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EFFECT OF RADIATION ON CONVECTIVE HEAT TRANSFER THROUGH A POROUS MEDIUM IN A HORIZONTAL WAVY CHANNEL WITH NON-UNIFORM TEMPERATURE

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ABSTRACT

We analyze the effect of radiation on mixed convective heat transfer of a viscous incompressible fluid through a horizontal channel bounded by wavy walls which are maintained at non-uniform temperature. Taking the slope (δ) of the boundary walls the equations governing the flow and heat transfer are solved by the perturbation technique. The velocity and temperature are analyzed for different variations of the governing parameters G, N_1, D^{-1}, R. The Shear stress and the rate of heat transfer on the boundary walls have been computed and numerically analyzed.

***Keywords**: Radiationeffect, HeatTransfer, Porous medium, Wavy channel.*

1. INTRODUCTION

It is well known that in order to harness maximal geothermal energy one should have complete and precise knowledge of the mechanism of initiating convection currents in geothermal fluids. For example, the use of thermal processes like injective heat into the under ground petroleum reservoirs is important in enhancing the recovery of hydrocarbons. In such recovery processes fluid flow takes place through a porous medium under convection. This lead the reservoir engineers and geophysicists to make a detailed study of the convection flow through porous media with internal heat sources. There has been an extensive literature on this topic.

Alagoa *et al.* [1] studied radiative and free convection effects on MHD flow through porous medium between infinite parallel plates with time-dependent suction. Bestman and Adjepong [2] analyzed unsteady hydromagnetic free convection flow with radiative heat transfer in a rotating fluid. Hossain and Ress [3] have analyzed the effects of combined buoyancy forces from mass and thermal diffusion by natural convection flow from a vertical wavy surface. Hossain and Takhar [4] investigated the radiation effects on mixed convection along a vertical plate with uniform surface temperature using the Keller Box finite difference method. The interaction of radiation with laminar free convection heat transfer has been discussed by several authors under different conditions [5, 6]. Arpaci considered a similar problem in both the optically thin and optically thick region and used the approximate integral technique and first-order profiles to solve the energy equation.

The study of buoyancy driven convection flows through a porous media has been stimulated by its applications in several geophysical and engineering problems. The two main configurations in which the heat transfer driven flow in a porous medium. This convection heat transfer potential flow through a porous medium is rapidly growing as an independent branch in Fluid Mechanics and Heat Transfer. This problem of combined buoyancy driven thermal and mass diffusion has been studied in parallel plate geometries by a few authors in the recent times, notably Lai F.C. [7, 8] Angirasa *et al.* [9] Abdul [10]. Natural convection in differentially heated vertical enclosures is of fundamental interest to many practical applications. Several investigators have presented analytical and experimental results on convection in the rectangular cavity with vertical walls at constant temperatures, the horizontal walls being insulated [11].

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2. FORMULATION OF THE PROBLEM

We analyse the effect of radiation on the steady convective flow of a viscous, incompressible through a porous medium confined in a horizontal channel bounded by wavy walls which are maintained at non-uniform temperature. The Boussinesque approximation is used so that the density variation will be considered only in the buoyancy term .The viscous dissipation is neglected in comparison to the transport of heat by conduction and convection. The kinematic viscosity and the thermal conductivity are treated as constants. We choose the Cartesian coordinate system O(x, y) with x-axis in the horizontal direction and y-axis normal to the walls. The walls of the channel are at y=±Lf. The equations governing the steady flow and heat transfer are

Equation of linear momentum

$$\rho_e(uu_x + vu_y) = -p_x + \mu\nabla^2 u - \left(\frac{\mu}{k_1}\right)u \tag{1}$$

$$\rho_e(uv_x + vv_y) = -p_y + \mu\nabla^2 v - \left(\frac{\mu}{k_1}\right)v - \rho g \tag{2}$$

Equation of Continuity

$$u_x + v_y = 0 \tag{3}$$

$$\rho_e C_p(uT_x + vT_y) = k\nabla T + Q - \frac{\partial(q_r)}{\partial y} \tag{4}$$

$$\rho - \rho_e = -\rho_e(\beta_1(T - T_e) \tag{5}$$

By using Rosseland approximation (Brewster (3a)) the radiation heat flux is given by

$$q_r = -\left(\frac{4\sigma^*}{3\beta_r}\right)\frac{\partial T'^4}{\partial y} \tag{6}$$

where σ^* is the Stefan-Boltzmann constant and β_r is the mean absorption coefficient. It should be noted that by using the Rosseland approximation the present analysis is limited to optically thick fluids. We assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about Te and neglecting higher order terms such that

$$T'^4 \approx 4T_e^3 T - 3T_e^4 \tag{7}$$

where ρe, Te are the density and temperature in the equilibrium state, (u, v) are the velocity components along O(x, y) directions, p is the pressure, T is the temperature in the flow region, ρ is the density of the fluid, k is the coefficient of thermal conductivity, β_1 is the coefficient of volume expansion, k_1 is the coefficient of permeability, and Q is the strength of the heat source..

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial y} - \rho_e g \tag{8}$$

where $p=p_d + p_e$, p_d being the hydrodynamic pressure.

The boundary conditions are

u = 0, v = 0

$$T - T_e = \gamma\left(\frac{\delta x}{L}\right) on \;\; y = \pm Lf(\delta x / L) \tag{9}$$

γ is chosen to be twice differential function, δ is a small parameter characterizing the slope of the boundary.

The flow is maintained by a constant imposed flux for which a characteristic velocity q is defined by

$$q = \frac{1}{L}\int_{-Lf}^{Lf} udy \tag{10}$$

We introduce the non-dimensional variables as

$$(x', y') = (x, y)/L \ , \ (u', v') = (u, v)/q$$

$$p' = p/\rho_e q^2 \ , \quad \theta' = \frac{T - T_e}{\Delta T_e},$$

$$(\Delta T_e = T_e(L) - T_e(-L))$$

Substituting these non-dimensional variables in equations (1)-(5) and eliminating the pressure, the momentum equations in terms of dimensionless stream function ψ reduces to

$$R\frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)} = \nabla^4\psi + (G/R)(\theta_x) - D_2^{-1}\nabla^2\psi \tag{11}$$

The energy equation in the non-dimensional from is

$$P_e(u\theta_x + v\theta_y) = \nabla^2\theta + \alpha + \frac{4}{3N_1}\frac{\partial^2\theta}{\partial y^2} \tag{12}$$

where

$$u = -\frac{\partial\psi}{\partial y}, \qquad v = \frac{\partial\psi}{\partial x}$$

$$G = \frac{\beta g \Delta T L^3}{\nu^2} \quad \text{(Grashof number)} \qquad D_2^{-1} = \frac{L^2}{k_1} \quad \text{(Darcy parameter)}$$

$$R = \frac{qL}{\nu} \quad \text{(Reynolds number)} \qquad \alpha = \frac{QL^2}{k} \quad \text{(Heat source parameter)}$$

$$N_1 = \frac{\beta_R\, k}{4\sigma^* T_e^3} \quad \text{(Radiation parameter)} \quad P = \frac{\mu C_p}{k} \quad \text{(Prandtl number)}$$

The boundary conditions in the non-dimensional form for ψ and θ are

$$\psi(+1) - \psi(-1) = -1$$

$$\frac{\partial\psi}{\partial y} = 0, \quad \frac{\partial\psi}{\partial x} =$$

$$\theta(x, y) = \gamma(\delta x) o \ \eta y = \pm f(\delta x) \quad (13)$$

The value of ψ on the boundary assures the constant volumetric flow in consistent with the hypothesis (10) .Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function γ .

3. SHEAR STRESS AND NUSSELT NUMBER

The shear stress on the boundary plates is given by

$$\tau = \frac{(\sigma_{x\eta}(1 - f'^2) + (\sigma_{\eta\eta} - \sigma_{xx}) f'^2}{(1 + f'^2)}$$

Where $\sigma_{ij} = -p\delta_{ij} + 2\mu e_{ij}, \sigma_{xx} = \frac{\partial u}{\partial x}, \sigma_{\eta\eta} = \frac{\partial v}{\partial \eta}, \sigma_{x\eta} = 0.5\left(\frac{\partial u}{\partial \eta} + \frac{\partial v}{\partial x}\right)$

and the corresponding expressions are $(\tau)_{\eta=1} = \frac{c_{15} + \delta c c_{16}}{c_{17}}, \ \ (\tau)_{\eta=-1} = \frac{c_{18} + \delta c c_{19}}{c_{20}}$

The rate of heat transfer (Nusselt Number) on the plates has been calculated using the formula

$$Nu = \frac{1}{f(\theta_m - \theta_w)}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=\pm 1}$$

where $\theta_m = \int_{-1}^{1} \theta \, d\eta$

and the corresponding expressions are

$$(Nu)_{\eta=1} = \frac{c_{23} + \delta c_{24}}{c_{21} + \delta c_{22}}, \quad (Nu)_{\eta=-1} = \frac{c_{26} + \delta c_{25}}{c_{21} + \delta c_{22}}$$

where c_1, c_2,…….,c_{26} are constants

4. DISCUSSION OF THE NUMERICAL RESULTS

The aim of this analysis is to discuss the combined effect of radiation on convective flow of a viscous, incompressible fluid through a porous medium in a horizontal channel bounded by wavy walls. The perturbation analysis is carried out to make use of the wall slope δ as a small parameter.

The variation of u with amplitude β of the boundary, Fig.1. shows that u continues to negative for all |β| except at β=0.5 at which we find a reversal flow in the entire fluid region, and this reversal flow which disappears for higher values of β. For β≤0.3 the axial velocity u decreases in the lowerhalf and enhances in the upper half and for higher β (0.5≤β≤0.7) the axial velocity experiences an enhancement remarkably in the entire flow region. But for β=0.7, |u| decreases in the flow region. For |β|≤0.3, the axial velocity u experiences an enhancement in the region -0.4≤ η ≤0.4 except in the region adjacent to η = ±1. The variation of u with radiation parameter N_1 is depicted in Fig.2. We find that u continues to be negative for all variations of N_1. We notice |u| enhances in the upper region and depreciates in the lower region with the radiation parameter N_1,

we observe that higher the dilation/constriction of the boundary, larger the variation of v in the entire flow region. The variation of v with the radiation parameter N_1 shows that For β>0 an increase in N_1≤0.3 the magnitude of v enhances in the lower region and reduces in the upper region, and for higher N_1. (Figs.3 and 4)

The behaviour of θ with amplitude α_1 of the boundary temperature on θ shows that the temperature experiences an enhancement in the flow region with an increase in amplitude α_1 (Fig. 5) .From Fig. 6 we observe that greater the dilation larger the temperature and higher the constriction, lesser the temperature in the flow region.

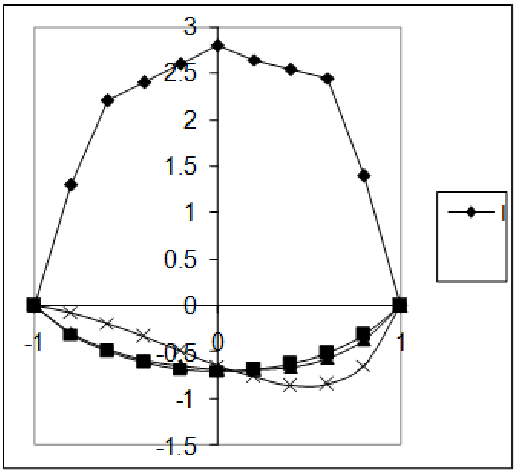

Fig.1: Profile for velocity (u) with β
$D^{-1} = 5x10^2$, R = 35, G = 10^3

	I	II	III	IV
β	0.1	0.3	0.5	0.7

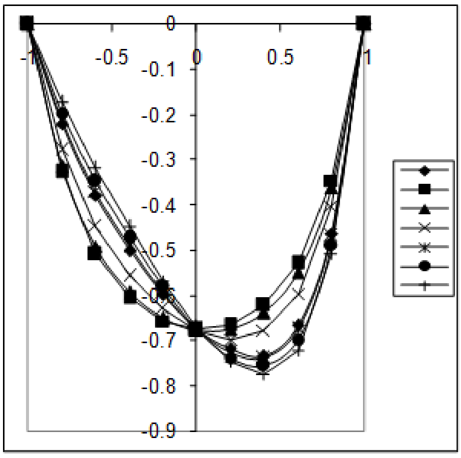

Fig.2: Profile for velocity (u) with N_1
$D^{-1} = 5x10^2$, R = 35, G = 10^3

	I	II	III	IV	V	VI	VII
N_1	0.1	0.3	1	4	5	10	100

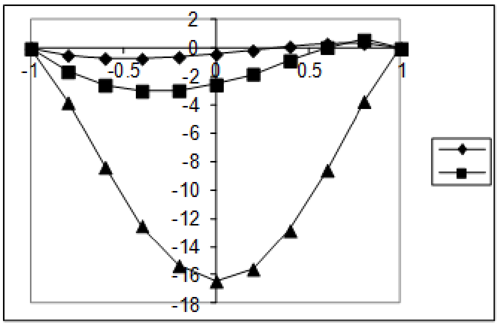

Fig.3: Profile for velocity (v) with β
$D^{-1} = 5x10^2$, $R = 35$, $G = 10^3$

	I	II	III	IV
β	0.1	0.3	0.5	0.7

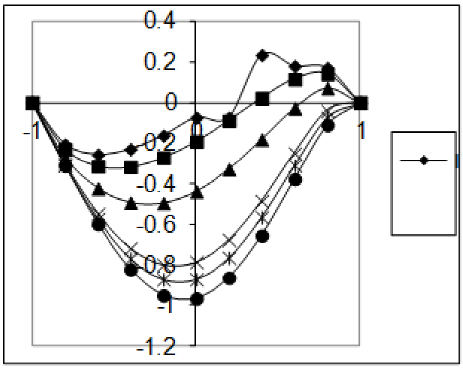

Fig.4: Profile for velocity (v) with N_1
$D^{-1} = 5x10^2$, $R = 35$, $G = 10^3$

	I	II	III	IV	V	VI	VII
N_1	0.1	0.3	1	4	5	10	100

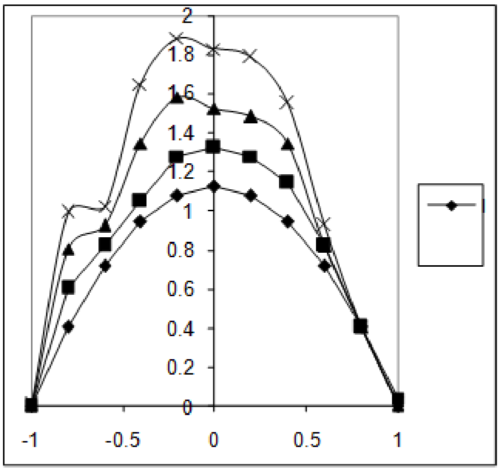

Fig.5: Profile for temeperature (θ) with β
$D^{-1} = 5x10^2$, $R = 35$, $G = 10^3$

	I	II	III	IV
β	0.1	0.3	0.5	0.7

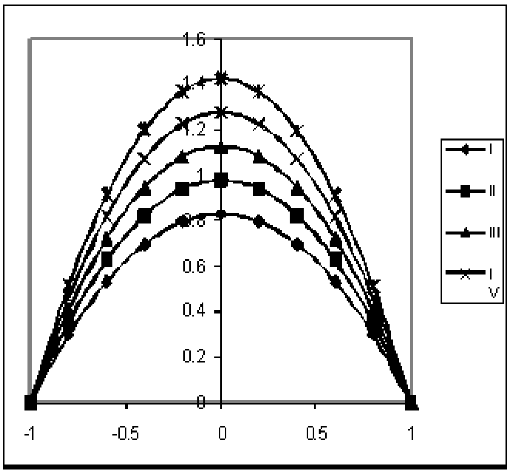

Fig.6: Profile for velocity (θ) with N_1
$D^{-1} = 5x10^2$, $R = 35$, $G = 10^3$

	I	II	III	IV	V	VI	VII
N_1	0.1	0.3	1	4	5	10	100

The shear stress at the boundaries have been evaluated for different values at the governing parameter in tables.1 and 2. The shear stress on the upper boundary η=1 is positive and is negative at the lower boundary η=-1 in both dilated and constricted cases.

From tables.1and 2 we find that the magnitude of τ enhances with radiation parameter N_1 in the heating case and reduces with N_1 in the cooling case, while a reversed effect is observed at η=-1. In the constricted case |τ| enhances with $N_1 \le 0.4$ and reduces with higher values of $N_1 \ge 1$ on η=1 and it experiences an enhancement with N_1 for all G>0, while a reversed effect is observed in the cooling case.

The Average Nusselt number(Nu) which measures the rate of heat transfer across the boundaries has been depicted in tables .3 and 4 for different values of the parameters. The Nusselt number Nu at the upper boundary η=1 is negative and is positive at the lower boundary at η=-1. The rate of heat transfer at η=-1 increases with Grashoff number G(>0) and reduces with |G|(<0) for β>0 while at the upper boundary η=1 the magnitude of Nu reduces in the heating case and enhances in the cooling case. An increase in R reduces Nu at η=±1 for β>0.

Table-1: Shear stress (τ) at y=1 P=0.71,R=35,α =2,β=0.5

G	I	II	III	IV	V
10^3	0.43084	0.43504	0.4408	0.4535	0.4572
2 x 10^3	0.43059	0.43678	0.4457	0.46611	0.47217
3 x 10^3	0.43473	0.4433	0.45596	0.48483	0.49348
-10^3	0.41535	0.39953	0.38867	0.35974	0.35195
-2 x 10^3	0.45938	0.46054	0.45431	0.44218	0.43794
-3 x 10^3	0.31222	0.28509	0.25704	0.19989	0.18487

N_1	0.1	0.4	1	5	10

Table-2: Shear stress (τ) at y=1 P=0.71, R=35, α=2, β= -0.5

G	I	II	III	IV	V
10^3	3.1549	3.1596	3.1543	3.1535	3.1533
2 x 10^3	3.1861	3.1873	3.1886	3.1907	3.1912
3 x 10^3	2.6653	2.6639	2.6621	2.6581	2.657
-10^3	11.5029	15.0257	11.5916	11.6821	11.7056
-2 x 10^3	-138.4716	-25.889	-139.9033	-141.361	-141.7397
-3 x 10^3	2406.71	496.651	2431.089	2455.91	2462.364

N_1	0.1	0.4	1	5	10

Table-3: Nusselt number (Nu) at y= 1 P=0.71, R=35

G	I	II	III	IV	V
10^3	-2.57734	-4.26698	-4.90334	-5.31344	-5.36699
2 x 10^3	2.57734	-4.26698	-4.90334	-5.31343	-5.36698
3 x 10^3	-2.57735	-4.26699	-4.90335	-5.31341	-5.36693
-10^3	-2.57734	-4.69362	-4.90339	-5.31345	-5.36700
-2 x 10^3	-2.57734	-4.26698	-4.90335	-5.31347	-5.36701
-3 x 10^3	-2.57735	-4.26708	-4.90336	-5.31349	-5.36703

Table-4: Nusselt number (Nu) at y=1 P=0.71,R=35

G	I	II	III	IV	V
10^3	-1.01941	-1.5499	-1.7299	-1.8445	-1.8595
2 x 10^3	-1.01941	-1.5497	-1.7295	-1.8441	-1.8594
3 x 10^3	-1.01942	-1.5494	-1.7294	-1.8440	-1.8592
-10^3	-1.01941	-1.6932	-1.7295	-1.8441	-1.8596
-2 x 10^3	-1.01941	-1.5494	-1.7291	-1.8441	-1.8597
-3 x 10^3	-1.01941	-1.5495	-1.7295	-1.8442	-1.8598

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