International Journal of Mathematical Archive-9(8), 2018, 1-6
Available online through www.ijma.info ISSN 2229 – 5046

On πg*β - Compact Spaces and πg*β - Connected Spaces in Topological spaces

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(Received On: 02-06-18; Revised & Accepted On: 01-08-18)

ABSTRACT

*This paper deals with πg*β -compact spaces and their properties by using nets, filter base and πg*β -complete accumulation points. The notion of πg*β-connectedness in topological spaces is also introduced and their properties are studied.*

***Keywords**: πg*β-open set, πg*β-closed sets, πg*β-compact spaces, πg*β-complete accumulation point and πg*β-connectedness.*

1. INTRODUCTION

The notions of compactness and connectedness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness and connectedness in Topological Spaces. The productivity and fruitfulness of these notions of compactness and connectedness motivated the researchers to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced and investigated. A. Devika and R.Vani in [2] introduce the concept of π generalized star beta closed (briefly, πg*β -closed) sets and π generalized star beta open (briefly, πg*β -open) sets in a topological spaces. They also defined in [3] π generalized star beta continuous (briefly, πg*β-continuous) functions and π generalized star beta irresolute (briefly, πg*β-irresolute) functions in topological spaces and studied some of their properties. The purpose of this paper is to introduce the concept of πg*β-compactness and πg*β-connectedness in topological spaces and is to give some characterizations of πg*β-compact spaces in terms of nets and filter bases. Further, the notion of πg*β-complete accumulation points is also introduce and is used to characterize πg*β – compactness and studied some of their properties.

2. PRELIMINARY NOTES

Throughout this paper (X, τ), (Y, σ) are topological spaces with no separation axioms assumed unless otherwise stated. Let $A\subseteq X$. The closure of A and the interior of A will be denoted by Cl(A) and Int(A) respectively.

Definition 2.1: [4] A set A is said to be regular open (briefly, r-open) (resp.regular closed(briefly, r-closed)) if A=int(cl(A)) (resp. A=cl(int(A))). The family of r-open (resp. r-closed) sets f a space X is denoted by RO(X) (resp. RC(X)).

Definition 2.2: [1] For any subset A of (X, τ), $RCl(A)=\cap\{G: G\supseteq A$, G is a regular closed subset of X}.

Definition 2.3: [2] A subset A of a topological space (X, τ) is called a generalized regular star closed set [briefly πg*β -closed] if $\beta cl(A)\subseteq U$ whenever $A\subseteq U$ and U is πg-open subset of X.

Definition 2.4: [2] For a subset A of a space X, πg*β -cl(A) = $\bigcap\{F: A\subseteq F$, F is πg*β closed in X} is called the πg*β -closure of A.

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Remark 2.5: [2] Every r-closed (π-closed) set in X is πg*β -closed in X.

Definition 2.6: [3] A function $f:(X,\tau)\to(Y,\sigma)$ is said to be πg*β-continuous if $f^{-1}(V)$ is πg*β-closed set in X for every closed set V in Y.

Definition 2.7: [3] A function $f:(X,\tau)\to(Y,\sigma)$ is said to be πg*β-irresolute if $f^{-1}(V)$ is πg*β-closed set in X for every πg*β-closed set V in Y.

3. πg*β -Compact Spaces

Definition 3.1:

(1) A collection $f:(X,\tau)\to(Y,\sigma)$ of πg*β -open sets in a topological space X is called πg*β -open cover of a subset B of X if $B\subset\cup\{A_\alpha:\alpha\in\nabla\}$ holds.

(2) A topological space X is called π generalized star beta compact (briefly, πg*β -compact) if every πg*β -open cover of X has a finite subcover.

(3) A subset B of X is called πg*β -compact relative to X if for every collection $\{A_\alpha:\alpha\in\nabla\}$ of πg*β -open subsets of X such that $B\subset\cup\{A_\alpha:\alpha\in\nabla\}$, there exist a finite subset $\nabla_\circ$ of ∇ such that $B\subset\cup\{A_\alpha:\alpha\in\nabla_\circ\}$.

(4) A subset B of X is said to be πg*β -compact if B is πg*β -compact as a subspace of X .

Theorem 3.2: Every πg*β -closed subset of πg*β -compact space X is πg*β -compact relative to X .

Proof: Let A be πg*β -closed subset of X , then A^C is πg*β -open. Let $O=\{G_\alpha:\alpha\in\nabla\}$ be a cover of A by πg*β -open subsets of X . Then $W=O\cup A^C$ is an πg*β -open cover X , i.e., $X=(\cup\{G_\alpha:\alpha\in\nabla\})\cup A^C$. By hypothesis, X is πg*β -compact. Hence W has a finite subcover of X say $(G_1\cup G_2\cup.....\cup G_n)\cup A^C$. But A and A^C are disjoint, hence $A\subset(G_1\cup G_2\cup.....\cup G_n)$. So O contains a finite subcover for A, therefore A is πg*β -compact relative to X .

Theorem 3.3: Let $f:X\to Y$ be a map:

(1) If X is πg*β -compact and f is πg*β -compact bijective, then Y is compact.

(2) If f is πg*β -irresolute, and B is πg*β -compact relative to X . Then $f(B)$ is πg*β -compact relative to Y .

(3) If X is compact and f is continuous surjective, then Y is πg*β -compact.

Proof:

(1) Let $f:X\to Y$ be an πg*β -continuous bijective map, and X be an πg*β -compact space. Let $\{A_\alpha:\alpha\in\nabla\}$ be open cover for Y , then $\{f^{-1}(A_\alpha):\alpha\in\nabla\}$ is an πg*β -open cover of X . Since X is πg*β -compact, it has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_1),....,\ f^{-1}(A_n)\}$ but f is surjective, so $\{A_1,A_2,\ldots,A_n\}$ is a finite subcover of Y . Therefore, Y is compact.

(2) Let $B\subset X$ be πg*β -compact relative to X , $\{A_\alpha:\alpha\in\nabla\}$ be any collection of πg*β -open subsets of Y such that $f(B)\subset\cup\{A_\alpha:\alpha\in\nabla\}$. Then $B\subset\cup\{f^{-1}(A_\alpha):\alpha\in\nabla\}$. By hypothesis, there exist a finite subset $\nabla_\circ$ of ∇ such that $B\subset\cup\{f^{-1}(A_\alpha):\alpha\in\nabla_\circ\}$. Therefore, we have $f(B)\subset\cup\{A_\alpha:\alpha\in\nabla_\circ\}$ which shows that $f(B)$ is πg*β -compact relative to Y .

(3) Let $A=\{A_\alpha:\alpha\in\nabla\}$ be an πg*β -open cover of Y . Since f is continuous, therefore $f^{-1}(A_\alpha)$ is open in X . The collection $W=\{f^{-1}(A_\alpha):\alpha\in\nabla\}$ is an open cover of X . Since X is compact, W has a finite subset say $\{f^{-1}(A_1), f^{-1}(A_2),...., f^{-1}(A_n)\}$ which cover X . Since $X=\{f(A_1)\cup f(A_2)\cup....\cup f(A_n)$ and f is surjective, therefore

$$Y = f(X) =$$
$$f(f^{-1}(A_1) \cup f^{-1}(A_2) \cup .... \cup f^{-1}(A_n)) = f(f^{-1}(A_1)) \cup f(f^{-1}(A_2)) \cup ... \cup f(f^{-1}(A_n)) \subset A_1 \cup A_2 \cup ... \cup A_n.$$

Thus $\{A_1, A_2, ..., A_n\}$ is a finite πg*β -open subcover of Y, and Y is πg*β -compact.

Definition 3.6: Let ⋀ be a directed set. A net ξ = { x_α: α∈⋀ } πg*β -accumulates at a point x∈X if the net is frequently in every U∈πg*β O(X, x), i.e., for each U∈πg*β O(X, x) and for each $\propto_0$∈⋀, there is some $\propto \geq \propto_0$ such that x_α∈U. The set ξ πg*β -converges to a point x of X if it is eventually in every U∈πg*β O(X, x).

Definition 3.7: We say that a filter base ⊖={$F_\propto$:∝∈Γ} πg*β -accumulates at a point x∈X if x∈D$\bigcap_{\propto\in\Gamma}$ πg*β Cl($F_\propto$). A filter base ⊖={$F_\propto$:∝∈Γ} πg*β -converges to a point x in X if for each U∈πg*β O(X, x), there exists an $F_\propto$ in ⊖ such that $F_\propto$⊂U.

Definition 3.8: A point x in a space X is said to be πg*β -complete accumulation point of a subset S of X if Card(S∩U)=Card(S) for each U∈πg*β O(X, x), where Card(S) denotes the cardinality of S.

Definition 3.9: In a topological space X, a point x is said to be a πg*β -adherent point of a filter base ⊖ on X if it lies in the πg*β -closure of all sets of ⊖.

Theorem 3.10: A space X is πg*β -compact if and only if each infinite subset of X has a πg*β -complete accumulation point.

Proof: Let the space X be πg*β -compact and let S be an infinite subset of X. Let K be the set of points x in X which are not πg*β -complete accumulation points of S. Now it is obvious that for each x in K, we are able to find U(x) ∈ πg*β O(X, x) such that Card(S∩U(x)) ≠ Card(S). If K is the whole space X, then ⊖={U(x): x∈X} is a πg*β -cover of X. By the hypothesis X is πg*β -compact, so there exists a finite sub-cover ψ={U(x_i)}, where i=1,2,…,n such that S⊂{U(x_i)∩S : i=1,2,…,n}. Then Card(S)=max{Card(U(x_i)∩S)}, where i=1,2,…,n which does not agree with what we assumed. This implies that S has πg*β -complete accumulation point.

Conversely, suppose that X is not πg*β -compact and that every infinite subset S⊂X has a πg*β -complete accumulation point in X. It follows that there exists a πg*β -cover E with no finite sub-cover. Set δ=min{Card(Φ) : Φ⊂E}, where Φ is a πg*β -cover of X. Fix ψ⊂E for which Card(ψ)=δ and ⋃{U:U∈ψ}=X. Let N denote the set of natural numbers. Then by hypothesis δ≥Card(N). By well ordering of ψ by some minimal well ordering ~ suppose that U is any member of ψ. By minimal well ordering ~ we have Card({V:V∈ψ, V~U})<Card{V:V∈ψ}). Since ψ can not have any sub-cover with cardinality less than δ, then for each U∈ψ we have X ≠ ⋃{V:V∈ψ, V~U}. For each U∈ψ choose a pont x(U)∈X-U{V∪{x(V)}: V∈ψ, V~U}. We are always able to do this if not one can choose a cover of smaller cardinality from ψ. If H={x(U): U∈ψ}, then to finish the proof we will show that H has no πg*β -accumulation points in X. Suppose that z is a point of X. Since ψ is a πg*β -cover of X, then z is a point of some set W in ψ. By the fact that U~W, we have x(U) ⊂ W. But Card(T)<δ. Therefore, Card(H∩W)<δ. But Card(H)=δ>Card(N), since for two distinct points U and W in ψ, we have x(U)≠x(W), This means that H has no πg*β -complete accumulation point in X which contradicts our assumptions. Therefore X is πg*β -compact.

Theorem 3.11: For a space X the following are equivalent

1. X is πg*β -compact.
2. Every net in X with a well ordered directed set as its domain πg*β -accumulates to some point of X.

Proof:

(1)=>(2): Suppose that (X, τ) is πg*β -compact and $\xi = \{x_a : \alpha \in \wedge\}$ a net with a well ordered directed set A as domain. Assume that ξ has no πg*β adherent point in X. Then for each point x is X, there exist $V(x) \in \pi g^* \beta O(X, x)$ and an $\alpha(x) \in \wedge$ such that $V(x) \cap \{x_\alpha : \alpha \geq \alpha(x)\} = \phi$. this implies that $\{x_\alpha : \alpha \geq \alpha(x)\}$ is a subset of X-V(x). Then the collection C = {V(x):x∈X} is a πg*β -cover of X. By hypothesis of theorem, X is πg*β -compact and so C has a finite subfamily {V(x_i)}, where i = 1,2,3,….,n such that X = ∪{V(x_i)} . Suppose that the corresponding elements of ∧ are {α(x_i)}, where i=1,2,3,…,n is finite, the largest element of {α(x_i)} exists. Suppose it is {α(x_i)}, then for γ ≥{α(x_i)}. We have

$\{x_\delta : \delta \geq \gamma\} \subset D \cap_{i=1}^{n} (X - V(x_i)) = X - \cup_{i=1}^{n} V(x_i) = \phi$ which is impossible. This shows that ξ has at least one πg*β -adherent point in X.

(2)=>(1): Now by the last Theorem 3.10, it is enough to prove that each infinite subset has a $\pi g^*\beta$ -complete accumulation point. Suppose that $S\subset X$ is an infinite subset of X. According to Zorn's lemma, the infinite set S can be

well ordered. This means that we can assume S to be a net with a domain, which is a well ordered index set. It follows that S has a $\pi g^*\beta$ -adherent point z. Therefore z is a $\pi g^*\beta$ -complete accumulation point of S. This shows that X is $\pi g^*\beta$ -compact.

Theorem 3.12: A space X is $\pi g^*\beta$ -compact if and only if each family of $\pi g^*\beta$ -closed subsets of X with the finite intersection properly has a non-empty intersection.

Proof: Given a collection $\mathcal{A}$ of subsets of X, let $\mathcal{C}$ ={X-A: A∈$\mathcal{A}$} be the collection of their complements. Then the following statements hold.

(a) $\mathcal{A}$ is a collection of $\pi g^*\beta$ -open sets if and only if $\mathcal{C}$ is a collection of $\pi g^*\beta$ -closed sets.

(b) The collection $\mathcal{A}$ covers X if and only if the intersection $\bigcap_{c\in\mathcal{C}} \mathcal{C}$ of all the elements of C is non-empty.

(c) The finite sub collection $\{A_n \ldots . A_n\}$ of $\mathcal{A}$ covers X if and only if the intersection of the corresponding elements $\mathcal{C}_i = X - A_i$ of $\mathcal{C}$ is empty.

The statement (a) is trivial, while the (b) and (c) follow from DeMorgan's law. $X - (D\bigcup_{\propto\in J} A\alpha) = D\bigcap_{\propto\in J} (X - A\alpha)$ The proof of the theorem now proceeds in two steps, taking the contrapositive of the theorem and then the complement.

The statement X is $\pi g^*\beta$ -compact is equivalent to: Given any collection $\mathcal{A}$ of $\pi g^*\beta$ -open subsets of X, if $\mathcal{A}$ covers X, then some finite sub collection of $\mathcal{A}$ covers X. This statement is equivalent to its contrapositive, which is the following.

Given any collection $\mathcal{A}$ of $\pi g^*\beta$ -open sets, if no finite sub-collection of $\mathcal{A}$ covers X, then $\mathcal{A}$ does not cover X. Letting $\mathcal{C}$ be as earlier, the collection {X-A: A∈$\mathcal{A}$} and applying (a) to (c), we see that this statement is in turn equivalent to the follwing:

Given any collection $\mathcal{C}$ of $\pi g^*\beta$ -closed sets, if every finite intersection of elements of $\mathcal{C}$ is non-empty, then the intersection of all the elements of $\mathcal{C}$ is non-empty. This is just the condition of our theorem.

Theorem 3.13: A space X is $\pi g^*\beta$ -compact if and only if each filter base in X has at least one $\pi g^*\beta$ -adherent point.

Proof: Suppose that X is $\pi g^*\beta$ -compact and $\theta=\{F_\alpha: \alpha\epsilon\Gamma\}$ a filter base in it. Since all finite intersecions of F_α's are non-empty, it follows that all finite intersections of $\pi g^*\beta\text{-Cl}(F_\alpha)$'s are also non-empty. Now it follows from Theorem 3.12 that $D\bigcap_{\propto\epsilon\Gamma} \pi g^*\beta\text{-Cl}(F_\alpha)$ is non-empty. This means that θ has at least one $\pi g^*\beta$ -adherent point.

Conversely, suppose θ is any family of $\pi g^*\beta$ -closed sets. Let each finite intersection be non-empty. The sets (F_α) with their finite intersection establish a filter base θ. Therefore $\pi g^*\beta$ -accumulates to some point z in X. It follows that $z\in D\bigcap_{\propto\epsilon\Gamma} (F_\alpha)$. Now by Theorem 3.12, we have that X is $\pi g^*\beta$ -compact.

Theorem 3.14: A space X is $\pi g^*\beta$ -compact if and only if each filter base on X with at most one $\pi g^*\beta$ -adherent point is $\pi g^*\beta$ -convergent.

Proof: Suppose that X is $\pi g^*\beta$ -compact, x is a point of X and a filter base on X. The $\pi g^*\beta$ -adherence of θ is a subset of {x}. Then the $\pi g^*\beta$ -adherence of θ is equal to {x} by Theorem 3.13. Assume that there exists $V\in\pi g^*\beta\ O(X, x)$ such that for all F∈θ, F∩(X-V) is non-empty. Then ψ={F-V: F∈θ} is a filter base on X. It follows that the $\pi g^*\beta$ -adherence of θ is non-empty. However, $D\bigcap_{F\epsilon\theta} \pi g^*\beta\ Cl(F - V) \subset (D\bigcap_{F\epsilon\theta} \pi g^*\beta\ Cl(F) \cap (X - V)) = \{x\} \cap \{X - V\} = \phi$. But this is a contradiction. Hence for each $V \in \pi g^*\beta\ O(X,x)$ there exists an F∈θ with F⊂V. This shows that $\pi g^*\beta$ -converges to x.

To prove the converse, it suffices to show that each filter base in X has at least one $\pi g^*\beta$ -accumuation point. Assume that θ is a filter base on X with no $\pi g^*\beta$ -adherent point. By hypothesis, $\pi g^*\beta$ -converges to some point z in X. Suppose F_α is an arbitrary element of θ. Then for each $V \in \pi g^*\beta\ O(X,z)$, there exits $F_\beta\in\theta$ such that $F_\beta\in V$. Since θ is a filter base, there exits a γ such that $F_\gamma \subset F_\alpha\cap F_\beta \subset F_\alpha\cap V$, where F_γ non-empty. This means that $F_\alpha\cap V$ is non-empty for every $V \in \pi g^*\beta\ O(X,z)$ and correspondingly for each α, z is a point of $\pi g^*\beta\text{-Cl}(F_\alpha)$ it follows that $z \in D\bigcap_\alpha \pi g^*\beta\text{-Cl}(F_\alpha)$. Therefore z is a $\pi g^*\beta$ -adherent point of θ which is contradiction. This shows that X is $\pi g^*\beta$ -compact.

4. πg*β -Connected Spaces

Definition 4.1: A space X is said to be π generalized star beta connected (briefly, $\pi g^*\beta$ -connected) if it cannot be written as a disjoint union of two non-empty $\pi g^*\beta$ -open sets, otherwise it said to be $\pi g^*\beta$ -disconnected. A subset of X is said to be $\pi g^*\beta$ -connected if it is $\pi g^*\beta$ -connected as a subspace of X.

Example 4.2: Let X={a, b} and let τ={X, ϕ, {a}}. Then it is $\pi g^*\beta$ -connected.

Remark 4.3: Every $\pi g^*\beta$ -connected space is connected but not converse need not be true in general, which follows from the following example.

Example 4.4: Let X={a, b} and τ={X, ϕ}. Clearly (X, τ) is connected. Then $\pi g^*\beta$ -open subsets of X are {X, ϕ,{a},{b}}. Therefore (X, τ) is not a $\pi g^*\beta$ -connected space, because X={a}∪{b} where {a} and {b} are non-empty $\pi g^*\beta$ -open sets.

Theorem 4.5: For a topological space X the following are equivalent.
(i) X is $\pi g^*\beta$ -connected.
(ii) X and ϕ are the only subsets of X which are both $\pi g^*\beta$ -open and $\pi g^*\beta$ -closed.
(iii) Each $\pi g^*\beta$ -continuous map of X into a discrete space Y with at least two points is a constant map.

Proof:
(i)⇒(ii): Let O be any $\pi g^*\beta$ -open and $\pi g^*\beta$ -closed subset of X. Then O^C is both $\pi g^*\beta$ -open and $\pi g^*\beta$ -closed. Since X is disjoint union of the $\pi g^*\beta$ -open sets O and O^C implies from the hypothesis of (i) that either O = ϕ or O = X.

(ii)⇒(i): Suppose that X=A∪B where A and B are disjoint non-empty $\pi g^*\beta$ -open subsets of X. Then A is both $\pi g^*\beta$ -open and $\pi g^*\beta$ -closed.

By assumption A=ϕ or X. Therefore X is $\pi g^*\beta$ -connected.

(ii)⇒(iii): Let f : X→ Y be a $\pi g^*\beta$ -continuous map. Then X is covered by $\pi g^*\beta$ -open and $\pi g^*\beta$ -closed covering $\{f^{-1}(Y) : y\in Y\}$,

By assumption $f^{-1}(y)=\phi$ or X for each y∈Y. If $f^{-1}(y)=\phi$ for all y∈Y, then f fails to be a map. Then there exits only one point y∈Y such that $f^{-1}(y)\neq\phi$ and hence $f^{-1}(y)=X$. This shows that f is a constant map.

(iii)⇒(ii): Let O be both $\pi g^*\beta$ -open and $\pi g^*\beta$ -closed in X. Suppose O≠ ϕ. Let f : X→ Y be a $\pi g^*\beta$ -continuous map defined by f(O)=y and $f(O^C)=\{w\}$ for some distinct points y and w in Y.

By assumption f is constant. Therefore we have O=X.

Theorem 4.6: If f : X → Y is a $\pi g^*\beta$ -continuous and X is $\pi g^*\beta$ -connected, then Y is connected.

Proof: Suppose that Y is not connected. Let Y=A∪B where A and B are disjoint non-empty open set in Y. Since f is $\pi g^*\beta$ -continuous and onto, $X=f^{-1}(A)\cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty $\pi g^*\beta$ -open sets in X. This contradicts the fact that X is $\pi g^*\beta$ -connected. Hence Y is connected.

Theorem 4.7: If f : X → Y is a $\pi g^*\beta$ -irresolute surjection and X is $\pi g^*\beta$ -connected, then Y is $\pi g^*\beta$ -connected.

Proof: Suppose that Y is not $\pi g^*\beta$ -connected.

Let Y=A∪B where A and B are disjoint non-empty $\pi g^*\beta$ -open sets in Y. Since f is $\pi g^*\beta$ -irresolute and onto, $X=f^{-1}(A)\cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty $\pi g^*\beta$ -open sets in X. This contradicts the fact that X is $\pi g^*\beta$ -connected. Hence Y is connected.

Theorem 4.8: In a topological space (X, τ) with at least two points, if πO(X, τ) = πCl(X, τ) then X is not $\pi g^*\beta$ -connected.

Proof: By hypothesis we have πO(X, τ) = πCl(X, τ) and by Remark 2.5 we have every π closed set is $\pi g^*\beta$ -closed, there exits some non-empty proper subset of X which is both $\pi g^*\beta$ -open and $\pi g^*\beta$ -closed in X. So by last Theorem 4.5 we have X is not $\pi g^*\beta$ -connected.

Theorem 4.9: If the $\pi g^*\beta$ -open sets C and D form a separation of X and if Y is $\pi g^*\beta$ -connected subspace of X, then Y lies entirely within C or D.

Proof: Since C and D are both $\pi g^*\beta$ -open in X the sets C∩Y and D∩Y are $\pi g^*\beta$ -open in Y these two sets are disjoint and their union is Y. If they were both non-empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or in D.

Theorem 4.10: Let A be a πg*β -connected subspace of X. If A⊂B⊂πg*β-Cl(A) then B is also πg*β –connected spaces.

Proof: Let A be πg*β -connected and let A⊂B⊂πg*β-Cl(A). Suppose that B=C∪D is a separation of B by πg*β -open sets. Then by Theorem 4.9 above A must lie entirely in C or in D. Suppose that A⊂C, then πg*β-Cl(A)⊆πg*β-Cl(C).

Since πg*β-Cl(C) and D are disjoint. B cannot intersect D. This contradicts the fact that D is non-empty subset of B. So D = ϕ which implies B is πg*β -connected.

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Source of support: Nil, Conflict of interest: None Declared.