

ON  $\# \alpha$ -REGULAR GENERALIZED OPEN SETS AND  $\# \alpha - RG$  NEIGHBOURHOODS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce a new class of open sets called the class of  $\# \alpha g$ -open and study their relationship with other open sets. Also we introduce of  $\# \alpha g$  neighbourhood (shortly  $\# \alpha g$ -nbhd) in topological spaces  $\# \alpha g$ -closure.

**Keywords:**  $\# \alpha g$ -open sets,  $\# \alpha g$ -nbhd,  $\# \alpha g$ -closure

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1. INTRODUCTION

Regular open sets and rw-open sets have been introduced and investigated by Stone [18] and Benchalli and Wali [1]. Maki [10] defined  $\alpha$ -generalized closed sets in 1994. Dontchev, Gnanambal and Palaniappan and Rao [5, 7, 13] introduce gsp closed sets. Syed Ali Fatima [19] defined  $\# rg$ -closed sets. In this paper we introduce new class of sets called:  $\# \alpha g$ -open sets which is properly placed in between the class of  $\# rg$ -open sets and the class of  $\alpha g$ -open sets.

Throughout this paper  $(X, \tau)$  represents a non-empty topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset  $A$  of a topological space  $X$ ,  $cl(A)$  and  $int(A)$  denote the closure of  $A$  and the interior of  $A$  respectively  $(X, \tau)$  will be replaced by  $X$  if there is no chance of confusion.  $X \setminus A$  or  $A^c$  denotes the complement of  $A$  in  $X$ . Let us recall the following definitions.

**Definition 1.1:** A Subset  $A$  of a space  $X$  is called

- (1) a preopen set [11] if  $A \subseteq int(cl(A))$  and a preclosed set if  $cl(int(A)) \subseteq A$
- (2) a semiopen set [8] if  $A \subseteq cl(int(A))$  and a semiclosed set if  $int(cl(A)) \subseteq A$
- (3) an  $\alpha$ -open set [21] if  $A \subseteq int(cl(int(A)))$  and an  $\alpha$ -closed set if  $cl(int(cl(A))) \subseteq A$
- (4) a regular open set [13] if  $A = int(cl(A))$  and a regular closed set if  $A = cl(int(A))$
- (5) a  $\pi$ -open set [1] if  $A$  is a finite union of regular open sets.
- (6) regular semi open [15] if there is a regular open  $U$  such  $U \subseteq A \subseteq cl(U)$

**Definition 1.2:** A subset  $A$  of a topological space  $(X, \tau)$  is called.

- (1) an  $\alpha$ -generalized closed set (briefly  $\alpha g$ -closed) [14] if  $acl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- (2) a generalized pre closed set (briefly gp-closed) [14] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- (3) a generalized semi pre closed set (briefly gsp-closed) [14] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- (4) a generalized  $\alpha$ -closed (i.e.,  $g \alpha$ -closed) set [21] if  $acl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open set in  $X$ .
- (5)  $\# rg$ -closed (19) if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is rw-open.
- (6)  $\# \alpha g$ -closed ( ) if  $acl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is rw-open,

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## 2. $\# \alpha$ -REGULAR GENERALIZED OPEN SETS

In this section we introduce the concept of  $\# arg - open$  sets and characterize their properties.

**Definition 2.1:** A subset  $A$  of a space  $X$  is called  $\# \alpha - regular$  generalized open (briefly  $\# arg - open$ ) set if its complement is  $\# arg - closed$ . We denote the set of all  $\# arg - open$  sets in  $X$  by  $\# \alpha RGO(X)$

**Remark 2.2:**  $acl(X \setminus A) = X \setminus aint(A)$

**Theorem 2.3:** A subset  $A$  of  $X$  is  $\# arg - open$  if and only if  $F \subseteq aint(A)$  whenever  $F$  is  $rw$ -closed and  $F \subseteq A$ .

**Proof:**

**Necessity:** Let  $A$  be  $\# arg - open$ . Let  $F$  be  $rw$ -closed and  $F \subseteq A$  then  $X \setminus A \subseteq X \setminus F$  Since  $X \setminus A$  is  $\# arg - closed$ ,  $acl(x \setminus A) \subseteq X \setminus F$  by Remark 2.2  $X \setminus aint(A) \subseteq X \setminus F$  that is  $F \subseteq aint(A)$ .

**Sufficiency:** Suppose  $F$  is  $rw$ -closed and  $F \subseteq A$  implies  $F \subseteq aint(A)$ . Let  $X \setminus A \subseteq U$  where  $U$  is  $rw$ -open. Then  $X \setminus U \subseteq A$  and  $X \setminus U$  is  $rw$ -closed. By hypothesis  $X \setminus U \subseteq aint(A)$ . That is  $X \setminus aint(A) \subseteq U$  by Remark 2.2  $acl(x \setminus A) \subseteq U$ . Hence  $X \setminus A$  is  $\# arg - closed$ . Thus  $A$  is  $\# arg - open$ .

**Theorem 2.4:** If  $aint(A) \subseteq B \subseteq A$  and  $A$  is  $\# arg - open$  then  $B$  is  $\# arg - open$ .

**Proof:** Let  $A$  be  $\# arg - open$  set and  $aint(A) \subseteq B \subseteq A$ . Now  $aint(A) \subseteq B \subseteq A$  implies  $X \setminus A \subseteq X \setminus B \subseteq X \setminus aint(A)$ . That is  $X \setminus A \subseteq X \setminus B \subseteq acl(x \setminus A)$ . Since  $X \setminus A$  is  $\# arg - closed$  then  $A$  is  $\# arg - open$ .  $X \setminus B$  is  $\# arg - closed$  then  $B$  is  $\# arg - open$ .

**Remark 2.5:** For any  $A \subseteq X$ ,  $aint(acl(A) \setminus A) = \phi$

**Theorem 2.6:** If  $A \subseteq X$  is  $\# arg - closed$  then  $acl(A) \setminus A$  is  $\# arg - open$ .

**Proof:** Let  $A$  be  $\# arg - closed$ . Let  $F$  be  $rw$ -closed set such that  $F \subseteq acl(A) \setminus A$ . Then by theorem 3.2 [21]  $F = \phi$ . So,  $F \subseteq aint(acl(A) \setminus A)$ . This shows that  $acl(A) \setminus A$  is  $\# arg - open$ .

**Theorem 2.7:** If a subset  $A$  of a space  $X$  is  $\# rg - open$  then it is  $\# arg - open$  but not conversely.

**Proof:** Let  $A$  be  $\# rg - open$  set in space  $X$ . Then  $X \setminus A$  is  $\# rg - closed$  set. By Theorem 3.2 [21]  $X \setminus A$  is  $\# arg - closed$ . Therefore  $A$  is  $\# arg - open$  set in  $X$ .

The converse of the theorem need not be true as seen from the following example.

**Example 2.8:** Let  $X = \{a, b, c, d\}$  be with topology  $\tau = \{X, \phi, \{a, b\}\}$  then the set  $A = \{a, b, c\}$  is  $\# arg - open$  but not  $\# rg - open$  in  $X$ .

**Corollary 2.9:** Every regular open set is  $\# arg - open$  but not conversely.

**Proof:** Follows from Fathima [20] and Theorem [2.7]

**Corollary 2.10:** Every  $\pi - open$  set is  $\# arg - open$  but not conversely.

**Proof:** Follows from Fathima [20] and Theorem [2.7]

**Theorem 2.11:** If a subset  $A$  of a topological space  $(X, \tau)$  is open that it is  $\# arg - open$  but not conversely.

**Proof:** Let  $A$  be an open set in a space  $X$ . Then  $X \setminus A$  is closed set. By Theorem 3.7 [21]  $X \setminus A$  is  $\# arg - closed$ . Therefore  $A$  is  $\# arg - open$  set in  $X$ . The converse of the theorem need not be true as seen from the following example.

**Example 2.12:** Let  $X = \{a, b, c, d\}$  be with topology  $\tau = \{X, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$  then the set  $\{b\}$  is  $\# arg - open$  but not open set in  $X$ .

**Theorem 2.13:** If a subset  $A$  of a topological Space  $(X, \tau)$  is  $\alpha$  – open then it is  $\#arg$  – open but not conversely.

**Proof:** Let  $A$  be  $\alpha$  – open set in  $X$ . Then  $X \setminus A$  is  $\alpha$  – closed set then by theorem 3.10 [21]  $X \setminus A$  is  $\#arg$  – closed set in  $X$ . Therefore  $A$  is  $\#arg$  – open set in  $X$ .

The converse of the theorem need not be true as seen from the following example.

**Example 2.14:** Let  $X = \{a, b, c, d\}$  be with topology  $\tau = \{X, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$  then the set  $\{a\}$  is  $\#arg$  – open but not  $\alpha$  – open set in  $X$ .

**Theorem 2.15:** If a subset  $A$  of  $X$  is  $\#arg$  – open then it is gp-open but not conversely.

**Proof:** Let  $A$  be  $\#arg$  – open set in a space  $X$ . Then  $X \setminus A$  is  $\#arg$  – closed set in  $X$  then by theorem 3.12 [21]  $X \setminus A$  is gp-closed set in  $X$ . Therefore  $A$  is gp-open in  $X$ .

The converse of the theorem need not be true as seen from the following example.

**Example 2.16:** Let  $X = \{a, b, c, d\}$  be with the topology  $\tau = \{X, \phi, \{a, b\}\}$  then the set  $A = \{a, d\}$  is gp-open but not  $\#arg$  – open set in  $X$ .

**Theorem 2.17:** If a subset  $A$  of a space  $X$  is  $\#arg$  – open then it is  $ag$  – open.

**Proof:** Let  $A$  be  $\#arg$  – open in  $X$ . Then  $X \setminus A$  is  $\#arg$  – closed set in  $X$ . Then By theorem 3.14 [21]  $X \setminus A$  is  $ag$  – closed set in  $X$ . Therefore  $A$  is  $ag$  – open in  $X$ .

The converse of the theorem need not be true as seen from the following example.

**Example 2.18:** Let  $X = \{a, b, c\}$  be with the topology  $\tau = \{X, \phi, \{c\}, \{a, b\}\}$  the set  $A = \{a, c\}$  is  $ag$  – open but not  $\#arg$  – open set in  $X$ .

**Theorem 2.19:** If a subset  $A$  of a space  $X$  is  $\#arg$  – open then it is gsp-open.

**Proof:** Let  $A$  be  $\#arg$  – open in  $X$ . Then  $X \setminus A$  is  $\#arg$  – closed set in  $X$ . Then by theorem 3.16 [21]  $X \setminus A$  is gsp-closed set in  $X$ . Therefore  $A$  is gsp-open in  $X$ .

The converse of the theorem need not be true as seen from the following example.

**Example 2.20:** Let  $X = \{a, b, c\}$  be with the topology  $\tau = \{X, \phi, \{c\}, \{a, b\}\}$  then the set  $A = \{a, c\}$  is gsp-open but not  $\#arg$  – open

**Theorem 2.21:** If  $A$  and  $B$  are  $\#arg$  – open set in a space  $X$ . The  $A \cap B$  is also  $\#arg$  – open set in  $X$ .

**Proof:** If  $A$  and  $B$  are  $\#arg$  – open sets in a space  $X$ . Then  $X \setminus A$  and  $X \setminus B$  are  $\#arg$  – closed sets in a space  $X$ . By theorem 3.20 [21]  $(X \setminus A) \cup (X \setminus B)$  is also  $\#arg$  – closed sets in  $X$ . Therefore  $A \cap B$  is  $\#arg$  – open set in  $X$ .

**Theorem 2.22:** The union of two  $\#arg$  – open sets in  $X$  is not a  $\#arg$  – open set in  $X$ .

**Example 2.23:** Let  $X = \{a, b, c, d\}$  be with topology  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  then the set  $A = \{b, c\}$  and  $B = \{b, d\}$  are  $\#arg$  – open set in  $X$  but  $A \cup B = \{b, c, d\}$  is not  $\#arg$  – open set in  $X$ .

**Theorem 2.24:** If a set  $A$  is  $\#arg$  – open set in  $X$  then  $G = X$  whenever  $G$  is rw-open and  $(\alpha \text{ int}(A) \cup (X \setminus A)) \subseteq G$ .

**Proof:** Suppose that  $A$  is  $\#arg$  – open set in  $X$ . Let  $G$  be rw-open and  $(\alpha \text{ int}(A) \cup (X \setminus A)) \subseteq G$ . This implies  $G^c \subseteq (\alpha \text{ int}(A) \cup (X \setminus A))^c = (\alpha \text{ int}(A))^c \cap A$ . That is  $G^c \subseteq (\alpha \text{ int}(A))^c \cap A$ . Thus  $G^c \subseteq \alpha \text{ cl}(A^c) \setminus A^c$ . Now  $G^c$  is rw-closed and  $A^c$  is  $\#arg$  – closed, by theorem 3.23 [21] it follows that  $G^c = \phi$ . Hence  $G = X$ .

**Theorem 2.25:** Every Singleton point set in a space is either  $\#arg$  – open or rw-closed.

**Proof:** Follows from theorem 3.27 [21]

### 3. $\# \alpha$ -REGULAR GENERALIZED NEIGHBOURHOODS

**Definition 3.1:** Let  $X$  be a topological space and let  $x \in X$ . A subset  $N$  of  $x$  is said to be  $\# \arg$  – Neighbourhood (briefly  $\# \arg - nbhd$ ) of  $x$  if and only if there exists a  $\# \arg$  – open set  $U$  such that  $x \in U \subset N$ .

**Definition 3.2:** A subset  $N$  of space  $X$  is called a  $\# \arg$  – neighbourhood (briefly  $\# \arg - nbhd$ ) of  $A \subset X$  if and only if there exists a  $\# \arg$  – open set  $U$  such that  $A \subset U \subset N$ .

**Theorem 3.3:** Every neighbourhood of  $N$  of  $x \in X$  is a  $\# \arg$  – Neighbourhood of  $X$ .

**Proof:** Let  $N$  be a neighbourhood of point  $x \in X$  by definition of neighbourhood, there exists an open set  $U$  such that  $x \in U \subset N$ . Since every open set is  $\# \arg$  – open set,  $U$  is  $\# \arg$  – open set such that  $x \in U \subseteq N$ . This implies  $N$  is  $\# \arg$  – Neighbourhood of  $X$ .

The converse of the above theorem need not be true as seen from the following example.

**Example 3.4:** Let  $X = \{a, b, c, d\}$  be with topology  $\tau = \{X, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$ . Then  $\# \alpha RGO(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b, c\}\}$ . The set  $\{b, d\}$  is  $\# \arg - nbhd$  of the point  $b$ , since the  $\# \arg$  – open set  $\{b\}$  is such that  $b \in \{b\} \subseteq \{b, d\}$ . However the set  $\{b, d\}$  is not a nbhd of the point  $b$ . Since no open set  $U$  exists such that  $b \in \{b\} \subseteq \{b, d\}$ . However the set  $\{b, d\}$  is not a nbhd of the point  $b$ . Since no open set  $U$  exists such that  $b \in U \subseteq \{b, d\}$ .

**Theorem 3.5:** If a subset  $N$  of a space  $X$  is  $\# \arg$  – open then  $N$  is  $\# \arg - nbhd$  of each of its point.

**Proof:** Suppose  $N$  is  $\# \arg$  – open let  $x \in N$ , For  $N$  is  $\# \arg$  – open set such that  $x \in N \subseteq N$ . Since ‘ $x$ ’ is an arbitrary point of  $N$ , it follows that  $N$  is a  $\# \arg - nbhd$  of each of its points. The converse of the theorem is not true as seen from the following example.

**Example 3.6:** Let  $X = \{a, b, c, d\}$  be with topology  $\tau = \{X, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$  then  $\# \alpha RGO(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b, c\}\}$ . The set  $\{b, c\}$  is a  $\# \arg - nbhd$  of the point  $b$ . Since the  $\# \arg$  – open set  $\{b\}$  is such that  $b \in \{b\} \subset \{b, c\}$ . Also the set  $\{b, c\}$  is  $\# \arg - nbhd$  of the point  $\{c\}$  since the  $\# \arg$  – open set  $\{c\}$  is such that  $c \in \{c\} \subseteq \{b, c\}$ . That is  $\{b, c\}$  is a  $\# \arg - nbhd$  of each of its points. However the set  $\{b, c\}$  is not  $\# \arg$  – open set in  $X$ .

**Theorem 3.7:** If  $F$  is a  $\# \arg$  – closed subset of  $X$  and  $x \in F^c$  then there exists a  $\# \arg - nbhd$   $N$  of  $x$  such that  $N \cap F = \phi$ .

**Proof:** Let  $F$  be  $\# \arg$  – closed subset of  $X$  and  $x \in F^c$  then  $F^c$  is  $\# \arg$  – open set of  $X$ . So by theorem 3.5  $F^c$  contains a  $\# \arg - nbhd$  of each of its points. Hence there exists a  $\# \arg - nbhd$   $N$  of  $x$  such that  $N \subseteq F^c$ . Hence  $N \cap F = \phi$ .

**Definition 3.8:** Let  $x$  be a point in a space  $X$ . The set of all  $\# \arg - nbhd$  of  $x$  is called  $\# \arg - nbhd$  system at  $X$  and is denoted by  $\# \arg - N(x)$ .

**Theorem 3.9:** Let  $X$  be a topological space and for each  $x \in X$ .  $\# \arg - N(x)$  be the collection of all  $\# \arg - nbhd$ s of  $x$ . Then we have the following results.

- (i)  $\forall x \in X, \# \arg - N(x) \neq \phi$
- (ii)  $N \in \# \arg - N(x) \Rightarrow x \in N$
- (iii)  $N \in \# \arg - N(x), N \subset M \Rightarrow M \in \# \arg - N(x)$
- (iv)  $N \in \# \arg - N(x), M \in \# \arg - N(x) \Rightarrow N \cap M \in \# \arg - N(x)$
- (v)  $N \in \# \arg - N(x) \Rightarrow$  there exists  $M \in \# \arg - N(x)$  such that  $M \subset N$  and  $M \in \# \arg - N(y)$  For every  $y \in M$ .

**Proof:**

- (i) Since  $X$  is a  $\# \arg$  – open set, it is  $\# \arg - nbhd$  of every  $x \in X$ . Hence there exists atleast one  $\# \arg - nbhd$  (namely  $X$ ) for each  $x \in X$ . Hence  $\# \arg - N(x) \neq \phi$  for every  $x \in X$ .
- (ii) If  $N \in \# \arg - N(x)$ , then  $N$  is a  $\# \arg - nbhd$  of  $x$ . So by the definition of  $\# \arg - nbhd$ ,  $x \in N$ .
- (iii) Let  $N \in \# \arg - N(x)$  and  $N \subset M$ . Then there is a  $\# \arg$ -open set  $U$  such that  $x \in U \subset N$  since  $N \subset M$ ,  $x \in U \subset N$  and so  $M$  is  $\# \arg - nbhd$  of  $x$ . Hence  $M \in \# \arg - N(x)$ .
- (iv) Let  $N \in \# \arg - N(x)$  and  $M \in \# \arg - N(x)$ . Then by definition of  $\# \arg - nbhd$  there exists  $\# \arg$ -open sets  $U_1$  and  $U_2$  such that  $x \in U_1 \subset N$  and  $x \in U_2 \subset M$ . Hence  $x \in U_1 \cap U_2 \subset N \cap M$ . Since  $U_1 \cap U_2$  is  $\# \arg$  – open set,  $N \cap M$  is a  $\# \arg - nbhd$  of  $x$ . Hence  $N \cap M \in \# \arg - N(x)$ .

- (v) If  $N \in \# \arg - N(x)$  then there exists a  $\# \arg - open$  set  $M$ . Such that  $x \in M \subset N$ . Since  $M$  is a  $\# \arg - open$  set, it is  $\# \arg - nbhd$  of each of its points. Therefore  $M \in \# \arg - N(y)$  for every  $y \in M$ .

#### 4. $\# \alpha$ -REGULAR GENERALIZED CLOSURE AND THEIR PROPERTIES.

**Definition 4.1:** For a subset  $A$  of  $X$   $\# \arg - cl(A) = \cap \{F : A \subseteq F, F \text{ is } \# \arg - closed \text{ in } X\}$

**Definition 4.2:** Let  $(X, \tau)$  be a topological space and  $\tau_{\# \arg} = \{V \subseteq X, \# \arg - cl(X \setminus V) = X \setminus V\}$ .

**Remark 4.3:** If  $A \subseteq X$  is  $\# \arg - closed$  then  $\# \arg - cl(A) = A$ . But the converse is not true, it is seen from the following example.

**Example 4.4:** Let  $X = \{a, b, c, d\}$  be with topology.  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  Let  $A = \{b, c\}$  then  $\# \arg - cl(A) = A$  but  $A$  is not  $\# \arg - closed$ .

**Remark 4.5:**  $\# \arg - cl(\phi) = \phi$  and  $\# \arg - cl(X) = X$ .

- (i)  $A \subseteq \# \arg - cl(A)$  for every subset  $A$  of  $X$ .
- (ii) Let  $A$  and  $B$  be subsets of  $X$  if  $A \subseteq B$  then  $\# \arg - cl(A) \subseteq \# \arg - cl(B)$ .

**Theorem 4.6:** Suppose  $\tau_{\# \arg}$  is a topology. If  $A$  is  $\# \arg - closed$  in  $(X, \tau)$  then  $A$  is  $\alpha - closed$  in  $(X, \tau_{\# \arg})$ .

**Proof:** Suppose  $A$  is  $\# \arg - closed$  in  $(X, \tau)$ ,  $\# \arg - cl(A) = A$ . This implies  $X \setminus A \in \tau_{\# \arg}$ . That is  $X \setminus A$  is  $\alpha - open$  in  $(X, \tau_{\# \arg})$ . Hence  $A$  is  $\alpha - closed$  in  $(X, \tau_{\# \arg})$ .

**Theorem 4.7:** For any  $x \in X$ ,  $x \in \# \arg - cl(A)$  if and only if  $\forall V \cap A \neq \phi$  for every  $\# \arg - open$  set  $V$  containing  $x$ .

**Proof:**

**Necessity:** Let  $x \in \# \arg - cl(A)$ . Suppose there exists a  $\# \arg - open$  set  $V$  containing  $x$  such that  $V \cap A = \phi$ . Since  $A \subseteq X \setminus V$ ,  $\# \arg - cl(A) \subseteq \# \arg - cl(X \setminus V)$  this implies that  $x \notin \# \arg - cl(A)$  a contradiction.

**Sufficiency:** Suppose  $x \notin \# \arg - cl(A)$ , then there exists a  $\# \arg - closed$  subset  $F$  containing  $A$  such that  $x \notin F$ . Then  $x \in X \setminus F$  and  $X \setminus F$  is  $\# \arg - open$ . Also  $(X \setminus F) \cap A = \phi$  a contradiction.

**Theorem 4.8:** Let  $A$  and  $B$  be subsets of  $X$  then  $\# \arg - cl(A \cap B) \subseteq \# \arg - cl(A) \cap \# \arg - cl(B)$ .

**Proof:** Since  $A \cap B \subseteq A$  and  $B$ , by Remark 4.5  $\# \arg - cl(A \cap B) \subseteq \# \arg - cl(A)$  and  $\# \arg - cl(A \cap B) \subseteq \# \arg - cl(B)$  thus  $\# \arg - cl(A \cap B) \subseteq \# \arg - cl(A) \cap \# \arg - cl(B)$ .

**Remark 4.9:**  $\# \arg - cl(A) \cap \# \arg - cl(B) \not\subseteq \# \arg - cl(A \cap B)$  it is seen from the following example.

**Example 4.10:** Let  $X = \{a, b, c, d\}$  be with topology  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{a\}$  and  $B = \{d\}$   $A \cap B = \emptyset$   $\# \arg - cl(A) = \{a, d\}$  and  $\# \arg - cl(B) = \{d\}$  Then  $\# \arg - cl(A) \cap \# \arg - cl(B) = \{d\} \not\subseteq \# \arg - cl(A \cap B)$

**Theorem 4.11:** If  $A$  and  $B$  are  $\# \arg - closed$  sets then  $\# \arg - cl(A \cup B) = \# \arg - cl(A) \cup \# \arg - cl(B)$ .

**Proof:** Let  $A$  and  $B$  be  $\# \arg - closed$  in  $X$ . Then  $A \cup B$  is also  $\# \arg - closed$ . Then  $\# \arg - cl(A \cup B) = A \cup B = \# \arg - cl(A) \cup \# \arg - cl(B)$ .

**Definition 4.12:** For any  $A \subseteq X$ ,  $\# \arg - int(A)$  is defined as the union of all  $\# \arg - open$  set contained in  $A$ .

**Theorem 4.13:**  $(X \setminus \# \arg - int(A)) = \# \arg - cl(X \setminus A)$

**Proof:** Let  $x \in X \setminus \# \arg - int(A)$  then  $x \notin \# \arg - int(A)$  That is every  $\# \arg - open$  set  $B$  containing  $x$  is such that  $B \not\subseteq A$ . This implies that every  $\# \arg - open$  set  $B$  containing  $x$  intersects  $X \setminus A$ . By theorem 4.6  $x \in \# \arg - cl(X \setminus A)$ . Hence  $(X \setminus \# \arg - int(A)) \subseteq \# \arg - cl(X \setminus A)$ . Conversely let  $x \in \# \arg - cl(X \setminus A)$ . Then every  $\# \arg - open$  set  $U$  containing  $x$  intersects  $X \setminus A$ . That is every  $\# \arg - open$  set  $U$  containing  $x$  is such that  $U \not\subseteq A$ , implies  $x \notin \# \arg - int(A)$ . Hence  $\# \arg - cl(X \setminus A) \subseteq (X \setminus \# \arg - int(A))$  Thus  $(X \setminus \# \arg - int(A)) = \# \arg - cl(X \setminus A)$ .

## CONCLUSION

In this paper we study  $\#$ arg- open sets and their basic properties relationship with some generalized sets in topological space also we have discuss  $\#$ arg-neighbourhood and  $\#$ arg-closure and their properties..

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