

COMBINED EFFECTS OF PRESSURE-DEPENDENT VISCOSITY AND MICROPOLAR FLUIDS
ON SQUEEZE FILM LUBRICATION BETWEEN PARALLEL STEPPED PLATES

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ABSTRACT

In this paper, a theoretical analysis of the effects of micropolar on the squeeze film lubrication between parallel stepped plates is presented. The modified Reynolds equation accounting for the viscosity variation in micropolar fluid is mathematically modelled. A closed form expressions are obtained for load carrying capacity and squeeze film time. According to the results obtained, the influence of micropolar enhances the squeeze film pressure, load carrying capacity and decreases the response time as compared to the classical Newtonian lubricant case. The load carrying capacity decreases as the step height increases.

Keyword: Squeeze film, Parallel stepped plates, Pressure-Dependent Viscosity, Micropolar fluid.

1. INTRODUCTION

The squeeze film lubrication phenomenon is widely observed in several applications such as gears, bearings, machine tools, rolling elements and auto motive engines. The squeeze film behaviour arises from the phenomenon of two lubricated surfaces approaching each other with a normal viscosity. The squeeze film phenomenon arises when the two lubricating surfaces move towards each other in the normal direction and generates a positive pressure and hence support a load. This phenomenon arises from the fact that a viscous lubricant present between the two surfaces cannot be instantaneously squeezed out when the two surfaces moving towards each other and this action provides a cushioning effect in bearings. The squeeze film lubrication between two infinitely long parallel plates is studied by Cameron [1].

The use of different liquids as lubricants under different circumstances has gained its importance with a development of modern machines. In most of these lubricating oils the additives of high molecular weight polymers are present as a kind of viscosity index improvers. The presence of these additives in the lubricant prevents the viscosity variation of the lubricants with a change in temperature. The lubricants with additives causes the non-Newtonian behaviour of the lubricating oils since the classical continuum mechanics of fluids neglects the size of fluid particles in the flow of fluids and hence several microcontinuum theories has been proposed to take into account of the intrinsic motion of material constituents [2, 3]. The theory of micropolar fluids introduced by Eringen [4] deals with a class of fluid which exhibits certain microscopic effects arising from the local structures and micro motion of fluid elements. These fluids can support stress moments and body moments and are influenced by the spin inertia. The flow of an incompressible fluid between two parallel plates due to normal motion of the plates is investigated by Bujurke *et al.* [5]. The unsteady flow between two parallel discs with arbitrary varying gap width was studied by Ishizawa [6]. The micropolar fluids are the subclass of micro fluids that includes the effects of local rotating inertia, couple stresses and inertial spin. Several investigators used the micropolar fluid theory for the study of several bearing systems. Allen and Kline [7] pointed out that micropolar fluid lubrication theory can be used as a first approximation for conventional lubricants that become contaminated with dirt and metal particles under general operating conditions and can be treated as fluid suspensions. The lubrication theory for micropolar fluid and its application to journal bearings is studied by Prakash and Sinha [8].

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Squeeze film theory for micropolar fluids is studied by Prakash and Sinha [9]. Lin *et al.*, [10] studied the effect of viscosity-pressure dependency on the non-Newtonian squeeze film of parallel circular plates. Squeeze film lubrication between parallel stepped plates with couplestress fluids is studied by Kashinath [11], in this paper he studied that the load carrying capacity decreases as the step height increases. Micropolar fluid poro-elastic squeeze film lubrication between a cylinder and a rough flat plate-a special reference to synovial joint lubrication is analysed by Naduvanamani and Savitramma [12]. Naduvanamani and Santosh analyzed theoretically micropolar fluid squeeze film lubrication of short partial porous journal bearing [13]. Naduvanamani and Marali presented the dynamic Reynolds equation for micropolar fluids and the analysis of plane inclined slider bearings with squeezing effect and have shown that the micropolar fluids provide an improved characteristics for both steady state and the dynamic stiffness and damping characteristics [14]. It is reported that the effect of viscosity-pressure dependency increases the load carrying capacity and lengthen the squeeze film time.

The viscosity of all liquids and particularly of hydrocarbon lubricants decreases with increasing temperature. This variation in viscosity with temperature is important in many practical applications where lubricants are required to function over a wide range of temperature. There is no fundamental mathematical relationship that will accurately predict the variation in the viscosity of oil with temperature. The formulae proposed for defining the viscosity temperature relationship are purely empirical and, for accurate calculations require the experimental data. In this paper, it is assumed that thermal equilibrium according to a given law. However, in order to apply this law, the temperature at each point should be known, which requires a complete thermal calculation. The viscosity- temperature relationship can be replaced by a relation between the viscosity and the film thickness. This is justified as it has been verified experimentally that the highest temperature occurs in zones where the film thickness is least.

In all the above analysis the lubricant viscosity μ is assumed to be a constant value. Barus and co-workers [15, 16] analyzed that the viscosity-pressure dependency using the relation

$$\mu = \mu_0 e^{\alpha p} \quad (1)$$

Where α denotes the coefficient of pressure-dependent viscosity (PDV) and μ_0 is the viscosity at ambient pressure and a constant temperature. The above relation indicates the lubricant viscosity is increasing exponentially and it could alter the predicted performance of squeeze film bearings.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The constitutive equations for micropolar fluids proposed by Eringen [2] simplify considerably under the usual assumptions of hydrodynamic lubrication. The resulting equations under steady-state conditions are.

$$\left(\mu + \frac{\chi}{2} \right) \frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial v_1}{\partial y} - \frac{\partial p}{\partial x} = 0 \quad (2)$$

$$\gamma \frac{\partial^2 v_1}{\partial y^2} - 2\chi v_1 + \chi \frac{\partial u}{\partial y} = 0 \quad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

Where (u, v) are the velocity components of the lubricant in the x , and y directions, respectively, (v_1, v_2) are micro rotational velocity components, χ is the spin viscosity, γ is the viscosity coefficient for micropolar fluids, and μ is the Newtonian viscosity coefficient.

The relevant boundary conditions are

- i) At the bearing surface ($y=0$)

$$u = 0, v = 0 \quad (5a)$$

$$v_1 = 0. \quad (5b)$$

- ii) At the upper surface ($y=h$)

$$u = 0, v = \frac{\partial h}{\partial t} \quad (6a)$$

$$v_1 = 0. \quad (6b)$$

The solution of the equation (3) using relation (1) with boundary conditions (5a) and (6a) is

$$u = \frac{1}{\mu_0 e^{\beta p}} \left(\frac{y^2}{2} \frac{\partial p}{\partial x} + A_1 y \right) - \frac{2N^2}{m} (A_2 \sinh my + A_3 \cosh my) + A_4 \quad (7)$$

$$v_1 = A_2 \cosh my + A_3 \sinh my - \frac{1}{2\mu_0 e^{\beta p}} \left(y \frac{\partial p}{\partial x} + A_1 \right) \quad (8)$$

where,

$$A_1 = 2\mu_0 e^{\beta p} A_2$$

$$A_2 = \frac{A_3 \sinh mh - \frac{h}{2\mu_0 e^{\beta p}} \frac{\partial p}{\partial x}}{1 - \cosh mh}$$

$$A_3 = \frac{h}{2\mu_0 e^{\beta p}} \frac{\partial p}{\partial x} \left\{ \frac{h}{2} (\cosh mh - 1) + h - \frac{N^2}{m} \sinh mh \right\} \frac{1}{A_5}$$

$$A_4 = \frac{2N^2}{m} A_3$$

$$A_5 = \frac{h}{\mu} \left\{ \sinh mh - \frac{2N^2}{mh} (\cosh mh - 1) \right\}$$

$$m = \frac{N}{l}, \quad N = \left(\frac{\chi}{\chi + 2\mu} \right)^{1/2}, \quad l = \left(\frac{\gamma}{4\mu} \right)^{1/2}$$

where, the symbols have their usual meaning as given in nomenclature.

The micropolar parameters N and l are assumed to be independent of viscosity variation for mathematical simplicity. It is assumed that χ and γ varies in the same way as μ varies.

$$\frac{\partial}{\partial x} \left\{ f(N, l, h) e^{-\beta p} \frac{\partial p}{\partial x} \right\} = 12 \frac{\partial h}{\partial t} \quad (9)$$

where,
$$f(N, l, h) = h^3 + 12l^2 h - 6Nlh^2 \coth \left(\frac{Nh}{2l} \right) \quad (10)$$

The volume flow rate of the lubricant is given by

$$Q = 2b \int_0^h u dy \quad (11)$$

Substituting the expression for u from equation (7) in equation (11) the volume flux is obtained in the form

$$Q = \frac{be^{-\beta p}}{6\mu_0} \frac{\partial p}{\partial x} f(N, l, h) \quad (12)$$

Introducing the non-dimensional variables.

$$r^* = \frac{r}{R}, \quad h^* = \frac{h}{h_2}, \quad p^* = \frac{ph_2^3}{\mu_0 R^2 (\partial h / \partial t)}, \quad l^* = \frac{l}{h_2}, \quad G = \frac{\beta \mu_0 R^2 (-dh / dt)}{h_2^3}$$

In equations (9) and (12) the non-dimensional modified Reynolds equation and the non-dimensional volume flow rate are obtained in the form

$$\frac{\partial}{\partial x^*} \left\{ e^{-Gp^*} f^*(N, h^*, l^*) \frac{\partial p^*}{\partial x^*} \right\} = -12 \quad (13)$$

$$Q^* = \frac{e^{-Gp^*}}{6} \frac{\partial p^*}{\partial x^*} f^*(N, h^*, l^*) \quad (14)$$

$$\text{where } f^*(N, h^*, l^*) = h^{*3} + 12l^{*2}h^* - 6Nl^*h^{*2}\text{Coth}\left(\frac{Nh^*}{2l^*}\right) \quad (15)$$

Reynolds equations in region I: ($0 \leq x^* \leq K$)

$$\frac{\partial}{\partial x^*} \left\{ e^{-Gp_1^*} f_1^*(N, h_1^*, l^*) \frac{\partial p_1^*}{\partial x^*} \right\} = -12 \quad (16)$$

$$\text{where } f_1^*(N, h^*, l^*) = h_1^{*3} + 12l^{*2}h_1^* - 6Nl^*h_1^{*2}\text{Coth}\left(\frac{Nh_1^*}{2l^*}\right) \quad (17)$$

Reynolds equations in region II: ($K \leq x^* \leq 1$)

$$\frac{\partial}{\partial x^*} \left\{ e^{-Gp_2^*} f_2^*(N, l^*) \frac{\partial p_2^*}{\partial x^*} \right\} = -12 \quad (18)$$

$$\text{where } f_2^*(N, l^*) = 1 + 12l^{*2} - 6Nl^*\text{Coth}\left(\frac{N}{2l^*}\right) \quad (19)$$

The pressure boundary conditions are

$$\frac{dP_1^*}{dr^*} = 0 \quad \text{at } x^* = 0 \quad (20)$$

$$P_2^* = 0 \quad \text{at } x^* = 1 \quad (21)$$

$$P_1^* = P_2^* \quad \text{at } x^* = R \quad (22)$$

$$Q_1^* = Q_2^* \quad \text{at } x^* = R \quad (23)$$

where Q_1^* and Q_2^* are the non dimensional volume flow rates in region I and region II respectively.

Solving equations (16) and (18) using the boundary conditions (20), (21), (22) and (23) gives

Pressure in region I: ($0 \leq x^* \leq K$)

$$p_1^* = -\frac{1}{G} \ln \left\{ \frac{6G(x^{*2} - K^2)}{f_1^*(N, h_1^*, l^*)} + \frac{6G(K^2 - 1)}{f_1^*(N, l^*)} + 1 \right\} \quad (24)$$

Pressure in region II: ($K \leq x^* \leq 1$)

$$p_2^* = -\frac{1}{G} \ln \left\{ \frac{6G(x^{*2} - 1)}{f_2^*(N, l^*)} + 1 \right\} \quad (25)$$

Integrating the film pressure, one can obtain the load-carrying capacity

$$W = 2b \int_0^{KR} p_1 dx + 2b \int_{KR}^1 p_2 dx \quad (26)$$

The non-dimensional load carrying capacity is obtained in the form

$$\begin{aligned} W^* &= \frac{Wh_2^3}{\mu_0 R^3 b (-dh/dt)} \\ &= -\frac{1}{G} \int_0^K \ln \left\{ \frac{6Gx^{*2}}{f_1^*(N, h_1^*, l^*)} - \frac{6G}{f_2^*(N, l^*)} + 1 \right\} dx^* - \frac{1}{G} \int_K^1 \ln \left\{ \frac{36(x^{*2} - 1)}{f_2^*(N, l^*)} + 1 \right\} dx^* \end{aligned} \quad (27)$$

The squeezing time for reducing the film thickness from an initial value $h_2^* = 1$ to a final value h_f^* is given by

$$T^* = \frac{Wth_0^2}{\mu_0 R^3 b}$$

$$= -\frac{1}{G} \int_{h_2^*}^1 \int_0^K \ln \left\{ \frac{6Gx^{*2}}{f_1^*(N, h_2^*, h_s^*, l^*)} - \frac{6G}{f_2^*(N, h_2^*, l^*)} + 1 \right\} dx^* dh_2^* \quad (28)$$

$$- \frac{1}{G} \int_{h_2^*}^1 \int_K^1 \ln \left\{ \frac{6G(x^{*2} - 1)}{f_2^*(N, h_2^*, l^*)} + 1 \right\} dx^* dh_2^*$$

where $f_1^*(h_2^*, h_s^*, N, l^*) = (h_2^* + h_s^*)^3 + 12l^{*2}(h_2^* + h_s^*) - 6Nl^*(h_2^* + h_s^*)^2 \text{Co} h\{N(h_2^* + h_s^*)/2l^*\}$
 $f_2^*(h_2^*, N, l^*) = h_2^{*3} + 12l^{*2}h_2^* - 6Nl^*h_2^{*2} \text{Coth}(Nh_2^*/2l^*)$
 $h_2^* = h_2/h_0, h_s^* = h_s/h_0, h_f^* = h_f/h_0, l^* = l/h_0$

3. RESULT AND DISCUSSIONS

On the basis of Barus [5] and Bartz and Ehert [6] analysis for pressure-dependent viscosity and the Eringens micropolar fluid model for lubricants, this paper predicts the squeeze film lubrication between parallel stepped plates with micropolar fluids. The results are analysed with respect to various non-dimensional parameters such as the coupling number, N, the additives length size parameter, l^* , the pressure-dependent viscosity parameter G and the step length K.

The following range of parameters is used for the discussion of squeeze film characteristics.

N=0, 0.2, 0.4 & 0.6;

$l^* = 0, 0.2, \& 0.4$;

G=0.0, 0.03 & 0.06;

3.1 Squeeze film pressure

The variation of non-dimensional squeeze film pressure P^* as a function of horizontal co-ordinate x^* for different values of G is depicted in figure 2. It is observed that P^* increases for the increasing values of l^* and G. Figure 3 depicts the variation of P^* with x^* for different values of N and G for the fixed values of $h_1^* = 1.2, k=0.7, l^*=0.4$. In this figure we observed that, P^* increases for the increasing values of N. Figure 4 shows the variation of non-dimensional maximum pressure P_{\max}^* with K for different values of l^* and G. In this figure we observed that maximum pressure is decreases for increasing values of K. Figure 5 the variation of non-dimensional P_{\max}^* with K maximum pressure increases for the increasing values of N and G for the fixed values of $h_1^* = 1.2, l^*=0.4$.

3.2 Load carrying capacity

The variation of non-dimensional load carrying capacity W^* with step height h_1^* for different values of l^* and G with K=0.7. It is observed that the non-dimensional load carrying capacity W^* decreases for increasing value of h_1^* . Figure 7 shows that W^* with h_1^* for different values of N and G with K=0.7. We observed that step height increases for decreasing values of W^* . Similarly figure 8 shows that W^* decreases for the increasing values of K for the fixed values of $h_1^* = 1.2$ and the different values of l^* and G. Figure 9 shows that the variation of W^* as a function K for different values of N and G. In this figure we observed that W^* is decreases for increasing the value of K.

The relative percentage increase in the non-dimensional load carrying capacity K_{w^*} is defined by

$$K_{w^*} = \left\{ \frac{(W_{PDV}^* - W_{Non-PDV}^*)}{W_{Non-PDV}^*} \right\} \times 100$$

3.3 Squeeze film time

The most important characteristics of the squeeze film bearing is the squeeze film time that is the time required for reducing the initial film thickness h_2 to h_0 to a final value h_f . Figure 10 represents the variation of non-dimensional squeeze film time T^* with h_f^* for different values of l^* and G with K=0.7. It is observed that the non-dimensional squeeze film time T^* decreases for increasing values of h_f^* . In the figure 11 shows that the variation of T^* with h_f^* for different values of N and G. It shows that the values of T^* increases for the decreasing values of h_f^* . In figure 12 it shows that T^* increases for the decreasing values of K. Figure 13 shows that the variation of non-dimensional squeeze film time T^* with K for different values of N and G for fixed values of $h_f^*=0.5, h_s^*=0.2$. It is observed that the non-dimensional squeeze film time T^* increases for decreasing values of K.

The relative percentage increase in the non-dimensional squeeze film time K_{T^*} is defined by

$$K_{T^*} = \left\{ \frac{(T_{PDV}^* - T_{Non-PDV}^*)}{T_{Non-PDV}^*} \right\} \times 100$$

4. CONCLUSIONS

The squeeze film lubrication between parallel stepped bearings with micropolar fluid as lubricant is studied on the basis of Eringen's micropolar fluid theory. On the basis of the numerical computations of the results presented. The squeeze film characteristics between parallel stepped plates can be improved by the use of lubricants with microstructure additives.

1. The squeeze film pressure decreases viscosity variation parameter increased.
2. The effect of Micropolar fluid enhances the load carrying capacity significantly.
3. When viscosity variation parameter increases the squeeze film time decreases.

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NOMENCLATURE

- h^* non-dimensional film thickness
 h_1 maximum film thickness
 h_2 minimum film thickness
 h_s^* step height ratio ($=h_s/h_2$)
 KL the position of the step $0 < KL < L$.
 l micropolar parameter $(\eta / \mu_0)^{1/2}$
 l^* non-dimensional micropolar parameter (l/h_2)
 p pressure in the film region
 p_1 fluid film pressure in the region $0 \leq x^* \leq KL$
 p_2 fluid film pressure in the region $KL \leq x^* \leq L$
 Q volume flow rate
 x, y Cartesian coordinate
 T time of approach
 T^* non-dimensional time of approach
 u, v velocity components
 W load carrying capacity
 W^* non-dimensional load carrying capacity
 β co efficient of pressure-dependent viscosity
 η material constant responsible for micropolar fluid
 μ dynamic viscosity
 μ_0 viscosity at atmospheric pressure

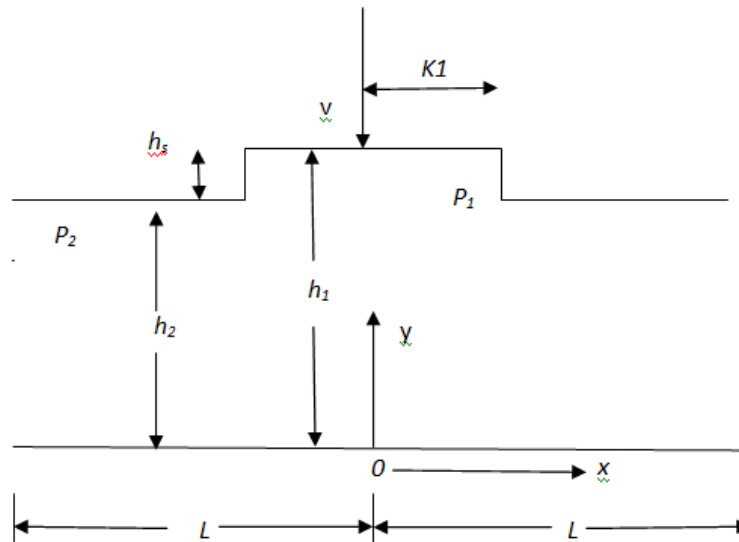


Figure-1: Squeeze film between parallel stepped plates

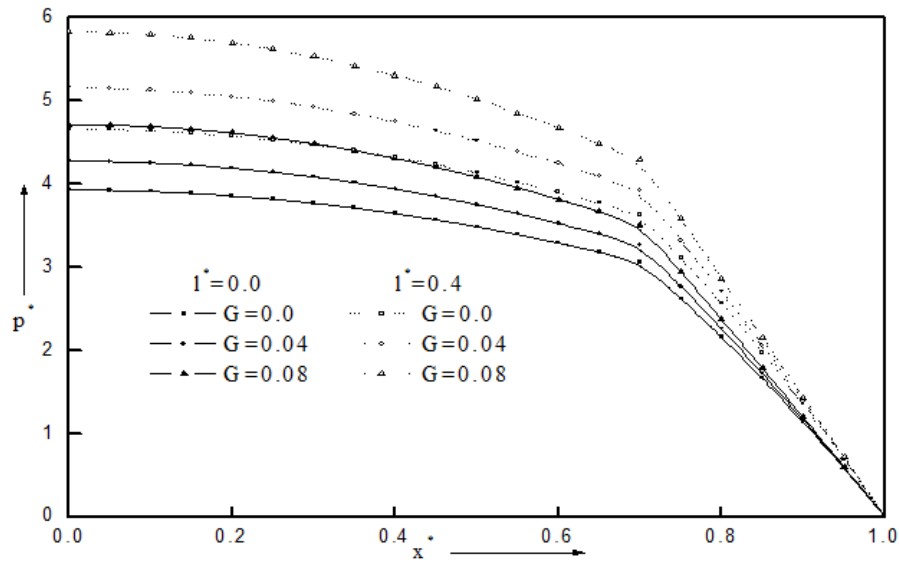


Figure 2 Variation of non-dimensional pressure p^* with x^* for different values of l^* and G with $h_1^* = 1.5$ and $K = 0.7$

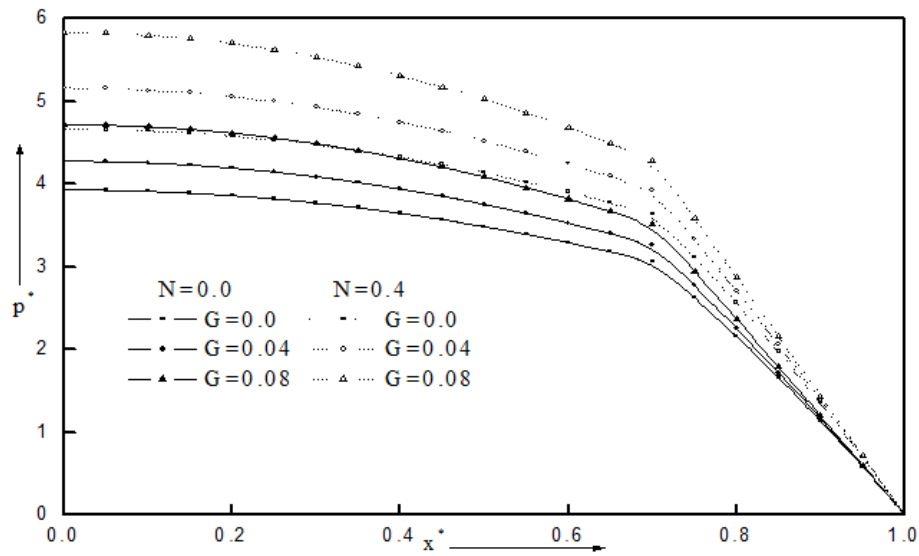


Figure 3 Variation of non-dimensional pressure p^* with x^* for different values of N and G with $h_1^* = 1.5$ and $K = 0.7$

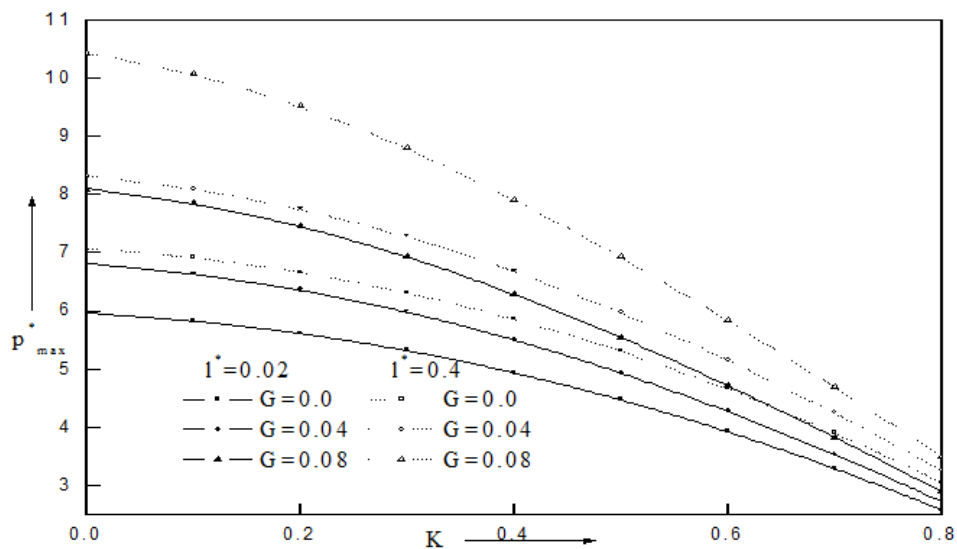


Figure 4 Variation of non-dimensional maximum pressure p^*_{max} with K for different values of l^* and G with $h_1^* = 1.5$

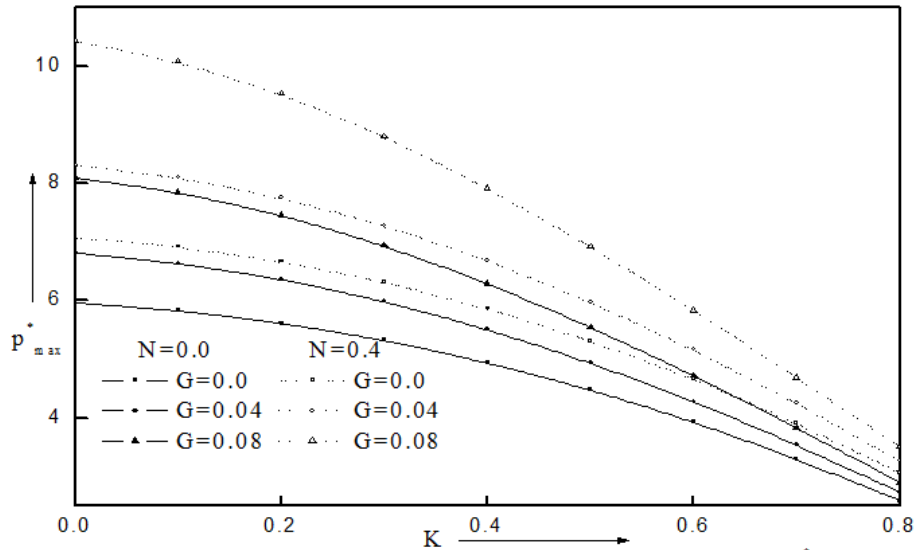


Figure 5 Variation of non-dimensional maximum pressure p^* with K for different values of N and G with $h_1^*=1.5$

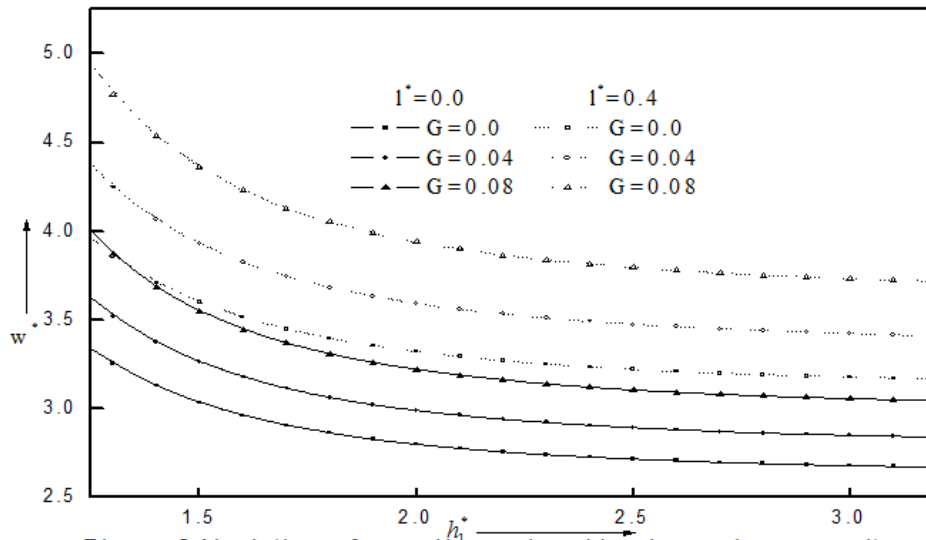


Figure 6 Variation of non-dimensional load carrying capacity w^* with h_1^* for different values of l^* and G with $K=0.7$.

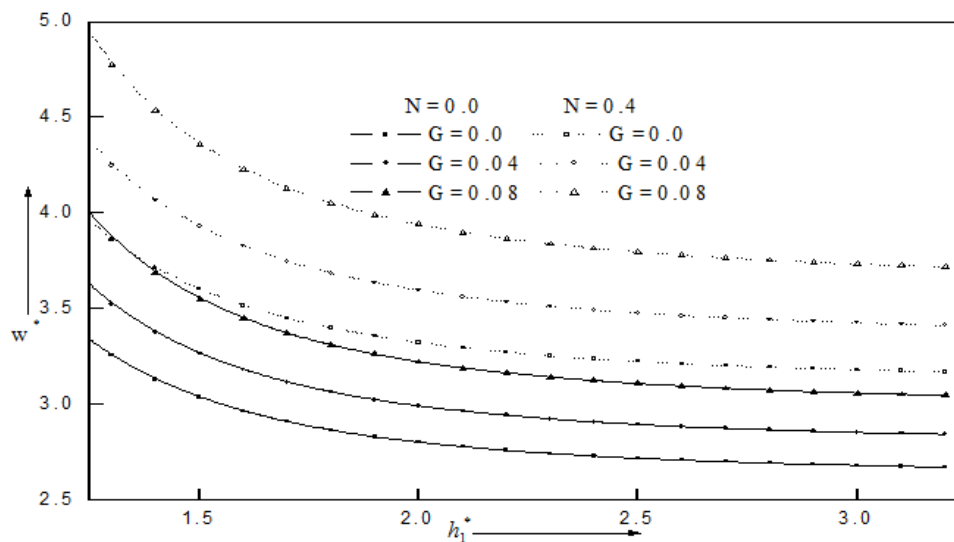


Figure 7 Variation of non-dimensional load carrying capacity w^* with h_1^* for different values of N and G with $K=0.7$

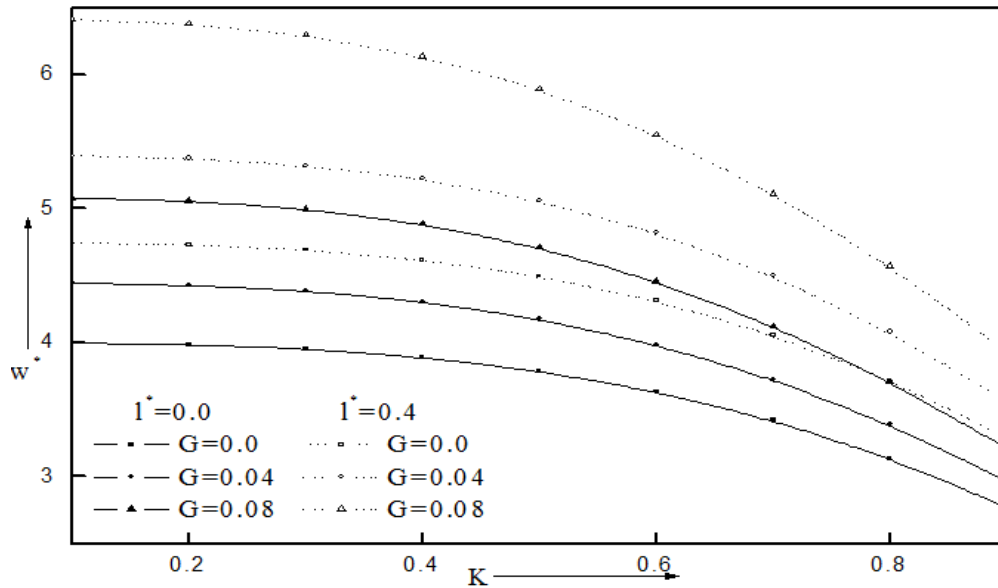


Figure 8 Variation of non-dimensional load carrying capacity w^* with K for different values of G and l^* with $h_1^* = 1.2$

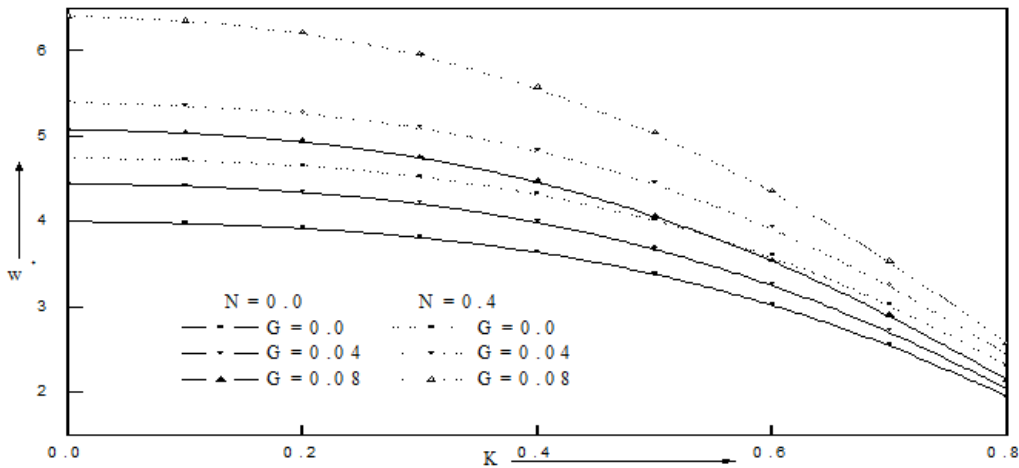


Figure 9 Variation of non-dimensional load carrying capacity w^* with K for different values of N and G .

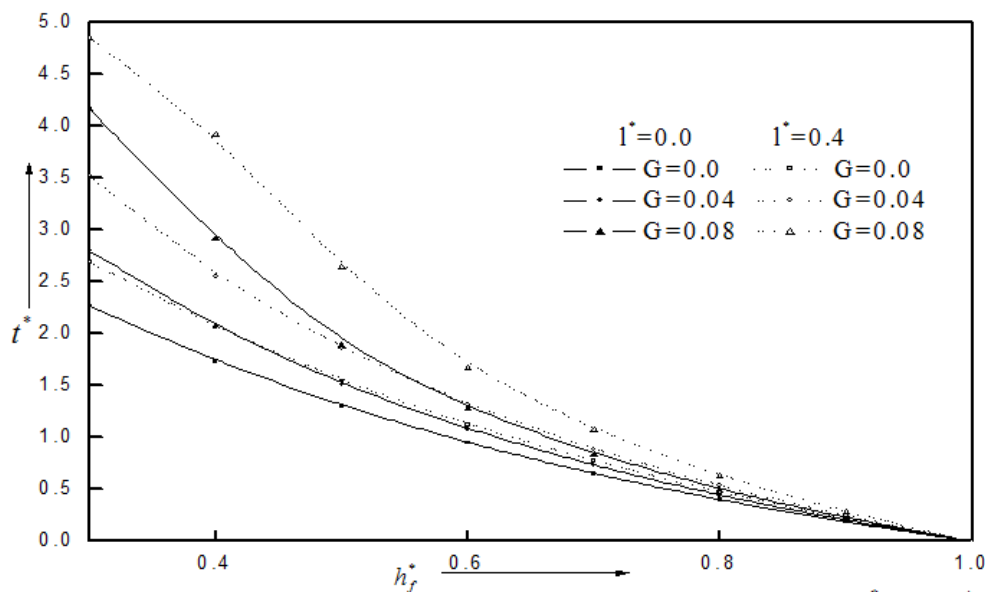


Figure 10 Variation of non-dimensional squeeze film time t^* with h_f^* for different values l^* and G with $K = 0.7$

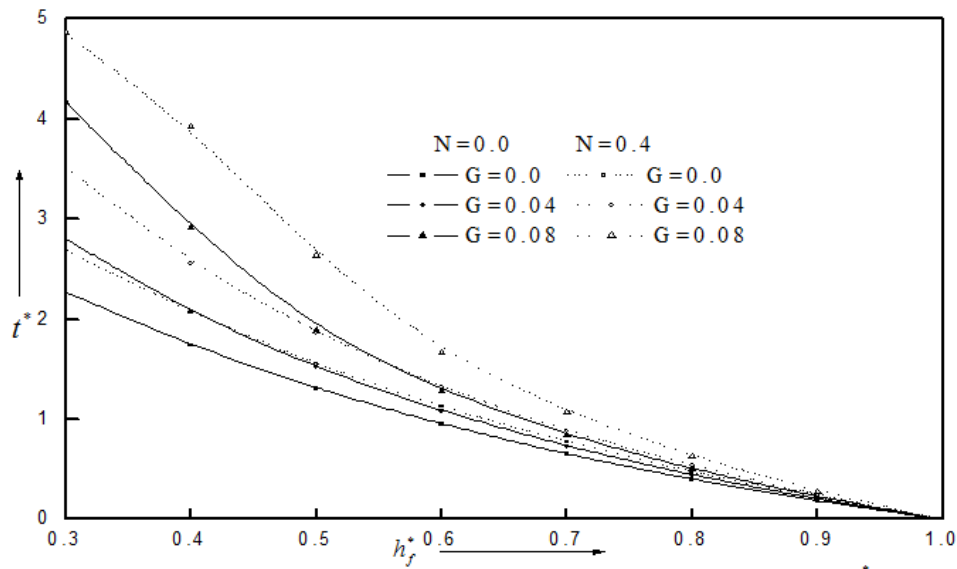


Figure 11 Variation on non-dimensional squeeze film time t^* with h_f^* for different values N and G with $K=0.7$ and $h_f^* = 0.5$, $h_i^* = 0.2$

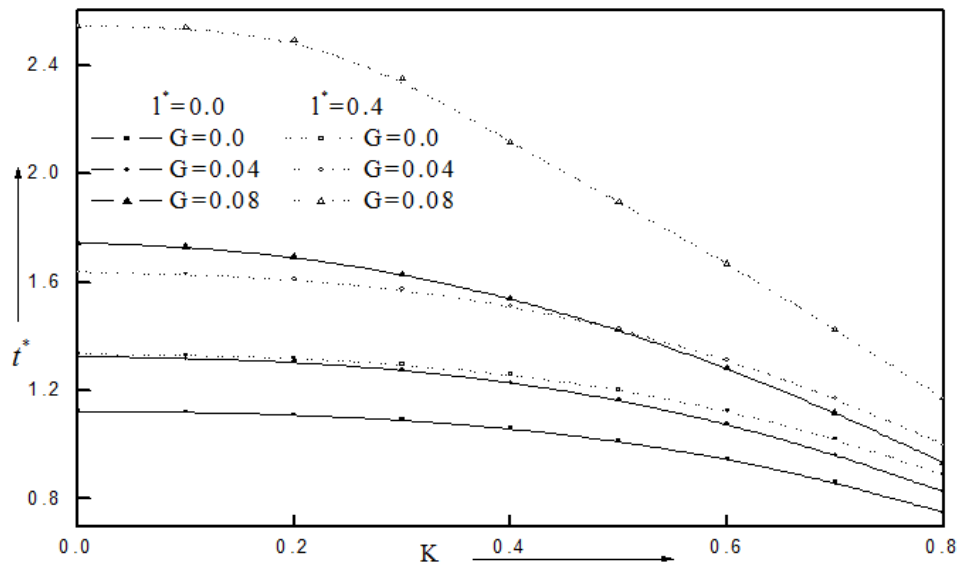


Figure 12 Variation of non-dimensional squeeze film time t^* with K for different values of l^* and G with $h_f^* = 0.5$, $h_i^* = 0.2$

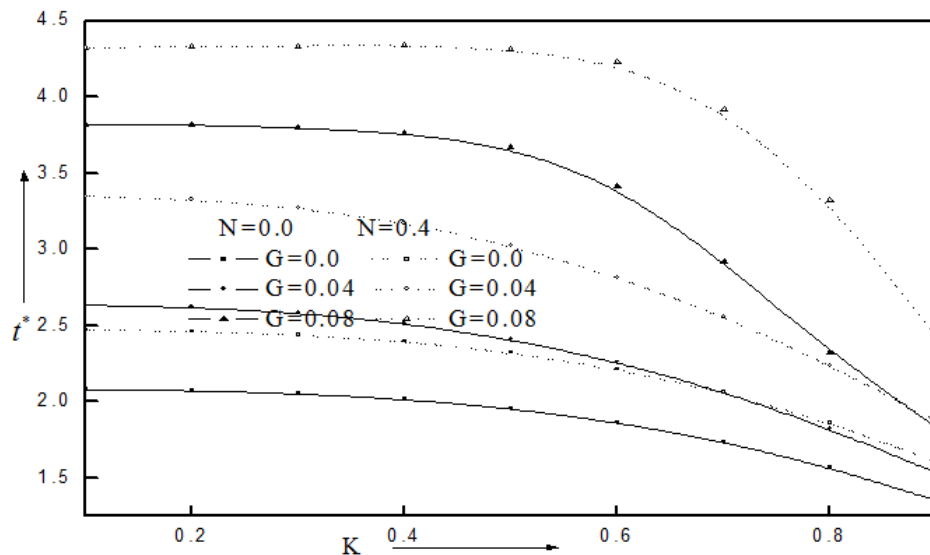


Figure 13 Variation of non-dimensional squeeze film time t^* with K for different values of N and G with $h_f^* = 0.5$, $h_i^* = 0.2$