ANTI Q-FUZZY B–IDEALS IN B–ALGEBRA

Dr. A. PRASANNA1, *M. PREMKUMAR2 AND HAJEE. Dr. S. ISMAIL MOHIDEEN3

1Assistant Professor, PG and Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli-620020, Tamilnadu, India.

2Research Scholar and Assistant Professor, Department of Mathematics, Mahendra Engineering College (Autonomous), Tiruchengode, Namakkal-637 503, Tamilnadu, India.

3Principal, Head and Associate Professor, PG and Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli-620020, Tamilnadu, India.

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ABSTRACT

In this paper, we introduce the notion of anti Q-fuzzy B–ideals of B-algebras, lower level B – Ideal and prove some results on these. We show that a Q-fuzzy subset of a B-algebra is a Q-fuzzy B-ideal if and only if the complement of this Q-fuzzy subset is an anti Q-fuzzy B-ideal.

Keywords: B–algebra, B–Ideal, Fuzzy B–Ideal, Anti Fuzzy B–Ideal, Q–Fuzzy B–Ideal, Anti Q–Fuzzy B–Ideal.

1. INTRODUCTION


2. PRELIMINARIES

In this section we give some basic definitions and preliminaries of B-algebras and introduce Q-fuzzy B-ideal.

Definition 2.1: (Jung R. Cho and H.S.Kim [4]) A B-algebra is a non-empty set X with a constant 0 and a binary operation “∗” satisfying the following axioms:

(i) $x \ast x = 0$
(ii) $x \ast 0 = x$
(iii) $(x \ast y) \ast z = x \ast (z \ast (0 \ast y)), \text{for all } x, y, z \in X$

For brevity we also call X a B-algebra. In X we can define a binary relation “≤” by $x \leq y$ if and only if $x \ast y = 0$.

Definition 2.2: (Jung R. Cho and H.S.Kim [4]) A non-empty subset $M$ of a B-algebra $X$ is called a sub-algebra of $X$ if $x \ast y \in M$ for any $x, y \in M$. 

Corresponding Author: *M. Premkumar

Research Scholar and Assistant Professor, Department of Mathematics, Mahendra Engineering College (Autonomous), Tiruchengode, Namakkal-637 503, Tamilnadu, India.
Definition 2.3: (Jiayin Peng [5]) Let $\alpha$ be a fuzzy set in a B-algebra. Then $\alpha$ is called a fuzzy subalgebra of $X$ if $\alpha(x \ast y) \geq \alpha(x) \land \alpha(y)$ for all $x, y \in X$.

Definition 2.4: (Jung R. Cho and H.S. Kim [4]) A non-empty subset $N$ of a B-algebra $X$ is called a B-ideal of $X$ if it satisfies for $x, y, z \in X$
(i) $0 \in N$
(ii) $(x \ast y) \in \text{Nand} (z \ast x) \in N$ implies $(y \ast z) \in N$

Definition 2.5: (L. A. Zadeh [1]) Let $X$ be a non-empty set. A fuzzy subset $\alpha$ of the set $X$ is a mapping $\alpha: X \rightarrow [0,1]$

Definition 2.6: (F. Adam and N. Hassan [1]) Let $Q$ and $G$ be any two sets. A mapping $\alpha: \{0,1\} \rightarrow [0,1]$ is called a fuzzy set in $G$.

Definition 2.7: (R. Muthuraj et al [7]) Let $\alpha$ be a Q-fuzzy set in B-algebra. Then $\alpha$ is called a Q-fuzzy sub-algebra of $X$ if $\alpha(x \ast y, q) \geq \min\{\alpha(x, q), \alpha(y, q)\}$ for all $x, y \in X \& q \in Q$

Definition 2.8: (R. Muthuraj et al [7]) Let $\alpha$ be a Q-fuzzy set in set $X$. Then the complement $\bar{\alpha}$ is the Q-fuzzy subset of $X$ given by $\bar{\alpha}(x, q) = 1 - \alpha(x, q)$ for all $x \in X \& q \in Q$.

Definition 2.9: (R. Muthuraj et al [6]) Let $\alpha$ be a Q-fuzzy set in a set $X$. For $S \subseteq [0,1]$, the set $\alpha_S = \{x \in \alpha(x, q) \geq S \text{ for all } q \in Q\}$ is called a level subset of $\alpha$.

Definition 2.10: (R. Muthuraj et al [7]) A Q-fuzzy set $\alpha$ in $X$ is called Q-fuzzy B-ideal of $X$ if it satisfies the following axioms:
(i) $\alpha(0,q) \geq \alpha(x,q)$
(ii) $\alpha(y \ast z, q) \geq \min\{\alpha(x \ast y, q), \alpha(z \ast x, q)\}$ for all $x, y \in X \text{ and } q \in Q$.

3. ANTI Q-FUZZY B-IDEALS

Definition 3.1: A Q-fuzzy set $\alpha$ of a B-algebra $X$ is called and Anti Q-Fuzzy subalgebra of $X$ if $\alpha(x \ast y, q) \leq \max\{\alpha(x, q), \alpha(y, q)\}$ for all $x, y \in X \text{ and } q \in Q$.

Definition 3.2: A Q-fuzzy set $\alpha$ in $X$ is called an anti Q-fuzzy B-ideal of $X$ if it satisfies the following axioms:
(i) $\alpha(0,q) \leq \alpha(x,q)$
(ii) $\alpha(y \ast z, q) \leq \max\{\alpha(x \ast y, q), \alpha(z \ast x, q)\}$ for all $x, y \in X \text{ and } q \in Q$

Theorem 3.3: Every anti Q-fuzzy B-ideal of a B-algebra $X$ is order preserving.

Proof: Let $\alpha$ be an anti Q-fuzzy B-ideal of a B-algebra $X$.
Let $x, y \in X$ and $q \in Q$ be such that $y \leq x$ if and only if $y \ast x = 0$

Now,
$$\alpha(y, q) = \alpha(0 \ast y, q)$$
$$\leq \max\{\alpha(x \ast 0, q), \alpha(y \ast x, q)\}$$
$$\leq \max\{\alpha(x, q), \alpha(0, q)\}$$
$$\leq \alpha(x, q)$$
$$\Rightarrow \alpha(y, q) \leq \alpha(y, q)$$

Theorem 3.4: A Q-fuzzy subset $\alpha$ of a B-algebra $X$ is a Q-fuzzy B-ideal of $X$ if and only if its complement $\bar{\alpha}$ is an anti Q-fuzzy B-ideal of $X$.

Proof: Let $\alpha$ be a Q-fuzzy B-ideal of $X$ and let $x, y, z \in X$ and $q \in Q$.
To prove:
$\bar{\alpha}$ is an anti Q-fuzzy B-ideal of $X$.
(i) $\bar{\alpha}(0,q) = 1 - \alpha(0,q)$
$$\leq 1 - \alpha(x,q)$$
$$= \bar{\alpha}(x,q)$$
(ii) $\bar{\alpha}(y \ast z, q) = 1 - \alpha(y \ast z, q)$
$$\leq 1 - \min\{\alpha(x \ast y, q), \alpha(z \ast x, q)\}$$
$$\leq 1 + \max\{-\alpha(x \ast y, q), -\alpha(z \ast x, q)\}$$
$$\leq \max\{1 - \alpha(x \ast y, q), 1 - \alpha(z \ast x, q)\}$$
$$\Rightarrow \bar{\alpha}(y \ast z, q) \leq \max\{\bar{\alpha}(x \ast y, q), \bar{\alpha}(z \ast x, q)\}$$
Thus, $\tilde{a}$ is an anti Q-fuzzy B-ideal of $X$.
The converse part can also be prepared similarly.
Hence, the proof.

**Definition 3.5:** Let $\alpha$ be a Q-fuzzy subset of a B-algebra $X$. For $S \in [0,1]$, the set $\alpha^S = \{ x \in X | \mu(x, q) \leq S \}$ is called a lower level cut of $\alpha$.

Clearly, $\alpha' = X$ and $\alpha_0^S \cup \alpha^S = X$ for $S \in [0,1]$. If $S_1 \leq S_2$ then $\alpha^{S_1} \subseteq \alpha^{S_2}$.

**Theorem 3.6:** Let $\alpha$ be a Q-fuzzy subset of a B-algebra $X$. If $\alpha$ is an anti Q-fuzzy B-ideal of $X$, then the lower level cut $\alpha^S$ is a B-ideal of $X$ for all $S \in [0,1], S \geq \alpha(0, q)$.

**Proof:** Let $\alpha$ be an anti Q-fuzzy B-ideal of $X$. Then for all $x, y \in X$ and $q \in Q$,

(i) $\alpha(0, q) \leq \alpha(x, q)$

(ii) $\alpha(y \ast z, q) \leq \max\{\alpha(x \ast y, q), \alpha(z \ast x, q)\}$

To prove: $\alpha^S$ is a B-ideal of $X$.

Let $x, y, z \in \alpha^S$

\[ \Rightarrow \alpha(x, q) \leq S \]

(i) Since $\alpha(0, q) \leq \alpha(x, q)$

\[ \Rightarrow \alpha(0, q) \leq S \]

\[ \Rightarrow 0 \in \alpha^S \]

(ii) $x \ast y \in \alpha^S \& z \ast x \in \alpha^S$

\[ \Rightarrow \alpha(x \ast y, q) \leq S \& \alpha(z \ast x, q) \leq S \]

\[ \alpha(y \ast z, q) \leq \max\{\alpha(x \ast y, q), \alpha(z \ast x, q)\} \]

\[ \leq \max\{S, S\} \]

\[ = S \]

\[ \Rightarrow \alpha(y \ast z, q) \leq S \]

\[ \Rightarrow y \ast z \in \alpha^S \]

Thus, $\alpha^S$ is a B-ideal of $X$.

Hence, the proof.

**Theorem 3.7:** Let $\alpha$ be a Q-fuzzy subset of a B-algebra $X$. If for each $S \in [0,1], S \geq \alpha(0, q)$ the lower level cut $\alpha^S$ is a B-ideal of $X$, then $S$ is an anti Q-fuzzy B-ideal of $X$.

**Proof:**

$\alpha^S$ is a B-ideal of $X$.

\[ 0 \in \alpha^S \]

\[ x \ast y \in \alpha^S \& z \ast x \in \alpha^S \Rightarrow y \ast z \in \alpha^S \]

To prove $\alpha^S$ is an anti Q-fuzzy B-ideal of $X$.

For all $x, y \in X$ and $q \in Q$

(i) $x \ast y \in \alpha^S \& z \ast x \in \alpha^S$

\[ \Rightarrow \alpha(x \ast y, q) \leq S \& \alpha(z \ast x, q) \leq S \]

Let $\alpha(x \ast y, q) = S \& \alpha(z \ast x, q) = S$

\[ y \ast z \in \alpha^S \]

\[ \Rightarrow \alpha(y \ast z, q) \leq S \]

\[ = \max\{S, S\} \]

\[ \leq \max\{\alpha(x \ast y, q), \alpha(z \ast x, q)\} \]

\[ \Rightarrow \alpha(y \ast z, q) \leq \max\{\alpha(x \ast y, q), \alpha(z \ast x, q)\} \]

(ii) $0 \in \mu^t$

Since $x \ast x = 0$

\[ \alpha(0, q) = \alpha(x \ast x, q) \leq \max\{\alpha(x, q), \alpha(x, q)\} \]

\[ = \alpha(x, q) \]

\[ \Rightarrow \alpha(0, q) \leq \alpha(x, q) \]

\[ \Rightarrow \alpha^S \text{ is an anti Q-fuzzy B-ideal of } S. \]

Hence, the proof.
4. CONCLUSION

This paper tried to define the Anti Q-Fuzzy B-Ideals on B-Algebra and proved some theorems on them. This Concept can further be generalized to normalization of Q-fuzzy B-Ideals in B-Algebra using n-fold translation and multiplication.

5. REFERENCES

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