

## $\Phi$ -GRACEFUL LABELING OF SOME GRAPHS

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(Received On: 21-05-18; Revised & Accepted On: 27-06-18)

### ABSTRACT

Let  $G$  be a graph. The  $\Phi$ -graceful labeling of a graph  $G(V, E)$  with  $p$  vertices and  $q$  edges is a injective a function  $f : V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$  such that the induced function  $f^* : E(G) \rightarrow N$  is given by  $f^*(u, v) = 2\{f(u) + f(v)\}$ , the resulting edge labels are distinct. In this paper we prove result  $\Phi$ -graceful labeling of Thorn ring graph, Cycle  $(C_n)$  with twin chords.

**Key Words:** Thorn ring graph, Cycle  $(C_n)$  with twin chords.

### 1. INTRODUCTION

we begin with all graphs are finite, simple and undirected graphs. Let  $G = (V, E)$  be a graph with vertex set  $V$  and edge set  $E$  the terminology and notations we follow Harry(2) Graph labeling has wide range of application in radar, communication network (design), x-ray crystallography etc. The definition and other information which are used for the present investigation are given.

### 2. DEFINITIONS

**Definition 2.1:  $\Phi$ -Graceful graph:** A graph  $G(V, E)$  with  $p$  vertices and  $q$  edges is a injective a function  $f : V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$  such that the induced function  $f^* : E(G) \rightarrow N$  is given by  $f^*(u, v) = 2\{f(u) + f(v)\}$  Detailed survey on graph labeling is given and up dated by Gallian (1)

### 3. RESULTS

**Theorem 3.1:** Every odd cycle graph  $(C_n)$  with twin chords is a  $\Phi$ -graceful graph if  $n \geq 7$ .

**Proof:** Let  $G$  be a cycle graph  $(C_n)$  with twin chords. Let  $\{v_1, v_2, v_3, \dots, v_n\}$  be the vertex set and  $\{e_1, e_2, e_3, \dots, e_n\}$  be the edge set. Consider  $\{e_n\}$  and  $\{e_{n-1}\}$  be the chords of the cycle  $(C_n)$

Now we define vertex labeling function as

$f : V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$  such that

$$f(v_1) = 0$$

$$f(v_{3+2i}) = 1 + i \text{ where } i = 0, 1, 2, \dots, \left\lfloor \frac{n-2}{2} \right\rfloor$$

$$f(v_{2+2i}) = \left( \frac{n+1}{2} \right) + i \text{ where } i = 0, 1, 2, \dots, \left\lfloor \frac{n-2}{2} \right\rfloor$$

Where  $n$  is the total number of vertices in  $G$ .

Continue the labeling until all vertices are labeled as shown in fig (1). Next the edge labeling function is defined as  $f^* : E(G) \rightarrow N$  is given by  $f^*(u, v) = 2\{f(u) + f(v)\}$  we get the edge labels are distinct. Thus  $f$  is  $\Phi$ -graceful of  $G$ . Hence cycle  $(C_n)$  with twin chords are  $\Phi$ -graceful graphs

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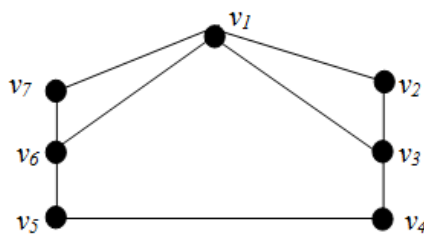


Figure-1

**Illustration:**  $\Phi$ - Graceful labeling of the graph cycle ( $C_n$ ) with twin chords is shown in fig.2

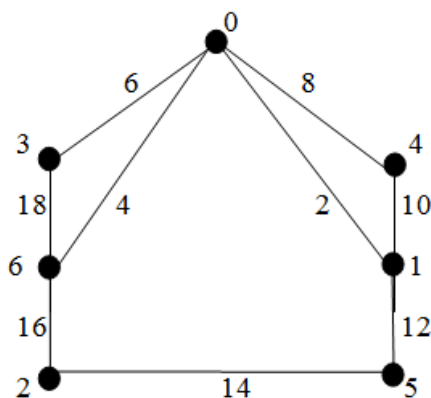


Figure-2

**Theorem 3.2:** Every odd thorn ring graph is a  $\Phi$  – graceful graph if  $n \geq 3$ .

**Proof:** Let  $G$  be a odd thorn ring graph. Let  $\{v_1, v_2, v_3, \dots, v_n\}$  be the vertex set and  $\{e_1, e_2, e_3, \dots, e_m\}$  be the edge set. Then  $|V(G)| = n$  and  $|E(G)| = m$

Now label the vertices of a thorn ring define by function as

$f : V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$  Such that

$f(v_1) = 1$

$f(v_2) = 0$

$f(v_{3+2i}) = 2 + i$  where  $i = 0, 1, 2, \dots, n-2$

Where  $n$  is the total number of vertices in  $G$ .

Continue the labeling until all vertices are labeled. Next the edge labeling function is defined as  $f^* : E(G) \rightarrow N$  is given by  $f^*(u, v) = 2 \{f(u) + f(v)\}$  we get the edge labels are distinct .Thus  $f$  is  $\Phi$  – graceful of  $G$  . Hence cycle ( $C_n$ ) with twin chords are  $\Phi$  – graceful graph.

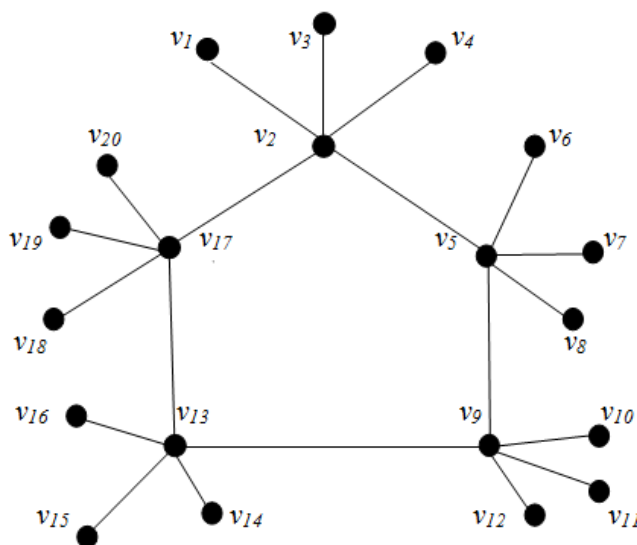


Figure-3

**Illustration:**  $\Phi$ - Graceful labeling of the odd thorn ring graph is shown in fig.4

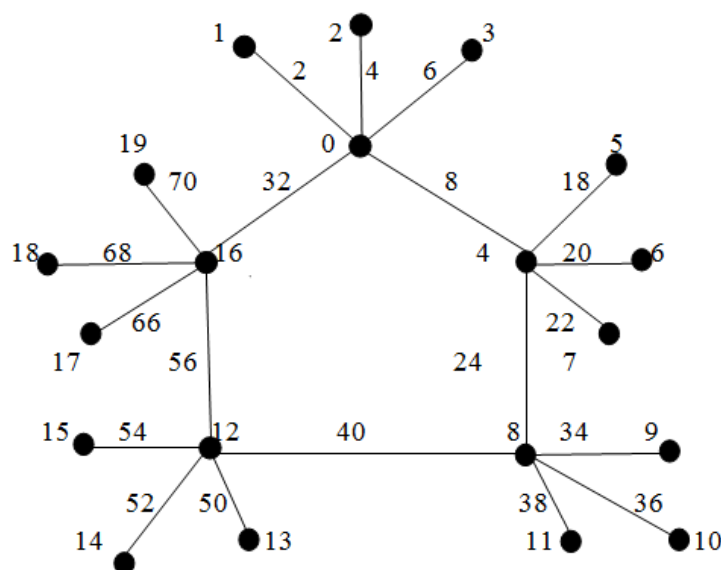


Figure-4

## CONCLUSION

In this paper we have shown that Thorn ring graph, Cycle ( $C_n$ ) with twin chords. are  $\Phi$  – graceful graph, are investigated it can also verified for some graphs.

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Source of support: Nil, Conflict of interest: None Declared.

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