sum_{i=1}^{N} x_{iv-1}x_{iv}^3 \\
\sum_{i=1}^{N} x_{i1}x_{i2} & \sum_{i=1}^{N} x_{i1}^2x_{i2} & \cdots & \sum_{i=1}^{N} x_{i1}x_{i2}x_{iv} & \sum_{i=1}^{N} x_{i1}^3x_{i2} & \cdots & \sum_{i=1}^{N} x_{i1}x_{i2}x_{iv}^2 & \sum_{i=1}^{N} x_{i1}^2x_{i2}^2 & \cdots & \sum_{i=1}^{N} x_{i1}x_{i2}x_{iv-1}x_{iv} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\sum_{i=1}^{N} x_{iv-1}x_{iv} & \sum_{i=1}^{N} x_{i1}x_{iv-1}x_{iv} & \cdots & \sum_{i=1}^{N} x_{iv-1}x_{iv}^2 & \sum_{i=1}^{N} x_{i1}^2x_{iv-1}x_{iv} & \cdots & \sum_{i=1}^{N} x_{iv-1}x_{iv}^3 & \sum_{i=1}^{N} x_{i1}x_{i2}x_{iv-1}x_{iv} & \cdots & \sum_{i=1}^{N} x_{iv-1}^2x_{iv}^2
\end{bmatrix}$$

The $V(\hat{Y}_u)$ is not in a simplified form as the elements in moment matrix are in higher order, hence imposing the restrictions on the moment matrix that all odd power summations are zero, towards reaching to near orthogonality, i.e. $\sum x_{ui}^{\delta 1} x_{uj}^{\delta 2} x_{uk}^{\delta 3} x_{ul}^{\delta 4} = 0$, where all the summations are over the design points u= 1, 2, …N and for distinct i, j, k, l = 1, 2, v. Let $\Sigma x^2_{ui} = N\lambda_2$; $\Sigma x^4_{ui} = CN\lambda_4$; $\Sigma x^2_{ui}x^2_{uj} = N\lambda_4$ then the moment matrix will be in the form

$$N^{-1}X'X = \begin{bmatrix} 1 & 0 & \lambda_2 J & 0 \\ 0 & \lambda_2 J & 0 & 0 \\ \lambda_2 J & 0 & [(c-1)I + J]\lambda_4 & 0 \\ 0 & 0 & 0 & \lambda_4 I \end{bmatrix} \text{ and } N(X'X)^{-1} = \begin{bmatrix} \lambda_4(c+v+1)\Delta^{-1} & 0 & (c+v-1)(c-1)J\Delta^{-1} & 0 \\ 0 & \lambda_2^{-1}I & 0 & 0 \\ -2\lambda_2 J\Delta^{-1} & 0 & Z_{VxV} & 0 \\ 0 & 0 & 0 & \lambda_2^{-1} \end{bmatrix}$$

where, $\Delta = [\lambda_4(c+v-1) - v\lambda_2^2] > 0$; $Z_{v\times v} = \dfrac{(c+v-1)I_v - J_{v,v}}{\lambda_4(c-1)(c+v-1)} + \dfrac{\lambda_2^2(c+v-1)(c-1)^2}{\lambda_4(c+v-1) - k\lambda_2^2} J_{k,k}$

Let $\sum_{i=1}^{v} x_{ui}^2 = \rho^2$; then $\sum_{i=1}^{v} x_{ui}^4 = \rho^4 - 2\sum_{i<j}^{v} x_{ui}^2 x_{uj}^2$. The variance of estimated response at the u[th] design point is

$$V(\hat{Y}_u) = V(\hat{\beta}_0) + \rho^2 V(\hat{\beta}_i) + [\rho^4 - 2\sum_{i<j}^{v} x_{ui}^2 x_{uj}^2]\, V(\hat{\beta}_{ii}) + \sum_{i<j}^{v} x_{ui}^2 x_{uj}^2\, V(\hat{\beta}_{ij}) + 2\,\mathrm{Cov}(\hat{\beta}_0, \hat{\beta}_{ii})\rho^2$$

$$+ 2\,\mathrm{Cov}(\hat{\beta}_{ii}, \hat{\beta}_{jj}) \sum_{i<j}^{v} x_{ui}^2 x_{uj}^2 \qquad (1.5)$$

where,

$V(\hat{\beta}_0) = [\lambda_4(c+k-1)/N\Delta]\,\sigma^2$]

$V(\hat{\beta}_i) = (1/N\lambda_2)\,\sigma^2$

$V(\hat{\beta}_{ij}) = (1/N\lambda_4)\,\sigma^2$

$\mathrm{Cov}(\hat{\beta}_0, \hat{\beta}_{ii}) = [-\lambda_2/N\Delta]\,\sigma^2$

$V(\hat{\beta}_{ii}) = [\{\lambda_4(c+k-2) - (k-1)\lambda_2^2\}/\{N\lambda_4(c-1)\Delta\}]\,\sigma^2$

$\mathrm{Cov}(\hat{\beta}_{ii}, \hat{\beta}_{jj}) = [(\lambda_2^2 - \lambda_4)/\{(c-1)N\lambda_4\Delta\}]\,\sigma^2$

$$V(\hat{Y}_u)=V(\hat{\beta}_0) + [V(\hat{\beta}_i) + 2Cov(\hat{\beta}_0, \hat{\beta}_{ii})]\rho^2 + V(\hat{\beta}_{ii})\rho^4 + [V(\hat{\beta}_{ij}) - 2V(\hat{\beta}_{ii}) + 2Cov(\hat{\beta}_{ii}, \hat{\beta}_{jj})] \sum_{i<j}^{v} x_{ui}^2 x_{uj}^2$$

$$\Rightarrow V(\hat{Y}_u) = \frac{\sigma^2}{N\Delta}\left[\left\{\frac{\Delta-\lambda_2^2}{\lambda_4(c-1)}\right\}\rho^4 + \left\{\frac{\Delta-2\lambda_2^2}{\lambda_2}\right\}\rho^2 - \left\{\Delta + v\lambda_2^2\right\}\right] + \left[\frac{(c-3)}{(c-1)N\lambda_4}\sigma^2\right]\sum_{i<j}^{v} x_{ui}^2 x_{uj}^2 \tag{1.6}$$

The condition $[\lambda_4(v+2) - v\lambda_2^2] > 0$ is a non-singularity condition necessary for the ${}^{v+2}C_v$ coefficients in the response function to be estimated. The condition can always be satisfied by mere addition of central points. The variance of the estimated response at any design point in the design is a function of ρ^2, i.e. the distance from design point to the origin. Such a design is called as Second Order Rotatable Design. From (1.6), when c = 3, the variance of estimated response can be expressed in the form of a function of ρ^2 as

$$V(\hat{Y}_u) = A\rho^4 + B\rho^2 + C \tag{1.7}$$

where, $A = \frac{\sigma^2}{N\Delta}\left[\frac{\Delta-\lambda_2^2}{\lambda_4(c-1)}\right]$; $B = \frac{\sigma^2}{N\Delta}\left[\frac{\Delta-2\lambda_2^2}{\lambda_2}\right]$; $C = \frac{\sigma^2}{N\Delta}\left[\Delta + v\lambda_2^2\right]$.

This paper provides new series for the construction of Second Order Rotatable Designs using Balanced Ternary Designs.

2. CONSTRUCTION OF NEW CLASS OF SORD

This section provides a new series for the construction Second Order Rotatable Design using Balanced Ternary Designs. The constructions are illustrated with suitable examples and presented.

Series 2.1: Second Order Rotatable Designs can be constructed using Balanced Ternary Designs provided by Tyagi and Rizwi (1979) are presented below

Step-1: Let N_1 be the incidence matrix of a Balanced Incomplete Block Design with parameters v, b, r, k, λ (assume $9\lambda^2 \geq 4r\lambda(v-1)$) and let $N_2 = I_v$, where I_v is the identity matrix of order 'v' and 'p' is a positive integer. The Balanced Ternary Designs are derived by adding the elements of j^{th} row of N_2 to those rows of N_1, which contain unity in the j^{th} column, constitute a Balanced Ternary Design with parameters V = v, B = vr, R = (k+1)r, K = k+1 and π = λ(k+2).

Step-2: Replace the elements 2 with α and 1 with β, then associate each block with an appropriate fraction of factorials (say 2^{k_1}) with levels ±1 such that no lower order interaction effects are confounded.

Step-3: Add n_0 ($n_0 > 0$) central design points (0, 0, … 0) to the resulting design, then total number design points in the design are: $N = 2^{k_1}$ vr + n_0.

Step-4: The levels 'α' and 'β' can be obtained such that $t = \alpha^2 / \beta^2$. Real roots for t can be obtained as $t = \frac{3\lambda \pm \sqrt{9\lambda^2 - 4r\lambda(v-1)}}{r}$. Choose the value for β then obtain the value of α as $\alpha^2 = t\beta^2$. The resulting design provides a v-dimensional Second Order Rotatable Design with five levels (± α, ± β, 0).

Theorem 2.1: A Second Order Rotatable Design exists with five levels (±α, ±β, 0) using a Balanced Ternary Design with parameters V = v, B = vr , R = (k+1)r, K = k+1 and π = λ(k+2) satisfying the condition $9\lambda^2 \geq 4r\lambda(v-1)$.

Proof: Let $\mathbf{N_{BXV}}$ be the incidence matrix of a balanced Ternary Design with parameters V = v, B = vr, R = (k+1)r, K = k+1 and π = λ(k+2). Each column of $\mathbf{N_{BxV}}$ contains 'r' times 2, λ(v-1) times 1 and (r-λ)(v-1) times 0. Every pair of columns contains the pairs (1, 2) or (2, 1)'s 2λ times and (2, 2) and (1, 1) pairs zero times. Replace the elements 1 with β and 2 with α. Associate each block with an appropriate fraction of factorials (say 2^{k_1}) for v factors, with levels ±1. After augmenting n_0 central points, the resulting design has $N = 2^{k_1}$ v(b-r) + n_0 design points. Then from the rotatable condition $\sum_{u=1}^{N} x_{ui}^4 = 3.\sum_{u=1}^{N} x_{ui}^2 x_{uj}^2$, we can obtain

$$2^{k_1}\left(r\alpha^4 + \lambda(v-1)\beta^4 \right) = 3.\ 2^{k_1}\left(2\lambda\alpha^2\beta^2\right) \tag{2.1}$$

$$\Rightarrow r\alpha^4 + \lambda(v-1)\beta^4 - 6\lambda\alpha^2\beta^2 = 0 \tag{2.2}$$

Let $t = \alpha^2/\beta^2$, then (2.2) can be expressed in the quadratic form as

$$r\,t^2 - 3\lambda t + \lambda(v-1) = 0 \tag{2.3}$$

The roots are real if $9\lambda^2 \geq r\lambda(v-1)$. Choose any real value for 'β', then the real value for 'α' can be obtained as $\alpha^2 = t\beta^2$. The resulting design D provides a v-dimensional Second Order Rotatable Design in five levels.

Example 2.1: Let N_1 be the incidence matrix of a Balanced Incomplete Block Design with parameters v = 4, b = 6, r = 3, k = 2 and λ = 1, and $N_2 = I_v$. The resulting Balanced Ternary Design is with parameters V = 4, B = 12, R = 9, K = 3 and π = 4. The Second Order Rotatable Design with four factors is presented in Table 2.1.

Table-2.1: Construction of Second Order Rotatable Design (SORD)

N_1	N_2	N	SORD
$\begin{bmatrix} 1&1&0&0\\ 1&0&1&0\\ 1&0&0&1\\ 0&1&1&0\\ 0&1&0&1\\ 0&0&1&1 \end{bmatrix}$	$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}$	$\begin{bmatrix} 2&1&0&0\\ 2&0&1&0\\ 2&0&0&1\\ 1&2&0&0\\ 0&2&1&0\\ 0&2&0&1\\ 1&0&2&0\\ 0&1&2&0\\ 0&0&2&1\\ 1&0&0&2\\ 0&1&0&2\\ 0&0&1&2 \end{bmatrix}$	$\begin{bmatrix} \pm\alpha&\pm\beta&0&0\\ \pm\alpha&0&\pm\beta&0\\ \pm\alpha&0&0&\pm\beta\\ \pm\beta&\pm\alpha&0&0\\ 0&\pm\alpha&\pm\beta&0\\ 0&\pm\alpha&0&\pm\beta\\ \pm\beta&0&\pm\alpha&0\\ 0&\pm\beta&\pm\alpha&0\\ 0&0&\pm\alpha&\beta\\ \pm\beta&0&0&\pm\alpha\\ 0&\pm\beta&0&\pm\alpha\\ 0&0&\pm\beta&\pm\alpha \end{bmatrix}$

The real solution for'‘t’ is 1. Select value for β arbitrarily and accordingly α can be evaluated.

Series 2.2: A series of Second Order Rotatable Designs can be constructed using Balanced Ternary Designs of Tyagi and Rizwi (1979) is like as follows.

Step-1: Let N_1 be the incidence matrix of a Balanced Incomplete Block Design with parameters v, b, r, k, λ and let $N_2 = 2I_v$, where I_v is the identity matrix of order ‘v’. The Balanced Ternary Designs are derived by adding the elements of j^{th} row of N_2 those rows of N_1 which contain zero in the j^{th} column, then v(b-r) blocks so formed constitute a Balanced Ternary Design with parameters V = v, B = v(b-r), R = (k+2) (b-r), K = k+2 and π = (r-λ)(4+k-1).

Step-2: Replace the elements 2 with α and 1 with β, then associate each block with an appropriate fraction of factorials (say 2^{k_1}) with levels ±1 such that no lower order interaction effects are confounded.

Step-3: Add n_0 ($n_0 > 0$) central design points (0, 0, … 0) to the resulting design, then total number design points in the design is $n = 2^{k_1}$ v(b-r) + n_0.

Step-4: The levels ‘α’ and ‘β’ can be obtained such that $t = \alpha^2 / \beta^2$. Real roots for t can be obtained as $t = \frac{3(b-2\lambda) \pm \sqrt{9(b-2\lambda)^2 - 4(b-r)[(v-1)(r-\lambda)-\lambda(v-k)]}}{2(b-r)}$. Choose the value for β then obtain the value of α as $\alpha^2 = t\beta^2$. The resulting design provides a v-dimensional Second Order Rotatable Design with five levels (± α, ± β, 0).

Theorem 2.2: A Second Order Rotatable Design exists with five levels (±α, ±β, 0) using a Balanced Ternary Design with parameters V = v, B = v(b-r), R = (k+2) (b-r), K = k+2 and π = (r-λ)(4+k-1).

Proof: Let N_{BXV} be the incidence matrix of a balanced Ternary Design with parameters V = v, B = v(b-r), R = (k+2) (b-r), K = k+2 and π = (r-λ)(4+k-1). Each column of $\mathbf{N_{BxV}}$ contains 2’s and 1’s ‘b-r-λ’ and ‘r-λ+λ(v-k)’ times. Every pair of columns contains the pairs (1,2) and (2,1) occurs ‘r-λ’ and ‘b-r-λ’ times. Replace the elements 1 with β and 2 with α. Associate each block with an appropriate fraction of factorials (say 2^{k_1}) for v factors, with levels ±1. After augmenting n_0 central points, the resulting design has N= 2^{k_1} v(b-r)+n_0 design points. Then from the rotatable condition $\sum_{u=1}^{N} x_{ui}^4 = 3. \sum_{u=1}^{N} x_{ui}^2 x_{uj}^2$, we can obtain

$$2^{k_1} \{(b-r)\,\alpha^4 + (r-\lambda)(v-1)\,\beta^4\} = 3.\ 2^{k_1} \{\lambda(v-k)\,\beta^4 + (b-2\lambda)\alpha^2\beta^2\} \quad (2.4)$$

From the rotatable condition, we obtain

$$r\alpha^4 - 3\,(b-2\lambda)\beta^4 - 3v\alpha^2\beta^2 = 0 \quad (2.5)$$

Let $t = \alpha^2/\beta^2$, then (2.5) can be expressed in the quadratic form as

$$(b-r)\, t^2 - 3\,(b-2\lambda)\, t + [(v-1)\,(r-\lambda) - \lambda(v-k)] = 0 \qquad (2.6)$$

The roots of the quadratic equation are $t = \dfrac{3(b-2\lambda) \pm \sqrt{9(b-2\lambda)^2 - 4(b-r).[(v-1)(r-\lambda) - \lambda(v-k)]}}{2(b-r)}$.

The roots are real if the discriminant in nonnegative. Choose any real value for 'β' then the real value for 'α' can be obtained as $\alpha^2 = t\beta^2$. The resulting design 'D' provides a v-dimensional Second Order Rotatable Design in five levels.

Example 2.2: Let N_1 be the incidence matrix of a Balanced Incomplete Block Design with parameters v = 4, b = 6, r = 3, k = 2 and λ = 1, and $N_2 = 2I_v$. The resulting Balanced Ternary Design is with parameters V = 4, B = 12, R = 12, K = 4 and π = 10. The Second Order Rotatable Design with four factors is presented in. Table 2.2

Table-2.2: Construction of Second Order Rotatable Design (SORD)

N_1	N_2	N	SORD
$\begin{bmatrix} 1&1&0&0\\ 1&0&1&0\\ 1&0&0&1\\ 0&1&1&0\\ 0&1&0&1\\ 0&0&1&1 \end{bmatrix}$	$\begin{bmatrix} 2&0&0&0\\ 0&2&0&0\\ 0&0&2&0\\ 0&0&0&2 \end{bmatrix}$	$\begin{bmatrix} 2&1&1&0\\ 2&1&0&1\\ 2&0&1&1\\ 1&2&1&0\\ 1&2&0&1\\ 0&2&1&1\\ 1&1&2&0\\ 1&0&2&1\\ 0&1&2&1\\ 1&1&0&2\\ 1&0&1&2\\ 0&1&1&2 \end{bmatrix}$	$\begin{bmatrix} \pm\alpha&\pm\beta&\pm\beta&0\\ \pm\alpha&\pm\beta&0&\pm\beta\\ \pm\alpha&0&\pm\beta&\pm\beta\\ \pm\beta&\pm\alpha&\pm\beta&0\\ \pm\beta&\pm\alpha&0&\pm\beta\\ 0&\pm\alpha&\pm\beta&\pm\beta\\ \pm\beta&\pm\beta&\pm\alpha&0\\ \pm\beta&0&\pm\alpha&\pm\beta\\ 0&\pm\beta&\pm\alpha&\pm\beta\\ \pm\beta&\pm\beta&0&\pm\alpha\\ \pm\beta&0&\pm\beta&\pm\alpha\\ 0&\pm\beta&\pm\beta&\pm\alpha \end{bmatrix}$

The real solution for 't' is 3.63. Select value for β arbitrarily and accordingly α can be evaluated.

ACKNOWLEDGEMENT

The authors are thankful to the referees for improving the version of manuscript. The first author is also thankful to the UGC for providing financial assistance in the form of Fellowship under BSR-RFSMS.

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Source of support: UGC, India., Conflict of interest: None Declared.