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OSCILLATIONS OF SECOND ORDER NONLINEAR NEUTRAL DELAY DIFFERENTIAL EQUATIONS

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ABSTRACT

Sufficient conditions for oscillations of second order nonlinear neutral delay differential equations of the form

$$\frac{d}{dt}\left\{r_1(t)\frac{d}{dt}\left\{m(t)y(t)+\frac{r(t)}{r(t-\tau)}y^{\alpha}(t-\tau)\right\}\right\}+f(t)y^{\alpha}(t-\sigma)=0,\quad t\geq t_0$$

are obtained ,where $r_1(t)$, $r(t), m(t)$ *are positive real valued continuous functions* $f(t)\geq 0$, *and* α *is the ratio of odd positive integers.*

Key words: *Oscillation, Second Order, Neutral Differential equation.*

1. INTRODUCTION

In this paper we consider the second order nonlinear neutral delay differential equation

$$\frac{d}{dt}\left\{r_1(t)\frac{d}{dt}\left\{m(t)y(t)+\frac{r(t)}{r(t-\tau)}y^{\alpha}(t-\tau)\right\}\right\}+f(t)y^{\alpha}(t-\sigma))=0 \tag{1}$$

where $r_1(t), m(t)\in C^{'}([t_0,\infty),(0,\infty))$, $r(t), f(t)\in C([t_0,\infty),[0,\infty))$.
Corresponding equation in the absence of neutral term is given by

$$\frac{d}{dt}\left\{r_1(t)\frac{d}{dt}\{m(t)y(t)\}\right\}+f(t)y^{\alpha}(t-\sigma))=0 \tag{2}$$

which is a delay differential equation and further if we take $m(t)=1, \sigma=0$ in equation (2) we get

$$\frac{d}{dt}(r_1(t)\frac{d}{dt}y(t))+f(t)y^{\alpha}(t))=0 \tag{3}$$

which is an ordinary differential equation.

The study of behavior of solutions of the equation (3) is voluminous. The differential equation (2) has been a subject of interest for several researchers. We mention the works of [1, 2, 10 and 12].Oscillatory behavior of delay differential equations is extensively studied by several authors [3- 9, 11, 13-16, 18, and 19]. In particular, differential equations of the form (1) and for special cases when $r_1(t)\equiv 1$, is a subject of intensive research.

Here we have some interesting results

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(i) Jiqin Deng [6]: Let $r_1(t) \equiv 1$ and $\alpha = 1$ If for large $t \in R$,

$$\int_t^\infty f(s)\,ds \geq \frac{\alpha_0}{t} \text{ where } \alpha_0 > \frac{1}{4}$$

then equation (3) is oscillatory.

(ii) Ch.G.Philos [10]: Let $r_1(t) \equiv 1$ and $\alpha = 1$ Let *n* be an integer with $n \geq 3$ and ρ be positive continuously differentiable function on the interval $[t_0, \infty)$ such that

$$Lim_{t\to\infty} \sup \frac{1}{t^{n-1}} \int_{t_0}^{t} \frac{(t-s)^{n-3}}{\rho(s)} \left[(n-1)\rho(s) - (t-s)\rho'(s)\right]^2 ds < \infty .$$

Then (3) is oscillatory if

$$Lim_{\ t\to\infty} \sup \frac{1}{t^{n-1}} \int_{t_0}^{t} (t-s)^{n-1} \rho(s) f(s)\,ds = \infty$$

Motivated by some of these works, we present oscillation criteria of the equations of the type (1) under certain ntegral conditions.

By a solution of equation (1) we mean a function $y(t) \in C([T_y, \infty))$ where $T_y \geq t_0$ which satisfies (1) on $[T_y, \infty)$. We consider only those solutions of *y(t)* of (1) which satisfy $Sup\{|y(t)| : t \geq T\} > 0$ for all $T \geq T_y$ and assume that (1) possesses such solutions.

A solution of equation (1) is called oscillatory if it has arbitrary large zeros on $[T_y, \infty)$; otherwise it is called nonoscillatory. Equation (1) is said to be oscillatory it all its solutions oscillate .Unless otherwise stated, when we write a functional inequality, it will be assumed to hold for sufficiently large t in our subsequent discussion.

II. MAIN RESULTS

We need the following in our discussion:

(H_1): $r_1(t), m(t), \in C^{'}([t_0, \infty), (0, \infty))$; $r_1(t), m(t) > 0$

(H_2): $r(t), f(t) \in C(t_0, \infty), [0, \infty))$, $f(t) > 0.$

(H_3): $0 < \alpha \leq 1,$ and α is the ratio of odd positive integers.

We set

$$z(t) = m(t)y(t) + \frac{r(t)}{r(t-\tau)} y^\alpha(t-\tau) \tag{4}$$

and

$$R(t) = \int_{t_0}^{t} \frac{1}{r_1(s)} ds = \infty \quad \text{as } t \to \infty \tag{5}$$

We have the following Lemma:

Lemma 2.1: Let $\alpha \geq 1$, be a ratio of odd positive integers. Then

$$-Cu^{\frac{\alpha+1}{\alpha}} + Du \leq \frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}} \frac{D^{\alpha+1}}{C^\alpha}, \qquad C > 0 \tag{6}$$

Proof: The proof can be found in [19].

Now we state our main result.

Theorem 2.2: Assume $(H_1)-(H_3)$ and (5) hold. If $\alpha \geq 1$ and there exists a positive non decreasing function $\rho \in C^{'}([t_0,\infty),R)$ such that

$$Lim_{t\to\infty} \sup \int_{t_1}^{t} \left[\rho(s) f(s) \left\{ \frac{1}{m(s-\sigma)} \left(1 - \frac{r(s-\sigma)}{r(s-\sigma-\tau)M^{1-\alpha}} \right) \right\}^{\alpha} - \frac{a(s-\sigma)(\rho'(s))^2}{4\alpha M^{\alpha-1}\rho(s)} \right] ds = \infty \tag{7}$$

then every solution of equation (1) is oscillatory.

Proof: Suppose to the contrary .And let *y(t)* be a nonoscillatory solution of equation (1).Without loss of generality we may assume that *y(t)* is eventually positive.

Since $$z(t) > 0, \quad z'(t) > 0, \quad (r_1(t)z'(t))' \leq 0; \text{ for } t \geq t_1 \tag{8}$$

From (8) and also since $t-\sigma \leq t$ we have

$$r_1(t)z'(t) \leq r_1(t-\sigma)z'(t-\sigma) \quad \text{for } t \geq t_1$$

Since $z'(t) > 0$, there exists a constant $M > 0$ such that $z(t) \geq M$ for all large t.

From the definition of z, we have

$$z(t) = m(t)y(t) + \frac{r(t)}{r(t-\tau)} y^{\alpha}(t-\tau)$$

$$m(t)y(t) = z(t) - \frac{r(t)}{r(t-\tau)} y^{\alpha}(t-\tau)$$

$$y(t) = \frac{1}{m(t)} \left[z(t) - \frac{r(t)}{r(t-\tau)} y^{\alpha}(t-\tau) \right]$$

$$\geq \frac{1}{m(t)} \left[z(t) - \frac{r(t)}{r(t-\tau)} z^{\alpha}(t-\tau) \right]$$

$$\geq \frac{1}{m(t)} \left[1 - \frac{r(t)}{r(t-\sigma)} \frac{z^{\alpha}(t)}{z(t)} \right] z(t)$$

$$y(t) \geq \frac{1}{m(t)} \left[1 - \frac{r(t)}{r(t-\sigma)} \frac{M^{\alpha}}{M} \right] z(t)$$

Hence $$y(t) \geq \frac{1}{m(t)} \left[1 - \frac{r(t)}{r(t-\sigma)M^{1-\alpha}} \right] z(t) \tag{9}$$

Define

$$\omega(t) = \rho(t) \frac{r_1(t)z'(t)}{z^{\alpha}(t-\sigma)}; \quad t \geq t_1 \tag{10}$$

Differentiating with respect to t we have

$$\omega'(t) = \rho'(t) \frac{r_1(t)z'(t)}{z^{\alpha}(t-\sigma)} + \rho(t) \left\{ \frac{r_1(t)z'(t)}{z^{\alpha}(t-\sigma)} \right\}'$$

$$= \rho'(t) \frac{r_1(t)z'(t)}{z^{\alpha}(t-\sigma)} + \rho(t) \left[\frac{z^{\alpha}(t-\sigma)\{r_1(t)z'(t)\}' - r_1(t)z'(t)\{z^{\alpha}(t-\sigma)\}'}{z^{2\alpha}(t-\sigma)} \right]$$

$$= \rho'(t) \frac{r_1(t)z'(t)}{z^{\alpha}(t-\sigma)} + \rho(t) \frac{\{r_1(t)z'(t)\}'}{z^{\alpha}(t-\sigma)} - \left[\rho(t) \frac{r_1(t)z'(t)\{z^{\alpha}(t-\sigma)\}'}{z^{2\alpha}(t-\sigma)} \right]$$

From (10), (1), and (9) we have

$$\omega'(t) \le \frac{\rho'(t)}{\rho(t)}\omega(t) - \rho(t) f(t)\left\{\frac{1}{m(t-\sigma)}\left(1-\frac{r(t-\sigma)}{r(t-\sigma-\tau)M^{\alpha-1}}\right)\right\}^{\alpha} - \left[\rho(t)\frac{r_1(t)z'(t)\{z^{\alpha}(t-\sigma)\}'}{z^{2\alpha}(t-\sigma)}\right]$$

$$\omega'(t) \le -\rho(t) f(t)\left\{\frac{1}{m(t-\sigma)}\left(1-\frac{r(t-\sigma)}{r(t-\sigma-\tau)M^{1-\alpha}}\right)\right\}^{\alpha} + \frac{\rho'(t)}{\rho(t)}\omega(t) - \frac{\beta M^{\alpha-1}\omega^2(t)}{\rho(t)a(t-\sigma)}.$$

Since

$$-Cu^{\frac{\alpha+1}{\alpha}} + Du \le \frac{\alpha^{\alpha}}{(\alpha+1)^{\alpha+1}}\frac{D^{\alpha+1}}{C^{\alpha}}, \qquad C > 0$$

We have

$$\omega'(t) \le -\rho(t) f(t)\left\{\frac{1}{m(t-\sigma)}\left(1-\frac{r(t-\sigma}{r(t-\sigma-\tau)M^{1-\alpha}}\right)\right\}^{\alpha} + .\frac{a(t-\sigma)(\rho'(t))^2}{4\alpha M^{\alpha-1}\rho(t)}.$$

Integrating the above inequality from t_1 to t we get,

$$.\int_{t_1}^{t}\left[\rho(s) f(t)\left\{\frac{1}{m(t-\sigma)}\left(1-\frac{r(t-\sigma)}{r(t-\sigma-\tau)M^{\alpha-1}}\right)\right\}^{\alpha} - \frac{a(t-\sigma)(\rho'(t))^2}{4\alpha M^{\alpha-1}\rho(t)}\right]ds < \omega(t_1)$$

which is a contradiction to equation (7) as $t \to \infty$. Thus the proof is completed.

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