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A COMMON FIXED POINT THEOREM IN INTUITIONISTIC MENGER (PQM) SPACE WITH USING PROPERTY (E.A.)

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ABSTRACT

In this paper deals a common fixed point theorem in intuitionistic menger space.

Key words: *fixed point, common fixed point theorem , menger space , intuitionistic menger space.*

INTRODUCTION

- There have been a number of generalizations of metric space. One such generalization is Menger space introduced in 1942 by Menger [98] who used distribution functions instead of nonnegative real numbers as values of the metric.
- In fact, he replaced the distance function $d: X \times X \to \mathfrak{R}^+$ with a distribution function $F_{p,q}: \mathfrak{R} \to [0,1]$ wherein for any number x, the value $F_{p,q}(x)$ describes the probability that the distance between p and q is less than.
- Schweizer and Sklar [118] studied this concept and then the important development of Menger space theory was due to Sehgal and Bharucha-Reid [122]. Sessa [123] introduced weakly commuting maps in metric spaces.
- Jungck [81] enlarged this concept to compatible maps. The notion of compatible maps in Menger spaces has been introduced by Mishra [99].
- Aamri and Moutawakil [1] and Liu *et al*. [92] respectively defined the property (E.A) and common property (E.A) and proved some common fixed point theorems in metric spaces.
- Imdad *et al.* [77] extended the results of Aamri and Moutawakil [1] to semi-metric spaces. Most recently, Kubiaczyk and Sharma [90] defined the property (E.A) in PM spaces and
- used the same to prove some results on common fixed points wherein authors claim their results for strict contractions which are in fact proved for contractions.
- Kutukcu *et al.* [91] defined the notion of intuitionistic Menger spaces with the help of t-norms and t−conorms as a generalization of Menger spaces due to Menger [96].On the other hand Rezaiyan et. al. [106] prove fixed point theorem for Menger (PQM) space which is modified by Mihet [98].
- The aim of this paper is to prove a hard and fast purpose theorem in Intuitionistic Menger (PQM) area mistreatment property E.A. for this 1st we have a tendency to offer some definitions and notable results that square measure utilized in this paper

Definition 7.1.1: A binary operation $T : [0,1] \times [0,1] \to [0,1]$, is a t-norm if T satisfies the following conditions:
71.1 (i) T is commutative and associative.
7.1.1 (ii) $T(a,1) = a$ for all $a \in [0,1]$.
7.1.1 (iii) $T(a,b) \le T(c,d)$ whenever $a \le c$ and $b \le d$
For $a, b, c, d \in [0,1]$.

Definition 7.1.2: A binary operation $S : [0,1] \times [0,1] \to [0,1]$ is a t−conorm if S satisfies the following conditions:
7.1.2 (i) S is commutative and associative.
7.1.2 (ii) $S(a,0) = a$ for all $a \in [0,1]$
7.1.2 (iii) $S(a,b) \le S(c,d)$ whenever $a \le c$ and $b \le d$
For $a, b, c, d \in [0,1]$.

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Remark 7.1.3: The concepts of t−norm T and t−conorm S are known as the axiomatic skeletons that we use $T : [0,1] \times [0,1] \rightarrow [0,1]$, for characterizing fuzzy intersections and unions respectively. Throughout this chapter, we will denote $R = (-\infty, \infty)$ and $R^+ = [0, \infty)$.

Definition 7.1.4: A distance distribution function is a function $F: R \rightarrow R^+$, which is left continuous on R, non-decreasing $T : [0,1] \times [0,1] \rightarrow [0,1]$, and $\inf_{t \in R} F(t) = 0$, $\sup_{t \in R} F(t) = 1$. We will denote by D, the family of all distance distribution functions and by H a special element of D defined by

$$H(t) = \begin{cases} 0, & if\ t \leq 0 \\ 1, & if\ t > 0 \end{cases}$$

If X is a nonempty set, $F: X \times X \rightarrow D$ is called a probabilistic distance on X.

Definition 7.1.5: A non-distance distrib $T : [0,1] \times [0,1] \rightarrow [0,1]$, ution function is a function $L: R \rightarrow R^+$, which is right continuous on R, non-increasing and $\inf_{t \in R} L(t) = 1$, $\sup_{t \in R} L(t) = 0$.

We will denote by E, the family of all non-distance distribution functions and by G a special element of E defined by

$$G(t) = \begin{cases} 1, & if\ t \leq 0 \\ 0, & if\ t > 0 \end{cases}$$

If X is a nonempty set, $L: X \times X \rightarrow D$ is called a probabilistic non-distance on X.

Definition 7.1.6 A triple $(X, F, L,)$ is said to be **an intuitionistic Menger(PQM)** space if X is a nonempty set, F is a probabilistic distance and L is probabilistic non-distance on X satisfying the following conditions:
For all $x, y, z \in X$ and $s, t > 0$

7.1.6 (i) $F_{x,y}(t) + L_{x,y}(t) \leq 1$
7.1.6 (ii) $F_{x,y}(t) = 0$
7.1.6 (iii) $F_{x,y}(t) = H(t)$for all $t > 0$ $if\ and\ only\ if\ x = y$
7.1.6 (iv) $F_{x,y}(t) = F_{y,x}(t)$
7.1.6 (v) $L_{x,y}(0) = 1$
71.6 (vi) $L_{x,y}(t) = G(t)$for all $t > 0$ $if\ and\ only\ if\ x = y$
7.1.6 (vii) $L_{x,y}(t) = L_{y,x}(t)$
If in addition, we have the triangle inequalities:
7.1.6 (viii) $F_{x,y}(t + s) \geq T\left(F_{x,z}(t), F_{z,y}(t)\right)$.
7.1.6 (ix) $L_{x,y}(t + s) \leq S(L_{x,z}(t), L_{z,y}(t))$.

Here T is a t−norm and S is a t−conorm. Then (X, F, L, T, S) is said to be an

INTUITIONISTIC MENGER (PQM) SPACE

The Functions $F_{x,y}(t)$ and $L_{x,y}(t)$ Denote The degree of nearness and degree of non-nearness between x and y with respect to t respectively.

Remark 7.1.7: Every Menger (PQM) space (X, F, T) is an intuitionistic Menger (PQM) space of the form $(X, F, 1 - F, T, S)$ such that t−norm T and t−conorm S are associated
i.e. $S(x, y) = 1 - T(1 - x, 1 - y)$ for any $x, y \in X$.

Example 7.1.8: (Induced intuitionistic Menger (PQM)space) Let (X, d)be a metric space. Then the metric d induces a distance distribution function F defined by

$$F_{x,y}(t) = H(t - d(x, y))$$

and a non-distance distribution function L defined by

$$L_{x,y}(t) = G(t - d(x, y)) \text{ for all } x, y, \in X \text{ and } t \geq 0.$$

Then (X, F, L) IS AN INTUITIONISTIC MENGER (PQM) SPACE.

We call this Intuitionistic Menger (PQM) space induced by a metric d the induced intuitionistic Menger (PQM) space. If t−norm T is $T(a, b) = \min\{a, b\}$ and t−conorm S is $S(a, b) = \min\{1, a + b\}$ for all $a, b \in [0, 1]$, then (X, F, L, T_M, S_M) is an intuitionistic Menger (PQM)space.

Remark 7.1.9: Note that the above examples hold even with the t−norm $T(a, b) = \min\{a, b\}$ and t−conorm $S(a, b) = \max\{a, b\}$, and hence (X, F, L, T, S) is an INTUITIONISTIC MENGER SPACE WITH RESPECT TO ANY T−NORM AND T−CONORM.Also note that, $T : [0,1] \times [0,1] \rightarrow [0,1]$, in the above example the t−norm T and t−conorm S are not associated.

Definition 7.1.10: Let (X, F, L, T, S) be a Intuitionistic Menger (PQM) space.
(i) A sequence $\{x_n\}$ in X is $T : [0,1] \times [0,1] \to [0,1]$, said to be convergent to x in X, if for every $\varepsilon > 0$, $\lambda > 0$, there exists positive integer N such that

$$F_{x_n,x}(\varepsilon) > 1 - \lambda \text{ and } L_{x_n,x}(\varepsilon) < \lambda \text{ whenever } n \geq N.$$

we write $x_n \to x$ as $n \to \infty$ or $\lim_{n\to\infty} x_n = x$.
(ii) A sequence $\{x_n\}$ in X is called cauchy sequence, if for every $\varepsilon > 0$, $\lambda > 0$,, there exists positive integer N such that

$$F_{x_n,x_m}(\varepsilon) > 1 - \lambda \text{ and } L_{x_n,x_m}(\varepsilon) < \lambda \text{ whenever } n \geq N \geq m \geq N.$$

(iii) A Menger (PQM) space $(X, F, T)$is said to be complete if and only if every cauchy sequence in X is convergent to a point in X.

Lemma 7.1.11: Let (X, F, L, T, S) be an intuitionistic Menger space. If there is a constant $k \in (0,1)$ such that for $x, y \in X$, $t > 0$, $F_{x,y}(kt) \geq F_{x,y}(t)$ and $L_{x,y}(kt) \leq L_{x,y}(t)$, then $x = y$.

Lemma 7.1.12: let (X, F, L, T, S) be an intuitionistic menger space. then $f_{x,y}(t)$ and $l_{x,y}(t)$ are continuous functions on $x \times x \to (0, \infty)$.

Definition 7.1.13: Let (X, F, L, T, S) be a intuitionistic menger (pqm) space such that the t-norm t and t−conorm s is continuous and P, Q be mappings from X into itself. Then, P and Q are said to be compatible if $\lim_{n\to\infty} F_{PQx_n,QPx_n}(x) = 1$ and $\lim_{n\to\infty} L_{PQx_n,QPx_n}(x) = 0$ for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$ for some $z \in X$.

Definition 7.1.14: Two self mappings P and Q are said to be weakly compatible if they commute at their
$T : [0,1] \times [0,1] \to [0,1]$, coincidence points that is $Px = Qx$. For some $x \in X$ implies $PQx = QPx$.

Definition 7.1.15: let P and Q be two self mappings of a menger space (X, F, L, T, S) we say that p and q satisfy the property (E.A) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$$

for some $z \in X$.

Example 7.1.16: Let $X = [0, +\infty)$. Define $P, Q : X \to X$ by

$$Px = \frac{7x}{5} \text{ and } x = \frac{3x}{8}, \forall x \in X.$$

Consider the sequence $x_n = \frac{1}{n}$.Clearly

$$\lim_{n\to\infty} x_n = Px_n = \lim_{n\to\infty} x_n = Qx_n = 0$$

Then P and Q satisfy (E, A).

Example 7.1.17: Let $X = [2, +\infty)$. Define $P, Q : X \to X$ by

$$Px = x + 1 \text{ and } x = 2x + 1 \ \forall x \in X.$$

Suppose that the property (E.A.) holds.

Then there is $T : [0,1] \times [0,1] \to [0,1]$, a sequence $\{x_n\}$ in X satisfying

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z \text{ for some } z \in X$$

Therefore $\lim_{n\to\infty} x_n = z - 1$ and $\lim_{n\to\infty} x_n = \frac{z-1}{2}$.

Thus, $z = 1$, which is a contradiction since $1 \notin X$.

Hence P and Q do not satisfy (E.A.). $T : [0,1] \times [0,1] \to [0,1]$,

7.2. COMMON FIXED POINT THEOREM IN INTUITIONISTIC MENGER SPACES

Theorem 7.2.1: Let (X, F, L, T, S) be a Intuitionistic Menger (PQM) space with
$T(x, y) = \min\{x, y\}$ and $S(x, y) = \max\{x, y\}$ for all $x, y \in [0,1]$. Let A, B, P and Q be mappings from X into itself such that:

7.2.1 (I) $A(X) \subset P(X)$ and $B(X) \subset Q(X)$.
7.2.1 (II) (A, Q) or (B, P) satisfies the property (E.A).
7.2.1 (III) There exists a number $k \in (0, 1)$ such that

$$\min\begin{Bmatrix}(F_{Au,Bv}\,kx)(F_{Qu,Pv}\,(x), F_{Qu,Bv}\,(x), F_{Pv,Bv}\,(x),\\ F_{Au,Qu}\,(x), F_{Au,Pv}\,(x), F_{Qu,Qu}(x)\end{Bmatrix}^2 \geq 0$$

$$\min\begin{Bmatrix}L_{Au,Bv}\,(kx)L_{Qu,Pv}\,(x), L_{Qu,Bv}\,(x), L_{Pv,Bv}\,(x),\\ L_{Au,Qu}\,(x), L_{Au,Pv}\,(x)L_{Qu,Qu}(x)\end{Bmatrix}^2 \leq 0$$

for all $u, v \in X$.

7.2.1 (IV) (A , Q) and (B , P) are weakly compatible,

7.2.1 (V) One of A(X), B(X), Q(X) or P(X) is a closed subset of X.

Then A, B , P and Q have a unique common fixed point in X.

Proof: Suppose that (B , P) satisfies the $T : [0,1] \times [0,1] \to [0,1]$, property (E.A). Then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = z$$

for some $z \in X$.

Since $B(X) \subset Q(X)$, there exists in X a sequence $\{y_n\}$ such that

$$Bx_n = Qy_n.$$

Hence $\lim_{n\to\infty} Qy_n = z$.

Let us show that $\lim_{n\to\infty} Ay_n = z$.

$$\min\begin{Bmatrix}F_{Ay_n,Bx_n}(kx)F_{Qy_n,Px_n}\,(x), F_{Qy_n,Bx_n}\,(x), F_{Px_n,Bx_n}\,(x),\\ F_{Ay_n,Qy_n}\,(x), F_{Ay_n,Px_n}\,(x)F_{Qy_n,Qx_n}\,(x)\end{Bmatrix}^2 \geq 0$$

$$\geq \min\begin{Bmatrix}F_{Ay_n,Bx_n}(kx)F_{Bx_n,Px_n}\,(x), F_{Px_n,Bx_n}\,(x),\\ F_{Ay_n,Bx_n}\,(x), F_{Ay_n,Px_n}\,(x)\end{Bmatrix}^2$$

$$\geq \{F_{Ay_n,Bx_n}(x)\}^2 \min\begin{Bmatrix}L_{Ay_n,Bx_n}(kx)L_{Qy_n,Px_n}\,(x), L_{Qy_n,Bx_n}\,(x), L_{Px_n,Bx_n}\,(x),\\ L_{Ay_n,Qy_n}\,(x), L_{Ay_n,Px_n}\,(x)L_{Qy_n,Qx_n}\,(x)\end{Bmatrix}^2 \leq 0$$

$$\leq \min\begin{Bmatrix}L_{Ay_n,Bx_n}(kx)L_{Bx_n,Px_n}\,(x), L_{Px_n,Bx_n}\,(x), & \leq 0\\ L_{Ay_n,Bx_n}\,(x), L_{Ay_n,Px_n}\,(x)\end{Bmatrix}^2$$

$$\leq \min\begin{Bmatrix}L_{Ay_n,Bx_n}(kx)L_{Bx_n,Px_n}\,(x), L_{Px_n,Bx_n}\,(x),\\ L_{Ay_n,Bx_n}\,(x), L_{Ay_n,Px_n}\,(x)\end{Bmatrix}^2$$

$$\leq \{L_{Ay_n,Bx_n}(x)\}^2 \{\text{k belong to (zero , one)}$$

Therefore with the Lemma (7.1.11) $Ay_n = Bx_n$.

Letting $n \to \infty$, we obtain

$$\min\begin{Bmatrix}L_{Ay_n,Bx_n}(kx)L_{Qy_n,Px_n}\,(x), L_{Qy_n,Bx_n}\,(x), L_{Px_n,Bx_n}\,(x),\\ L_{Ay_n,Qy_n}\,(x), L_{Ay_n,Px_n}\,(x)L_{Qy_n,Qx_n}\,(x)\end{Bmatrix}^2 \leq 0$$

$$\leq \min\begin{Bmatrix}L_{Ay_n,Bx_n}(kx)L_{Bx_n,Px_n}\,(x), L_{Px_n,Bx_n}\,(x),\\ L_{Ay_n,Bx_n}\,(x), L_{Ay_n,Px_n}\,(x)\end{Bmatrix}^2 \leq 0$$

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Ay_n = z. \{\text{k belong to (zero, one)}$$

Suppose Q(X)is a closed subset of X. Then $z = Qu$ for some $u \in X$.

Subsequently, we have

$$\min\begin{Bmatrix}L_{Ay_n,Bx_n}(kx)L_{Qy_n,Px_n}\,(x), L_{Qy_n,Bx_n}\,(x), L_{Px_n,Bx_n}\,(x),\\ L_{Ay_n,Qy_n}\,(x), L_{Ay_n,Px_n}\,(x)L_{Qy_n,Bx_n}\,(x)\end{Bmatrix}^2 \leq 0$$

$$\min\begin{Bmatrix}L_{Ay_n,Bx_n}(kx)L_{Bx_n,Px_n}\,(x), L_{Px_n,Bx_n}\,(x),\\ L_{Ay_n,Bx_n}\,(x), L_{Ay_n,Px_n}\,(x)\end{Bmatrix}^2 \leq 0 \{\text{k belong to (zero, one)}$$

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

We have

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

$$\min\begin{Bmatrix}F_{Au,Bx_n}(kx)F_{Qu,Px_n}(x), F_{Qu,Bx_n}(x), F_{Px_n,Bx_n}(x), \\ F_{Au,Qu}(x), F_{Au,Px_n}(x)F_{Qy_n,Qx_n}(x)\end{Bmatrix}^2 \geq 0$$

$$\min\begin{Bmatrix}L_{Au,Bx_n}(kx)L_{Qu,Px_n}(x), L_{Qu,Bx_n}(x), L_{Px_n,Bx_n}(x), \\ L_{Au,Qu}(x), L_{Au,Px_n}(x)L_{Qy_n,Qx_n}(x)\end{Bmatrix}^2 \leq 0 \text{ \{k belong to (zero, one)}$$

Letting $n \to \infty$, we obtain

$$\{F_{Au,Su}(kx)F_{Au,Su}(x)\}^2 \geq 0$$
$$\{L_{Au,Su}(kx)L_{Au,Su}(x)\}^2 \leq 0$$

Therefore with the Lemma (7.1.11) we have

$$Au = Qu.$$

The weak compatibility of A and Q implies that

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$
$$AQu = QAu$$

and then $AAu = AQu = QAu = QQu$.

On the other hand, since $A(X) \subset P(X)$, there exists a point $v \in X$, such that $Au = Pv$.

We claim that $Pv = Bv$.

We have

$$\min\begin{Bmatrix}F_{Au,Bv}(kx)F_{Qu,Pv}(x), F_{Qu,Bv}(x), F_{Pv,Bv}(x), \\ F_{Au,Qu}(x), F_{Au,Pv}(x)F_{Qy_n,Qx_n}(x)\end{Bmatrix}^2 \geq 0$$
$$\geq \{F(Au, Bv)(x)\}^2 \text{ \{k belong to (zero , one)}$$
$$\min\begin{Bmatrix}L_{Au,Bv}(kx)\, L_{Qu,Pv}(x), L_{Qu,Bv}(x), L_{Pv,Bv}(x), \\ L_{Au,Qu}(x), L_{Au,Pv}(x)F_{Qy_n,Qx_n}(x)\end{Bmatrix}^2 \leq 0$$
$$\leq \{ L_{Au,Bv}(x)\}.^2 \text{\{k belong to (zero, one)}$$

Therefore, with the Lemma (7.1.11) we have $Au = Bv$.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Thus $Au = Qu = Pv = Bv$.

The weak compatibility of B and P implies that

$$BPv = PBv \text{ and } PPv = PBv = BPv = BBv.$$

Let us show that Au is a common fixed point of A, B, Pand Q.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

We have

$$\text{Min}\begin{Bmatrix}F_{AAu,Bv}(kx)\, F_{QAu,Pv}(x), F_{QAu,Bv}(x), F_{Pv,Bv}(x), \\ F_{AAu,QAu}(x), F_{AAu,Pv}(x)F_{Qy_n,Qx_n}(x)\end{Bmatrix}^2 \geq 0 \quad \text{\{k belong to (zero , one)}$$
$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$
$$\{F_{Au,AAu}(kx)\, F_{AAu,Av}(x)\}^2 \geq 0$$
$$\min\begin{Bmatrix}L_{AAu,Bv}(kx)\, L_{QAu,Pv}(x), L_{QAu,Bv}(x), L_{Pv,Bv}(x), \\ L_{AAu,QAu}(x), L_{AAu,Pv}(x)L_{Qy_n,Qx_n}(x)\end{Bmatrix}^2 \leq 0 \text{ \{k belong to (zero, one)\}}$$
$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$
$$\{L_{Au,AAu}(kx)\}^2 \quad \leq \{L_{AAu,Av}(x)\}^2$$

Therefore, we have

$$Au = AAu = QAu$$

That is Au is a common fixed point of A and Q.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Similarly, we can prove that Bv is a common fixed point of B and P.

Since $Au = Bv$, we conclude that Au is a common fixed point of A, B, P and Q.

The proof is similar when P(X) is assumed to be a closed subset of X.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

The cases in which A(x) or B(x) is closed subset of X are similar to the cases in which P(X) or Q(X), respectively, is closed

Since $A(X) \subset P(X)$ and $B(X) \subset Q(X)$.

If $Au = Bu = Su = Lu = u$ and $Av = Bv = Sv = Lv = v$.

We have

$$\min\left\{\begin{matrix} F_{Au,Bv}(kx)\, F_{Qu,Pv}(x), F_{Qu,Bv}(x), F_{Pv,Bv}(x), \\ F_{Au,Qu}(x), F_{Au,Pv}(x) F_{Qy_n,Qx_n}(x) \end{matrix}\right\}^2 \geq 0$$

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu \geq \{F_{u,v}(x)\}^2.$$

$$\min\left\{\begin{matrix} L_{Au,Bv}(kx)\, L_{Qu,Pv}(x), L_{Qu,Bv}(x), L_{Pv,Bv}(x), \\ L_{Au,Qu}(x), L_{Au,Pv}(x) L_{Qy_n,Qx_n}(x) \end{matrix}\right\}^2 \leq 0$$

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu \leq \{L_{u,v}(x)\}^2.$$

Thus we have $u = v$ and the common fixed point is unique.

This completes the proof of the theorem.

For three mapping, we have the following result:

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Corollary 7.2.2: Let (X, F, L, T, S) be an Intuitionistic Menger (PQM) space with $T(x, y) = \min\{x, y\}$ and $S(x, y) = \max\{x, y\}$ for all $x, y \in [0, 1]$.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Let A, B and P be mappings from X into itself such that:

7.2.2 (I) $A(X) \subset P(X)$ and $B(X) \subset P(X)$

7.2.2 (II) (A, P) or (B, P) satisfies the property (E.A.),

7.2.2 (III) There exists a number $k \in (0,1)$ such that

$$\min\left\{\begin{matrix} F_{Au,Bv}(kx) F_{Pu,Pv}(x), F_{Pu,Bv}(x), \\ F_{Pv,Bv}(x), F_{Au,Pu}(x), F_{Au,Pv}(x) F_{Qy_n,Qx_n}(x) \end{matrix}\right\} \geq 0$$

$$\min\left\{\begin{matrix} L_{Au,Bv}(kx)\, L_{Pu,Pv}(x), L_{Pu,Bv}(x), \\ L_{Pv,Bv}(x), L_{Au,Pu}(x), L_{Au,Pv}(x) F_{Qy_n,Qx_n}(x) \end{matrix}\right\} \leq 0$$

for all $u, v \in X$.

7.2.2 (IV) (A, P) and (B, P) are weakly compatible,

7.2.2 (V) One of $A(X), B(X)$ or $P(X)$ is a closed subset of X.

Then A, B and P have a unique common fixed point in X.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

Corollary 7.2.3: Let (X, F, L, T, S) be a Intuitionistic Menger (PQM) space with $T(x, y) = \min\{x, y\}$ and $S(x, y) = \max\{x, y\}$ for all $x, y \in [0, 1]$.

Let Aand P be mappings from X into itself such that:

7.2.3 (I) $A(X) \subset P(X)$.

7.2.3 (II) (A, P) satisfies the property (E.A),

7.2.3 (III) There exists a number $k \in (0,1)$ such that

$$\min\left\{\begin{matrix} F_{Au,Av}(kx) F_{Pu,Pv}(x), F_{Pu,Av}(x), \\ F_{Pv,Av}(x), F_{Au,Pu}(x), F_{Au,Pv}(x) F_{Qy_n,Qx_n}(x) \end{matrix}\right\}^2 \geq 0$$

$$\min\begin{Bmatrix} L_{Au,Av}(kx)L_{Pu,Pv}(x), L_{Pu,Av}(x), \\ L_{Pv,Av}(x), L_{Au,Pu}(x), L_{Au,Pv}(x)L_{Qy_n,Qx_n}(x) \end{Bmatrix}^2 \leq 0$$

for all $u, v \in X$.

7.2.3 (IV) (A, P) be weakly compatible,

7.2.3 (V) One of $A(X)$ or$P(X)$ is a closed subset of X.

Then A and P have a unique common fixed point in X.

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qy_n = Qu$$

REFERENCES

1. M. Aamri and D. El Moutawakil, Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl. 270 (2002) 181–188.
2. C. Alaca, D. Turkoglu and C. Yildiz, Fixed points in Intuitionistic fuzzy metric spaces, Chaos Solitons and Fractals 29 (2006) 1073–1078.
3. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1986) 87–96.
4. D. Coker, An introduction to Intuitionistic Fuzzy topological spaces, Fuzzy Sets Syst. 88 (1997) 81–89.
5. J. X. Fang and Y. Gao, Common fixed point theorems under strict contractive conditions in Menger spaces, Nonlinear Anal. 70 (2009) 184–193.
6. A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets Syst. 64 (1994) 395–399.
7. M. Imdad, M. Tanveer and M. Hasan, Some common fixed point theorems in Menger PM-spaces, Fixed Point Theory and Applications Volume 2010, Article ID 819269, 14 pages.
8. G. Jungck, Compatible mappings and common fixed points, Internat. J. Math. Math. Sci. 9 (1986) 771–779.
9. G. Jungck and B.E. Rhoades, Fixed points for set valued functions without continuity, Indian J. Pure Appl. Math. 29 (1998) 227–238.
10. I. Kramosil and J. Michalek, Fuzzy metric and statistical spaces, Kybernetica 11 (1975) 336–344.
11. S. Kumar, Common fixed point theorems in Intuitionistic fuzzy metric spaces using property (E.A), J. Indian Math. Soc. 76 (2009) 94–103.
12. S. Kumar and B. Fisher, A common fixed point theorem in fuzzy metric space using property (E. A.) and implicit relation, Thai J. Math. 8 (2010) 439–446.
13. W. Lui, J. Wu and Z. Li, Common fixed points of single-valued and multivalued maps, Int. J. Math. Math. Sci 19 (2005) 3045–3055.
14. K. Menger, Statistical metrics, Proc. Nat. Acad. Sci. (USA) 28 (1942) 535–537.
15. D. Mihet, Fixed point theorems in fuzzy metric spaces using property E.A., Nonlinear Anal. 73 (2010), 2184–2188.
16. R.P. Pant, Common fixed points of contractive maps, J. Math. Anal. Appl. 226 (1998) 251–258.
17. J.H. Park, Intuitionistic fuzzy metric spaces, Chaos Solitions and Fractals 22 (2004) 1039–1046. 8 Manro
18. B. Schweizer and A. Sklar, Probabilistic Metric Spaces, North Holland Amsterdam, 1983
19. S. Sharma and B. Deshpande, Compatible mappings of type (I) and (II) on Intuitionistic fuzzy metric spaces in consideration of common fixed point, Commun. Korean Math. Soc. 24 (2009) 197–214. [20] L.A. Zadeh, Fuzzy sets, Infor. and Control. 8 (1965) 338–353

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